ACSC/STAT 3720, Life Contingencies I
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Homework Sheet 5
Model Solutions

Basic Questions

1. *An insurance company offers a 10-year endowment policy with benefit \$700,000 payable either at the end of the year of death, or at the end of the 10 years. The premium for this policy for a select individual aged 44 for whom the lifetable in Table 1 is appropriate, is \$70,000, payable at the start of each year. If the current interest rate is $i = 0.03$, what is the probability that the present value of future loss for this policy exceeds \$150,000?*

   If the payment is made at the end of the nth year, the present value of the payment is $700000(1.03)^{-n}$ and the present value of the premiums received is $70000\frac{1.03-(1.03)^{1-n}}{0.03}$. The present value of future loss is therefore $\left(700000 + 70000\frac{1.03}{0.03}\right)(1.03)^{-n} - 70000\frac{1.03}{0.03}$. This exceeds \$150,000 if

$$\begin{aligned}
\left(700000 + 70000\frac{1.03}{0.03}\right)(1.03)^{-n} - 70000\frac{1.03}{0.03} &> 150000\\
\left(700000 + 70000\frac{1.03}{0.03}\right)(1.03)^{-n} &> 70000\frac{1.03}{0.03} + 150000\\
(1.03)^{-n} &> \frac{70000\frac{1.03}{0.03} + 150000}{700000 + 70000\frac{1.03}{0.03}}\\
n &< -\frac{\log\left(\frac{70000\frac{1.03}{0.03}+150000}{700000+70000\frac{1.03}{0.03}}\right)}{\log(1.03)}
\end{aligned}$$

   Which gives $n < 6.599628292$, so if the person dies within the first 6 years, the present value of future loss exceeds \$150,000. The probability of this is ${}_6q_{[44]} = 1 - \frac{9887.98}{9931.96} = 0.004428129$.

2. *An insurance company offers a 10-year term policy with death benefit \$600,000 payable at the end of the year of death. If the interest rate is $i = 0.05$, calculate the annual premium for this policy for a select individual aged 36, using the lifetable in Table 1 and the equivalence principle.*

   From the lifetable we calculate

$$
\begin{aligned}
A_{46:\overline{0}|} &= 1\\
A_{45:\overline{1}|} &= 0.952381\\
A_{44:\overline{2}|} &= 0.907059\\
A_{43:\overline{3}|} &= 0.86392\\
A_{42:\overline{4}|} &= 0.822853\\
A_{41:\overline{5}|} &= 0.783757\\
A_{40:\overline{6}|} &= 0.746533\\
A_{39:\overline{7}|} &= 0.711091\\
A_{[36]+2:\overline{8}|} &= 0.677327\\
A_{[36]+1:\overline{9}|} &= 0.645161\\
A_{[36]:\overline{10}|} &= 0.614517
\end{aligned}
$$

$d = 1 - \frac{1}{1.05} = \frac{1}{21}$, so $\ddot{a}_{[36]:10} = \frac{1-0.614517}{\frac{1}{21}} = 8.095143$.

The probability of surviving to the end of the term is $\frac{9923.26}{9971.50}$, so we have $A^1_{[36]:\overline{10}|} = A_{[36]:\overline{10}|} - \frac{9923.26}{9971.50}(1.05)^{-10} = .0035737$.

Under the equivalence principle, the premium is given by $8.095143P = 0.0035737 \times 600000$ or $P = \frac{0.0035737\times 600000}{8.095143} = \$264.88$.

3. *The current interest rate is $i = 0.04$. A life aged 65 to whom the ultimate part of the lifetable in Table 1 applies, wants to purchase an annuity with annual payments of \$23,000 starting immediately. The initial costs for the insurance company of setting up this annuity are \$3,600 plus 3% of the cost of the annuity, and the renewal costs are \$300 per year. What is the Gross premium for this annuity? (It is purchased with a single lump sum premium.)*

   Using the standard recurrence, we calculate $A_{65} = 0.340726$. $i = 0.04$ gives $d = \frac{0.04}{1.04} = \frac{1}{26}$, which gives $\ddot{a}_{65} = 26(1 - 0.340726) = 17.141124$. The EPV of benefits is therefore 23000×17.141124, and the EPV of costs is $0.03P + 3300 + 300 \times 17.141124$, where P is the cost of the annuity. (\$300 from the initial costs is transfered to renewal costs, so that the renewal costs start immediately.) This gives the equation $0.97P = 23300 \times 17.141124 + 3300$, so $P = \frac{23300\times 17.141124+3300}{0.97} = \$415,142.46$.

