ACSC/STAT 3720, Life Contingencies I
Winter 2015
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Homework Sheet 3
Model Solutions

Basic Questions

1. *The lifetable in Table 1 applied 5 years ago. The following is an excerpt from the ultimate part of an updated lifetable:*

x	l_x	d_x
39	*10000.00*	*4.22*
40	*9995.78*	*4.68*
41	*9991.10*	*5.06*
42	*9986.04*	*5.52*
43	*9980.52*	*5.99*

Calculate the reduction factor used for each age between 39 and 43.

age	old q_x	new q_x	reduction factor
39	$\frac{4.38}{9962.82} = 0.0004396$	$\frac{4.22}{10000.00} = 0.0004220$	0.9599
40	$\frac{4.75}{9958.44} = 0.0004770$	$\frac{4.68}{9995.78} = 0.0004682$	0.9816
41	$\frac{5.14}{9953.69} = 0.0005164$	$\frac{5.06}{9991.10} = 0.0005065$	0.9808
42	$\frac{5.57}{9948.55} = 0.0005599$	$\frac{5.52}{9986.04} = 0.0005528$	0.9873
43	$\frac{6.04}{9942.98} = 0.0006075$	$\frac{5.99}{9980.52} = 0.0006002$	0.9880

2. *Calculate the expected benefit of a whole life insurance sold to an individual aged 93, if the death benefit is \$1,400,000 at the end of the year of death, the lifetable is Table 1, and the interest rate is $i = 0.05$.*

Using the recursion, we have that $A_{125} = 1$, $A_{124} = \frac{1}{1.05} = 0.9523$, $A_{123} = \frac{0.1111111 \times A_{124} + 0.8888888}{1.05} = 0.9473$. $A_{122} = \frac{\frac{0.09}{0.37} \times A_{123} + \frac{0.28}{0.37}}{1.05} = 0.9402$. Continuing in this way gives:

$$
\begin{aligned}
A_{121} &= 0.934961\\
A_{120} &= 0.929669\\
A_{119} &= 0.924079\\
A_{118} &= 0.918198\\
A_{117} &= 0.911935\\
A_{116} &= 0.905316\\
A_{115} &= 0.898316\\
A_{114} &= 0.890918\\
A_{113} &= 0.883122\\
A_{112} &= 0.874926\\
A_{111} &= 0.86632\\
A_{110} &= 0.857306\\
A_{109} &= 0.84788\\
A_{108} &= 0.838046\\
A_{107} &= 0.827808\\
A_{106} &= 0.81717\\
A_{105} &= 0.806142\\
A_{104} &= 0.794732\\
A_{103} &= 0.782952\\
A_{102} &= 0.770816\\
A_{101} &= 0.758339\\
A_{100} &= 0.745539\\
A_{99} &= 0.732435\\
A_{98} &= 0.719046\\
A_{97} &= 0.705396\\
A_{96} &= 0.691506\\
A_{95} &= 0.677402
\end{aligned}
$$

So $A_{95} = 0.677402$, and the EPV of the death benefit is $0.677 \times 1,400,000 = \$948,362.80$.

3. *Calculate the expected benefit, and the variance of the benefit of a 10-year endowment policy with benefit \$200,000 either at the end of 10 years or at the end of year of death of the policyholder. The lifetable for this policy is Table 1, and the interest rate is $i = 0.03$. The policy is sold to an individual aged 32.*

   For the endowment policy, we have $A_{42:0} = 1$, so $A_{41:1} = \frac{1}{1.03} = 0.9708737864$ and $A_{40:2} = \frac{q_{40}+p_{40}A_{41:1}}{1.03} = \frac{.0004769823+.9995230177\times 0.9708737864}{1.03} = .9426093970$. Continuing we get

$$
\begin{aligned}
A_{39:3} &= 0.915179 \\
A_{38:4} &= 0.888557 \\
A_{37:5} &= 0.862718 \\
A_{36:6} &= 0.837637 \\
A_{35:7} &= 0.813291 \\
A_{34:8} &= 0.789657 \\
A_{33:9} &= 0.766715 \\
A_{32:10} &= 0.744442
\end{aligned}
$$

4. *A select individual aged 42 purchases a 5-year term insurance with a death benefit of \$100,000. Force of interest is $\delta = 0.028$ and the benefit is payable immediately upon the death of the individual. Using a uniform distribution of deaths assumption, calculate the expected benefit from this policy.*

   Using the standard recursion, we calculate $A^{\,1}_{[42]:\overline{5}|} = 0.033825984$. Under the uniform distribution of deaths we have

$$
\overline{A}^{\,1}_{[42]:\overline{5}|} = \frac{e^{\delta}-1}{\delta} A^{\,1}_{[42]:\overline{5}|} = \frac{0.033825984 \times 0.028395684}{0.028} = 0.034303998
$$

   This means that the EPV of the benefit is $0.034303998 \times 100000 = \$3,430$.

