ACSC/STAT 3703, Actuarial Models I

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Homework Sheet 1

Model Solutions

Basic Questions

1. *A customer has utility function $u(x) = \log(x)$. The customer's current wealth is \$18,000. The customer's car has a value of \$11,300. The probability of the car being stolen is 0.01. How much would the customer be willing to pay for insurance against the car being stolen?*

   Without insurance, the customer has wealth \$18,000 if the car is not stolen (probability 0.99) and \$6,700 if the car is stolen (probability 0.01). The customer's expected utility without insurance is therefore $0.99\log(18000)+0.01\log(6700) = 9.78824439456$. This is the same as the utility from wealth $e^{9.78824439456} = 17822.9885494$. The customer would therefore be willing to pay up to $18000 - 17822.9885494 = \$177.011$ for the insurance.

2. *Which of the following risks are insurable? For risks which are not insurable, explain why they are not insurable.*

   (i) The risk of an individual being killed by a meteorite.

   (ii) The risk that an airline will have fewer flights in winter than in summer.

   (iii) The risk that poor weather will adversely affect a farmer's crops.

   (iv) The risk that a couple will divorce.

   (v) The risk that a \$5 child's toy will be broken.

   (vi) The risk that a debt will not be repaid.

   (vii) The risk that you fail this course.

   (viii) The risk that investors will not make enough money on the stock market.

   (i) This is not insurable as there is not sufficient data to estimate the probability.

   (ii) This is not insurable as the loss is not random.

(iii) This is an insurable risk. Indeed, this kind of agricultural insurance is sold.

(iv) This is not insurable as the loss is not well-defined.

(v) This is not insurable as it is not economically feasible.

(vi) This is an insurable risk. However, it would often be dealt with on financial markets using options.

(vii) This is not insurable as the risks are not homogeneous.

(viii) This is not insurable as losses from different policyholders are not independant.

3. *A homeowner's house is insured at $470,000. The insurer requires 80% coverage for full insurance. The home sustains $12,800 damage from wind. The policy has a deductible of $5,000, which decreases linearly to zero when the total cost of the loss is $15,000. The insurance company reimburses $8,840. What value are they using for the house's value?*

   The deductible is $\frac{15000-12800}{10000} \times 5000 = \$1100.$. Thus if the home were fully insured, the insurer would pay $12800 - 1100 = \$11,700$. Thus the home has $\frac{8840}{11700} = 0.755555555556$ coverage. This means that for full coverage a value of $\frac{470000}{0.755555555556} = 622058.823529$ is required. Since this is 80% of the home's value, the home's value must be $\frac{622058.823529}{0.8} = \$777573.53$.

4. *A tennant's insurance policy has a deductible of $1,000, a policy limit of $20,000 and co-insurance such that the policyholder pays 30% of the remaining claim. How much does the insurer pay if the loss is:*

   (i) $800

   (ii) $6,200

   (iii) $21,400

   (i) This is less than the deductible, so the insurer pays $0.

   (ii) $0.7(6200 - 1000) = \$3,640$.

   (iii) $0.7(21400 - 1000) = \$14,280$.

Standard Questions

5. *An insurer charges a loading of 28% on its policies with limit $1,000,000, and a loading of 26% on its policies with limit $500,000. It purchases stop-loss reinsurance of $500,000 over $500,000 for a loading of 45%. What percentage of the insurer's premiums for a policy with limit $1,000,000 are paid to the reinsurer?*

Let x be the expected losses limited to \$500,000 and let y be the expected losses limited to \$1,000,000. The insurer's premium for a policy with limit \$500,000 is $1.26x$, and the insurer's premium for a policy with limit \$1,000,000 is $1.28y$. The reinsurer's premium is $1.45(y-x)$. Since this is the difference between the insurers two premiums, we have

$$\begin{aligned}1.26x + 1.45(y-x) &= 1.28y\\ 0.17y &= 0.19x\\ y &= 1.11764705882x\end{aligned}$$

The insurer collects $1.28y = 1.43058823529x$ in premiums, and pays $1.45(y-x) = 0.170588235289x$ to the reinsurer. The percentage paid to the reinsurer is therefore $\frac{0.170588235289}{1.43058823529} = 11.92\%$.

6. *Policyholders are assumed to have a utility function* $u(x) = -e^{-\frac{x}{15000}} - e^{-\frac{x}{30000}}$. *Policyholder wealth is assumed to follow an exponential distribution with mean \$10,000. An insurance company is considering selling an insurance policy which covers a risk which causes a loss of \$3,000 with probability 0.02. The expenses for this policy are \$2 million plus \$2 per policy sold. If there are 2 million policyholders who might buy the policy, what will the expected profit on this policy be for the insurance company if they set the premium for each policy at \$65?*

   For a policyholder with wealth x, the expected utility without insurance is

$$\begin{aligned}\mathbb{E}(u(x)) &= -\left(0.98e^{-\frac{x}{15000}} + 0.98e^{-\frac{x}{30000}} + 0.02e^{-\frac{x-3000}{15000}} + 0.02e^{-\frac{x-3000}{30000}}\right)\\ &= -e^{-\frac{x}{15000}}\left(0.98 + 0.02e^{\frac{3000}{15000}}\right) - e^{-\frac{x}{30000}}\left(0.98 + 0.02e^{\frac{3000}{30000}}\right)\\ &= -1.00442805516^{-\frac{x}{15000}} - 1.00210341836e^{-\frac{x}{30000}}\\ &= -e^{-\frac{x-66.2742025389}{15000}} - e^{-\frac{x-63.0362781846}{30000}}\end{aligned}$$

   If the policy has premium $p < 66.2742025389$, then a policyholder will buy it if

$$\begin{aligned}-e^{-\frac{x-p}{15000}} - e^{-\frac{x-p}{30000}} &> -e^{-\frac{x-66.2742025389}{15000}} - e^{-\frac{x-63.0362781846}{30000}}\\ e^{-\frac{x}{15000}}\left(e^{\frac{p}{15000}} - e^{\frac{66.2742025389}{15000}}\right) &< e^{-\frac{x}{30000}}\left(e^{\frac{63.0362781846}{30000}} - e^{\frac{p}{30000}}\right)\\ e^{-\frac{x}{30000}} &> \frac{e^{\frac{63.0362781846}{30000}} - e^{\frac{p}{30000}}}{e^{\frac{p}{15000}} - e^{\frac{66.2742025389}{15000}}}\end{aligned}$$

   [The inequality is reversed in the final line because the value we divided by is negative.]

In particular for $p = 65$, the policyholder will buy it if

$$\begin{aligned} e^{-\frac{x}{30000}} &> \frac{e^{\frac{65}{30000}} - e^{\frac{63.0362781846}{30000}}}{e^{\frac{66.2742025389}{15000}} - e^{\frac{65}{15000}}} \\ &= 0.768843437175 \\ x &< -30000\log(0.768843437175) \\ &= 7886.03768865 \end{aligned}$$

Since X follows an exponential distribution with mean 10000, the probability

$$P\left(e^{-\frac{X}{30000}} > a\right) = P\left(X < -30000\log(a)\right) = 1-e^{-\frac{-30000\log(a)}{10000}} = 1-e^{3\log(a)} = 1-a^3$$

In particular

$$P\left(e^{-\frac{X}{30000}} > 0.768843437175\right) = 1 - 0.768843437175^3 = 0.545521089901$$

so the expected number of buyers is $2000000 \times 0.545521089901 = 1091042.1798$.

The expected profit is therefore $1091042.1798 \times (65 - 62) - 2000000 = \$1,273,126.54$.