Standard Questions

4. *A select individual aged 43, to whom the lifetable in Table 1 applies, wants to purchase a whole life insurance policy. She can afford to pay annual premiums of \$1,300 from now until age 80 (so she pays the last premium at age 79). The interest rate is $i = 0.04$, which gives $A_{[43]} = 0.159678$ and $A_{[77]+3} = 0.526062$. Using the equivalence principal to calculate net premiums, what is the largest death benefit that she can afford to purchase?*

   The EPV of her annual premiums is

$$1300\left(\ddot{a}_{[43]} - \ddot{a}_{[77]+3}(1.04)^{-37}\,{}_{37}p_{[43]}\right) = 1300\left(\frac{1-A_{[43]}}{\frac{1}{26}} - \frac{1-A_{[77]+3}}{\frac{1}{26}}(1.04)^{-37}\frac{8423.00}{9938.39}\right) = \$25,221.94$$

We need to match this to the EPV of the death benefit. If the death benefit is X, then its EPV is $A_{[43]}X = 0.159678X$, so we have $X = \frac{25221.94}{0.159678} = \$157,955.03$.

5. *An individual aged 42 is paying premiums of \$700 a month for a whole life insurance policy which pays benefits at the end of the month of death. If the individual's mortality follows the ultimate part of Table 1, and the interest rate is $i^{(12)} = 0.06$, calculate the equivalent annual premiums (for a policy which pays benefits at the end of the year of death) using:*

   (a) Uniform distribution of deaths

   We have $i = 1.005^{12} - 1 = 0.0616778$. This gives $d = 0.05809466$, and $d^{(12)} = 12\left(1 - \frac{1}{1.005}\right) = 0.05970149$. From the lifetable we calculate $A_{42} = 0.0673059$ and therefore $\ddot{a}_{42} = \frac{1-0.0673059}{0.05809466} = 16.05473$. Under UDD, we have $A_{42}^{(12)} = \frac{0.0616778}{0.06}A_{42} = 0.0691880$. This gives $\ddot{a}_{42}^{(12)} = \frac{1-0.0691880}{0.05970149} = 15.591102$. The death benefit for the policy should therefore be $\frac{12\times700\times15.591102}{0.0691880} = \$1,892,889.72$. For an annual policy with this benefit, the premium should be $\frac{1892889.72\times0.0673059}{16.05473} = \$7,935.52$.

   (b) Woolhouse's formula

   We estimate $\mu_{42} = \frac{1}{2}(q_{41} + q_{42}) = \frac{1}{2}\left(\frac{5.14}{9953.69} + \frac{5.57}{9948.55}\right) = 0.000538136$. Force of interest is $12\log(1.005) = 0.0598505$. Using Woolhouse's formula, we have $\ddot{a}_{42}^{(12)} = \ddot{a}_{42} - \frac{11}{24} - \frac{143}{1728}(0.0598505+0.000538136) = 15.5913995$. We have $A_{42}^{(12)} = 1-d^{(12)}\ddot{a}_{42}^{(12)} = 0.06917018$, so the death benefit of this policy should be $\frac{12\times700\times15.5913995}{0.06917018} = \$1,893,413.55$. For an annual policy, the premium should be $\frac{1893413.55\times0.0673059}{16.05473} = \$7,937.72$.