5. *An individual aged 39 wants to purchase whole life insurance that pays a benefit at the end of the year of death. The interest rate is $i = 0.06$. The individual has a number of dangerous hobbies and uses the special lifetable:*

x	l_x	d_x
39	*10000.00*	*4.80*
40	*9995.20*	*4.86*
41	*9990.34*	*4.93*
42	*9985.41*	*5.01*
43	*9980.40*	*5.09*
44	*9975.31*	*5.18*
45	*9970.13*	*5.29*

   After age 45, the individual will be too old to participate in these hobbies and will use a standard lifetable, which will give the value $A_{45} = 0.1761$. Calculate the EPV of the benefit for this individual from a whole-life policy which has a death benefit of \$200,000.

   We use the recursion $A_{44} = \frac{\frac{5.18}{9975.31} + \frac{9970.13}{9975.31} A_{45}}{1.06} = 0.1660458060$. Similarly,

$$
\begin{aligned}
A_{43} &= 0.1567332264 \\
A_{42} &= 0.1478706417 \\
A_{41} &= 0.1395138162 \\
A_{40} &= 0.1316335173 \\
A_{39} &= 0.1242026256
\end{aligned}
$$

So the expected present value of the benefit is $200000 \times 0.1242026256 = \$24,840.52$.

Standard Questions

6. *An insurance company has used Makeham's formula with a constant factor to discount for selected lives — that is $\mu_{[x]+s} = D^{3-s}\mu_{x+s}$ to construct a lifetable for female smokers. The lifetable is given below.*

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
40	*9946.06*	*9927.38*	*9899.70*	*9858.68*
41	*9897.42*	*9876.17*	*9844.66*	*9798.00*
42	*9842.05*	*9817.87*	*9782.05*	*9729.00*
43	*9779.05*	*9751.57*	*9710.85*	*9650.58*
44	*9707.41*	*9676.19*	*9629.95*	*9561.53*
45	*9626.00*	*9590.58*	*9538.11*	*9460.53*

Find the parameters used in the model to produce the table. [It was constructed using the approximation $q_{[x]+s} = \mu_{[x+0.5]+s}$.]

We have for example, $\mu_{[41.5]+2} = 1 - \frac{l_{[41]+3}}{l_{[41]+2}}$ and $\mu_{[40.5]+3} = 1 - \frac{l_{[40]+4}}{l_{[40]+3}}$. We also have that $\mu_{[41.5]+2} = D\mu_{[40.5]+3}$, so we get

$$D = \frac{\left(1 - \frac{l_{[41]+3}}{l_{[41]+2}}\right)}{\left(1 - \frac{l_{[40]+4}}{l_{[40]+3}}\right)} = \frac{\left(\frac{46.66}{9844.66}\right)}{\left(\frac{60.68}{9858.68}\right)} = 0.77$$

For the ultimate part of the model, we have $\mu_{[40.5]+3} = 1 - \frac{9798.00}{9858.68} = 0.0061549823$, $\mu_{[41.5]+3} = 1 - \frac{9729.00}{9798.00} = 0.0070422536$, and $\mu_{[42.5]+3} = 1 - \frac{9650.58}{9729.00} = 0.0080604379$. That is:

$$\begin{aligned} A + BC^{43.5} &= 0.0061549823 \\ A + BC^{44.5} &= 0.0070422536 \\ A + BC^{45.5} &= 0.0080604379 \end{aligned}$$

Multiplying the first equation by C and subtracting from the second, and Multiplying the second equation by C and subtracting from the third gives:

$$\begin{aligned} AC &= 0.0061549823C - 0.0070422536 \\ AC &= 0.0070422536C - 0.0080604379 \end{aligned}$$

Combining these gives $0.0061549823C - 0.0070422536 = 0.0070422536C - 0.0080604379$, so $C = \frac{0.0080604379 - 0.0070422536}{0.0070422536 - 0.0061549823} = 1.1475$.

Since $AC = 0.0061549823C - 0.0070422536$, this gives $A = 0.0061549823 - \frac{0.0070422536}{1.1475} = .000018$. Finally, we substitute these numbers to get $B = \frac{0.0061549823 - .000018}{1.1475^{43.5}} = 0.0000154$.

[Since the numbers in the table are rounded, it is possible that using different numbers or slightly different methods to calculate the parameters could result in slightly different results.]

