ACSC/STAT 3703, Actuarial Models I

WINTER 2023
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Homework Sheet 5

Due: Wednesday 15th March: 11:30

Note: This homework assignment is only valid for WINTER 2023. If you find this homework in a different term, please contact me to find the correct homework sheet.

Basic Questions

1. A distribution of a random loss X has density function

$$f_X(x) = \begin{cases} \frac{Ce^{-\frac{x}{5}}}{(x+1)(x+2)} & \text{if } x > 0 \\ 0 & \text{if} x \leqslant 0 \end{cases}$$

   for some constant C. After two years, there has been 15% inflation, so the loss distribution is now the distribution of $1.15X$. What is the density function for this distribution?

2. Calculate the distribution of X^6 when X follows a Pareto distribution with $\alpha = 3$ and $\theta = 13$.

3. Let T be the time until a claim is processed. The moment generating function of T is $M_T(t) = \frac{192}{(3-t)(4-t)^3}$. Inflation is at an annual rate of 5%. What is the variance of the random variable 1.05^T?

4. X is a mixture of 3 distributions:

   - With probability 0.3, X follows a gamma distribution with $\alpha = 0.3$ and $\theta = 20$.
   - With probability 0.6, X follows a Pareto distribution with $\alpha = 5$ and $\theta = 60$.
   - With probability 0.1, X follows a Weibull distribution with $\theta = 20$ and $\tau = 4$.

   The moments of these distributions are given in the following table:

	Distribution 1	Distribution 2	Distribution 3
Probability	0.3	0.6	0.1
μ	6	15	18.12805
μ_2	120	75	25.86457
μ_3	4800	2250	-11.47475
μ_4	331200	253125	1838.22388
μ_2'	156	300	354.49077
μ_3'	7176	9000	7352.50021
μ_4'	473616	540000	160000.00000

(a) What is the coefficient of variation of X?

(b) [bonus] What is the kurtosis of X?

5. For a particular claim, the insurance company has observed the following claim sizes:

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1.1  1.9  3.0  7.3  10.9  12.8  14.8  15.0  25.6  39.2
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Using a kernel smoothing model with a Gaussian kernel with variance 4, calculate the probability that the next claim size is between 14 and 24.

Standard Questions

6. An insurance company models the claims of an individual (in dollars) as following a Pareto distribution with $\theta = 1000$ and α varying between individuals. For a random individual, α is assumed to follow a Gamma distribution with shape α and scale θ.

From the insurer's data, 5% of claims exceed \$700 and 1% of claims exceed \$5,500. Which of the following values of α would achieve this, and what is the corresponding value of θ? Justify your answer.

(i) $\alpha = 3.48762$.

(ii) $\alpha = 7.42930$.

(iii) $\alpha = 11.09824$.

(iv) $\alpha = 18.14619$.

7. The time until failure of a product has hazard rate $\lambda(t) = 2(1-a) + \frac{t^2}{16}$ where a is a measure of the quality of the product, and is modelled as following a distribution with density $f_A(a) = 7.5a^2 - 4.5a + 0.75$ for $0 \leqslant a \leqslant 1$. The product has a two-year waranty. What is the probability that it will be replaced under this waranty?

8. An insurance company models claims as following a log-normal distribution with $\mu = 4$ and $\sigma^2 = 3$. They want to transform the claims by raising to a power in order to make the kurtosis of the distribution equal to 6. What power should they use? [You may need to use numerical methods to solve the necessary equations.]