6. *An insurance company provides a regular annual premium annuity contract to a select individual aged 47, using the lifetable in Table 1. The interest rate is $i = 0.05$. This gives that $A_{[62]+3} = 0.270592$, $A_{[72]+3} = 0.388846$ and $A_{[47]} = 0.128316$. The individual will pay annual net premiums until age 65 (so the last premium will be at age 64). These premiums are calculated using the equivalence principle. From age 65, they will receive an annuity of \$20,000 at the start of each year. This annuity is guaranteed for 10 years. What is the probability that the insurance company makes a net profit on this policy?*

   The EPV of the benefits is $20000\left((1.05)^{-17}a_{\overline{10}|0.05} +_{28} p_{[47]}\ddot{a}_{[72]+3}(1.05)^{-28}\right) = 126639.00$. If P is the annual premium, the EPV of the premiums is $P\frac{1-A_{[47]:\overline{18}|}}{d} = P\frac{1-A_{[47]}-{}_{18}p_{47}(1.05)^{-18}(1-A_{[62]+3})}{d} = 21(1-0.128316-\frac{9568.61}{9909.11}(1.05)^{-18}(1-0.270592))P = 12.15931P$. Matching expected present values, we get $P = \frac{126639.00}{12.15931} = \$10,414.98$.

   If the life survives to age 65, the accumulated value of the premiums is $10414.98(1.05)s_{\overline{18}|0.05} = 10414.98\frac{(1.05)^{19}-1.05}{0.05} = 307648.13$. The company makes a profit if this is greater than the present value of the benefits paid from the policy. That is if $20000\ddot{a}_{\overline{n}|0.05} < 307648.13$. We solve this:

$$\begin{aligned}
20000\frac{1-1.05^{-n}}{\frac{1}{21}} &< 307648.13\\
1-1.05^{-n} &< \frac{307648.13}{20000\times21} = 0.73249560.7432436\\
1.05^{-n} &> 0.2675044\\
n &< \frac{-\log(0.2675044)}{\log(1.05)} = 27.02633
\end{aligned}$$

That is, the policy makes a profit if the life dies before age 93. The probability of this is $1-\frac{5680.73}{9909.11}=0.426733$.

On the other hand, if the life dies before 65, the guaranteed benefit is $(1.05)^{-18}\ddot{a}_{\overline{10}|0.05}$, so the insurance company only makes a profit if the premiums received have larger present value than this. That is if $10414.98\ddot{a}_{\overline{n}|0.05} \geqslant 20000(1.05)^{-17}a_{\overline{10}|0.05}$

$$\begin{aligned}
1-(1.05)^{-n} &\geqslant \frac{20000}{10414.98}(1.05)^{-18}(1-(1.05)^{-10}) \\
(1.05)^{-n} &\leqslant 1-\frac{20000}{10414.98}(1.05)^{-18}(1-(1.05)^{-10}) \\
n &\geqslant -\frac{\log\left(1-\frac{20000}{10414.98}(1.05)^{-18}(1-(1.05)^{-10})\right)}{\log(1.05)}=7.54804
\end{aligned}$$

where n is the number of premiums received. So they only make a profit if the life dies between ages 55 and 93. The probability of this is $\frac{9823.08}{9909.11}-\frac{5680.73}{9909.11}=\frac{4142.35}{9909.11}=0.4180345$