7. *A select individual aged 27 has whole life insurance with a death benefit of \$140,000 payable at the end of the year of death. The individual wants to convert this to a 5-year term insurance policy. If the current interest rate is $i = 0.04$, what benefit for the term insurance policy would have the same EPV as the whole life policy? [The company has already calculated that $A_{[29]+3} = 0.106825$ and $A_{[27]} = 0.0885419$.]*

   We have that $A_{[27]} = A^1_{[27]:5} + (1.04)^{-5}_5 p_{[27]} A_{[27]+5}$, since the first term on the right corresponds to insurance that pays only if the life dies in the first 5 years, and the second term corresponds to insurance that pays only if the life dies in at the end of 5 years. Equating the EPV of the death benefits means

$$140000A_{[27]} = XA^1_{[27]:5} = X\left(A_{[27]} - (1.04)^{-5} \times \frac{9985.80}{9995.14} A_{[27]+5}\right)$$

   This gives

$$X = \frac{140000 \times 0.0885419}{0.0885419 - (1.04)^{-5} \times .9990655458 \times 0.106825} = 15087760.9379537727$$

   So the death benefit with the same EPV is \$15.1 million.

8. *A woman aged 30 buys a house with a mortgage of \$200,000. She amortises this amount with monthly payments over a period of 25 years at $i^{(2)} = 6\%$. She takes out mortgage insurance, which pays off the outstanding balance (principle plus interest) of the mortgage at the end of the month in which she dies. [Assume that the mortgage company does not charge a penalty for early repayment in this case.] If the insurance company uses an interest rate $i = 5.6\%$ and the life table from Table 1, and the woman is select at age 30, calculate the expected present value of the benefit on this policy. [Use the uniform distribution of deaths assumption. You are given the following values: At interest rate $i = 0.056$, $A^1_{30:\overline{25}|} = 0.007131791$, while at interest rate $i = -0.00461872$, $A^1_{30:\overline{25}|} = 0.01813134$.]*

   $i^{(2)} = 6\%$, so the interest rate each month is $(1.03)^{\frac{1}{6}} - 1 = 0.004938622$. This means that the monthly payments are given by solving $Ra_{\overline{300}|0.004938622} = 200000$, so $R = \frac{200000 \times 0.004938622}{1-1.03^{-50}} = 1279.62$. After n payments, the outstanding balance is approximately $1279.62a_{\overline{300-n}|0.004938622}$ [This is approximate because it does not take into account the reduced final payment. The final payment is reduced by \$4.63, so the outstanding balance is actually $1279.62a_{\overline{300-n}|0.004938622} - 4.63(1.004938622)^{n-300}$.]. The expected present value of this benefit is given by

$$\sum_{n=0}^{299} {}_{\frac{n}{12}|\frac{1}{12}}q_{30} \left(1279.62a_{\overline{300-n}|0.004938622} - 4.63(1.004938622)^{n-300}\right)(1.056)^{-\frac{n}{12}}$$

   We can evaluate this as

$$\sum_{n=0}^{299} {}_{\frac{n}{12}|\frac{1}{12}}q_{30} \left(259104.67 - 259100.04(1.0609)^{\frac{n-300}{12}}\right)(1.056)^{-\frac{n}{12}}$$

$$\sum_{n=0}^{299} {}_{\frac{n}{12}|\frac{1}{12}}q_{30} \left(200000 - 59102.55(1.0609)^{\frac{n}{12}}\right)(1.056)^{-\frac{n}{12}}$$

$$\sum_{n=0}^{299} {}_{\frac{n}{12}|\frac{1}{12}}q_{30} 200000(1.056)^{-\frac{n}{12}} - \sum_{n=0}^{299} {}_{\frac{n}{12}|\frac{1}{12}}q_{30} 59102.55\left(\frac{1.0609}{1.056}\right)^{\frac{n}{12}}$$

The first sum corresponds to the EPV of a death benefit of \$200,000. The second term corresponds to the EPV of a death benefit of \$59,102.55 at an interest rate of $\frac{1.056}{1.0609} - 1 = -0.00461872$.

Converting to monthly, for $i = 0.056$, we have $\frac{i}{i^{(m)}} = 1.025414167$, so $A^{(12)}{}^{\,1}_{30:\overline{25|}} = 0.007131791 \times 1.025414167 = 0.00731304$. Similarly, for $i = -0.00461872$, we have $\frac{i}{i^{(m)}} = 0.997881317$, so $A^{(12)}{}^{\,1}_{30:\overline{25|}} = 0.01813134 \times 0.997881317 = 0.018092925$. The EPV of the benefit from the mortgage insurance is therefore $200000 \times 0.00731304 - 59102.55 \times 0.018092925 = \$393.27$.

