ACSC/STAT 3703, Actuarial Models I (Further Probability with Applications to Actuarial Science)
Winter 2015
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Homework Sheet 7
Due: Wednesday 8th April: 12:30 PM

Basic Questions

1. Individual policy holders are each assigned a risk factor Θ, which follows a Pareto distribution with $\alpha = 2$ and $\theta = 100$. For a policy holder with risk factor $\Theta = \theta$, the size of a claim follows a log-logistic distribution with $\gamma = 2$ and this value of θ. Calculate the probability that a random claim exceeds $30,000. [Hint: Calculate

$$\frac{3}{\theta^2 + 30000} - \frac{3}{(\theta + 100)^2} + \frac{200}{(\theta + 100)^3}$$

You may also need $\int_0^\infty \frac{1}{1+u^2}\,du = \frac{\pi}{2}$.]

2. An insurance company divides claims into three intervals: claims less than $2,000; claims between $2,000 and $20,000; and claims larger than $20,000. It uses the following distributions to model claim size on these three intervals:

Interval	Probability	Distribution of claims in this interval	mean	variance
$[0, 2000]$	0.6	Uniform	1000	$\frac{1000000}{3}$
$[2000, 20000]$	0.3	Gamma, $\alpha = 3$, $\theta = 1200$	4314.076	3520532
$[20000, \infty]$	0.1	Pareto, $\alpha = 4$, $\theta = 1500$	$\frac{81500}{3}$	102722222

(These distributions are all truncated to their intervals). Calculate the expected value and variance of a random claim.

3. The number of claims in a given year follows a compound Poisson-Poisson distribution with parameters 4 and 2. Calculate the probability that there are more than 2 claims in a given year.

Standard Questions

4. Individual policy holders are each assigned a risk factor Θ, which follows a Pareto distribution with $\alpha = 2$ and $\theta = 1600$. For a policy holder with risk factor $\Theta = \theta$, the size of a claim follows a Pareto distribution with $\alpha = 2$ and this value of θ. The insurance company buys reinsurance on each policy. This reinsurance pays the portion of any claim above

\$50,000. The premium of this reinsurance is set as 1.2 times the expected reinsurance payment. Calculate this premium. [Hint: $\int_0^\infty \frac{x^2}{(x+a)^3(x+b)}\,dx = -\frac{b^2}{(a-b)^3}\log\left(\frac{b}{a}\right) - \frac{b}{(a-b)^2} - \frac{1}{2(a-b)}$.]

5. The number of claims from 1200 policies in one year follows a compound Poisson-truncated ETNB distribution with the Poisson distribution having $\lambda = 0.4$ and the truncated ETNB distribution having $\beta = 1.5$ and $r = -0.4$.

   (a) The following year The number of policies increases to 2100. Calculate the probability that the number of claims the following year is exactly 2.

   (b) The company wants to ensure that the number of claims is at most 2 with probability at least 0.9. How many policies can it issue while maintaining this condition?

   (i) 969

   (ii) 1356

   (iii) 1760

   (iv) 1987