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66
26	9997.00	9995.83	9994.40	9992.66
27	9995.14	9993.90	9992.38	9990.52
28	9993.16	9991.84	9990.22	9988.24
29	9991.05	9989.65	9987.92	9985.80
30	9988.81	9987.30	9985.46	9983.18
31	9986.40	9984.80	9982.82	9980.38
32	9983.83	9982.11	9979.99	9977.37
33	9981.07	9979.23	9976.95	9974.13
34	9978.11	9976.13	9973.68	9970.64
35	9974.93	9972.79	9970.16	9966.88
36	9971.50	9969.20	9966.36	9962.82
37	9967.80	9965.33	9962.25	9958.44
38	9963.81	9961.14	9957.82	9953.69
39	9959.50	9956.61	9953.02	9948.55
40	9954.84	9951.71	9947.82	9942.98
41	9949.79	9946.41	9942.19	9936.94
42	9944.32	9940.66	9936.08	9930.38
43	9938.39	9934.41	9929.45	9923.26
44	9931.96	9927.64	9922.25	9915.52
45	9924.97	9920.28	9914.42	9907.10
46	9917.37	9912.28	9905.91	9897.94
47	9909.11	9903.58	9896.65	9887.98
48	9900.13	9894.11	9886.57	9877.13
49	9890.36	9883.80	9875.59	9865.30
50	9879.71	9872.57	9863.63	9852.42
51	9868.12	9860.34	9850.59	9838.38
52	9855.48	9847.01	9836.39	9823.08
53	9841.72	9832.48	9820.90	9806.39
54	9826.71	9816.64	9804.02	9788.18
55	9810.34	9799.37	9785.60	9768.33
56	9792.49	9780.52	9765.51	9746.67
57	9773.03	9759.97	9743.60	9723.05
58	9751.79	9737.56	9719.69	9697.28
59	9728.63	9713.10	9693.62	9669.17
60	9703.36	9686.43	9665.17	9638.51
61	9675.80	9657.33	9634.15	9605.07
62	9645.73	9625.59	9600.31	9568.61
63	9612.94	9590.98	9563.42	9528.85
64	9577.18	9553.24	9523.19	9485.52
65	9538.19	9512.09	9479.35	9438.30
66	9495.69	9467.25	9431.58	9386.86
67	9449.37	9418.39	9379.54	9330.85
68	9398.90	9365.17	9322.87	9269.88
69	9343.95	9307.23	9261.20	9203.55
70	9284.12	9244.18	9194.11	9131.43
71	9219.03	9175.59	9121.17	9053.07
72	9148.24	9101.03	9041.91	8967.97
73	9071.30	9020.03	8955.85	8875.63

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
74	8987.73	8932.10	8862.49	8775.52
75	8897.04	8836.71	8761.27	8667.10
76	8798.69	8733.34	8651.66	8549.78
77	8692.13	8621.41	8533.09	8423.00
78	8576.81	8500.36	8404.95	8286.16
79	8452.13	8369.60	8266.68	8138.66
80	8317.52	8228.53	8117.67	7979.93
81	8172.36	8076.57	7957.35	7809.41
82	8016.08	7913.13	7785.15	7626.56
83	7848.11	7737.67	7600.54	7430.89
84	7667.89	7549.66	7403.05	7221.99
85	7474.92	7348.64	7192.27	6999.51
86	7268.77	7134.21	6967.86	6763.22
87	7049.07	6906.07	6729.62	6513.04
88	6815.55	6664.05	6477.46	6249.02
89	6568.09	6408.10	6211.48	5971.42
90	6306.70	6138.35	5931.96	5680.73
91	6031.59	5855.15	5639.41	5377.67
92	5743.19	5559.08	5334.61	5063.27
93	5442.15	5250.97	5018.61	4738.86
94	5129.44	4931.97	4692.79	4406.12
95	4806.33	4603.54	4358.89	4067.08
96	4474.39	4267.51	4018.96	3724.10
97	4135.60	3926.04	3675.44	3379.91
98	3792.25	3581.66	3331.11	3037.57
99	3447.02	3237.23	2989.05	2700.39
100	3102.90	2895.94	2652.63	2371.88
101	2763.19	2561.21	2325.37	2055.64
102	2431.39	2236.61	2010.90	1755.27
103	2111.15	1925.80	1712.81	1474.18
104	1806.12	1632.34	1434.48	1215.44
105	1519.82	1359.55	1178.94	981.65
106	1255.46	1110.36	948.70	774.71
107	1015.81	887.14	745.58	595.71
108	802.96	691.49	570.56	444.87
109	618.23	524.17	423.71	321.41
110	462.04	385.00	304.13	223.65
111	333.80	272.80	210.00	149.10
112	231.99	185.53	138.71	94.62
113	154.19	120.34	87.07	56.74
114	97.30	73.90	51.50	31.84
115	57.78	42.55	28.41	16.52
116	31.92	22.69	14.43	7.81
117	16.15	11.04	6.63	3.30
118	7.34	4.79	2.69	1.21
119	2.90	1.79	0.93	0.37
120	0.95	0.55	0.26	0.09
121	0.23	0.13	0.05	0.01
122	0.03	0.02	0.01	0.00