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66
26	9997.00	9995.83	9994.40	9992.66
27	9995.14	9993.90	9992.38	9990.52
28	9993.16	9991.84	9990.22	9988.24
29	9991.05	9989.65	9987.92	9985.80
30	9988.81	9987.30	9985.46	9983.18
31	9986.40	9984.80	9982.82	9980.38
32	9983.83	9982.11	9979.99	9977.37
33	9981.07	9979.23	9976.95	9974.13
34	9978.11	9976.13	9973.68	9970.64
35	9974.93	9972.79	9970.16	9966.88
36	9971.50	9969.20	9966.36	9962.82
37	9967.80	9965.33	9962.25	9958.44
38	9963.81	9961.14	9957.82	9953.69
39	9959.50	9956.61	9953.02	9948.55
40	9954.84	9951.71	9947.82	9942.98
41	9949.79	9946.41	9942.19	9936.94
42	9944.32	9940.66	9936.08	9930.38
43	9938.39	9934.41	9929.45	9923.26
44	9931.96	9927.64	9922.25	9915.52
45	9924.97	9920.28	9914.42	9907.10
46	9917.37	9912.28	9905.91	9897.94
47	9909.11	9903.58	9896.65	9887.98
48	9900.13	9894.11	9886.57	9877.13
49	9890.36	9883.80	9875.59	9865.30
50	9879.71	9872.57	9863.63	9852.42
51	9868.12	9860.34	9850.59	9838.38
52	9855.48	9847.01	9836.39	9823.08
53	9841.72	9832.48	9820.90	9806.39
54	9826.71	9816.64	9804.02	9788.18
55	9810.34	9799.37	9785.60	9768.33
56	9792.49	9780.52	9765.51	9746.67
57	9773.03	9759.97	9743.60	9723.05
58	9751.79	9737.56	9719.69	9697.28
59	9728.63	9713.10	9693.62	9669.17
60	9703.36	9686.43	9665.17	9638.51
61	9675.80	9657.33	9634.15	9605.07
62	9645.73	9625.59	9600.31	9568.61
63	9612.94	9590.98	9563.42	9528.85
64	9577.18	9553.24	9523.19	9485.52
65	9538.19	9512.09	9479.35	9438.30
66	9495.69	9467.25	9431.58	9386.86
67	9449.37	9418.39	9379.54	9330.85
68	9398.90	9365.17	9322.87	9269.88
69	9343.95	9307.23	9261.20	9203.55
70	9284.12	9244.18	9194.11	9131.43
71	9219.03	9175.59	9121.17	9053.07
72	9148.24	9101.03	9041.91	8967.97
73	9071.30	9020.03	8955.85	8875.63

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
74	8987.73	8932.10	8862.49	8775.52
75	8897.04	8836.71	8761.27	8667.10
76	8798.69	8733.34	8651.66	8549.78
77	8692.13	8621.41	8533.09	8423.00
78	8576.81	8500.36	8404.95	8286.16
79	8452.13	8369.60	8266.68	8138.66
80	8317.52	8228.53	8117.67	7979.93
81	8172.36	8076.57	7957.35	7809.41
82	8016.08	7913.13	7785.15	7626.56
83	7848.11	7737.67	7600.54	7430.89
84	7667.89	7549.66	7403.05	7221.99
85	7474.92	7348.64	7192.27	6999.51
86	7268.77	7134.21	6967.86	6763.22
87	7049.07	6906.07	6729.62	6513.04
88	6815.55	6664.05	6477.46	6249.02
89	6568.09	6408.10	6211.48	5971.42
90	6306.70	6138.35	5931.96	5680.73
91	6031.59	5855.15	5639.41	5377.67
92	5743.19	5559.08	5334.61	5063.27
93	5442.15	5250.97	5018.61	4738.86
94	5129.44	4931.97	4692.79	4406.12
95	4806.33	4603.54	4358.89	4067.08
96	4474.39	4267.51	4018.96	3724.10
97	4135.60	3926.04	3675.44	3379.91
98	3792.25	3581.66	3331.11	3037.57
99	3447.02	3237.23	2989.05	2700.39
100	3102.90	2895.94	2652.63	2371.88
101	2763.19	2561.21	2325.37	2055.64
102	2431.39	2236.61	2010.90	1755.27
103	2111.15	1925.80	1712.81	1474.18
104	1806.12	1632.34	1434.48	1215.44
105	1519.82	1359.55	1178.94	981.65
106	1255.46	1110.36	948.70	774.71
107	1015.81	887.14	745.58	595.71
108	802.96	691.49	570.56	444.87
109	618.23	524.17	423.71	321.41
110	462.04	385.00	304.13	223.65
111	333.80	272.80	210.00	149.10
112	231.99	185.53	138.71	94.62
113	154.19	120.34	87.07	56.74
114	97.30	73.90	51.50	31.84
115	57.78	42.55	28.41	16.52
116	31.92	22.69	14.43	7.81
117	16.15	11.04	6.63	3.30
118	7.34	4.79	2.69	1.21
119	2.90	1.79	0.93	0.37
120	0.95	0.55	0.26	0.09
121	0.23	0.13	0.05	0.01
122	0.03	0.02	0.01	0.00