ACSC/STAT 3720, Life Contingencies I
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Homework Sheet 6
Model Solutions

Basic Questions

1. *An insurer issues 5,000 whole life insurance policies to select lives aged 51. The appropriate interest rate is $i = 0.06$. The company calculates $A_{[51]} = 0.111865$ and ${}^2A_{[51]} = 0.0276334$. If the death benefit is \$350,000, what annual premium should the company charge using the portfolio percentile method with a 98% probability of making a profit?*

   If the premium is P and a life dies in the nth year, then the present value of the loss on the policy is $350000(1.06)^{-n} - P\ddot{a}_{n} = 350000(1.06)^{-n} - P(1.06)\frac{1-1.06^{-n}}{0.06} = \left(350000 + \frac{1.06}{0.06}P\right)(1.06)^{-n} - \frac{1.06}{0.06}P$. The expected value of future loss is therefore

   $$\left(350000 + \frac{1.06}{0.06}P\right)A_{[51]} - \frac{1.06}{0.06}P$$

   and the variance of the future loss is

   $$\left(350000 + \frac{1.06}{0.06}P\right)^2 \left({}^2A_{[51]} - {A_{[51]}}^2\right) = 0.0151196\left(350000 + \frac{1.06}{0.06}P\right)^2$$

   We expand the expected present value of future loss as

   $$39152.75 - \frac{0.888135 \times 1.06}{0.06}P$$

   For a portfolio of 5,000 policies, the expected average present value of future loss is

   $$39152.75 - \frac{0.888135 \times 1.06}{0.06}P$$

   while the variance is

   $$\frac{0.0151196}{5000}\left(350000 + \frac{1.06}{0.06}P\right)^2$$

   The probability that the portfolio makes a loss overall is therefore

   $$\Phi\left(\frac{\frac{0.888135\times 1.06}{0.06}P - 39152.75}{\sqrt{\frac{0.0151196}{5000}\left(350000 + \frac{1.06}{0.06}P\right)^2}}\right)$$

We set this equal to 0.98 to get

$$\begin{aligned}
\frac{\frac{0.888135\times1.06}{0.06}P-39152.75}{\sqrt{\frac{0.0151196}{5000}}\left(350000+\frac{1.06}{0.06}P\right)}&=2.05\\
\frac{0.888135\times1.06}{0.06}P-39152.75&=2.05\sqrt{\frac{0.0151196}{5000}}\left(350000+\frac{1.06}{0.06}P\right)\\
0.888135P-2216.193396&=.00173894(19811.320755+P)\\
.886396&=2250.644138\\
P&=2539.10
\end{aligned}$$

So the annual premium should be \$2,539.10.

2. *Using the lifetable in Table 1, and interest rate $i=0.05$, calculate the net annual premium for a 5-year term insurance policy with Death benefit \$250,000, sold to a life aged 32, if:*

   (a) The life is an impaired life, and is treated as being 8 years older than she is [but is still treated as being select at the start of the policy].

   This is the premium for the same policy sold to a select life aged 40. Using the standard recurrence, we have $A_{[40]:5}=0.783690$, and therefore $\ddot{a}_{[40]:5}=21(1-0.783690)=4.542504$. Since the survival probability is $\frac{9930.38}{9954.84}=0.997543$, we have that $A^1_{[40]:5}=0.783690-0.997543(1.05)^{-5}=0.00208933$ The premium is therefore given by $\frac{250000\times0.00208933}{4.542504}=\$114.99$.

   (b) The life works in a hazardous environment, and has mortality 0.008 higher than normal.

   To calculate the premium in this case, we increase the force of interest by 0.008. This gives an interest rate $e^{log(1.05)+0.008}-1=0.05843369$. At this interest rate the standard recurrence gives us $A_{[32]:5}=0.7529045$. We calculate $d=1-\frac{1}{1.05843369}=0.0552077$ $\ddot{a}_{[32]:5}=\frac{1-0.753187}{0.0552077}=4.475743$. At the true interest rate of 0.05, we get $A_{[32]:5}=1-\frac{0.05}{1.05}\times4.475743=0.7868694$ and the probability of surviving 5 years is $\frac{9970.64}{9983.83}e^{-0.04}=0.9595201$, so $A^1_{[32]:5}=0.7868694-0.9595201(1.05)^{-5}=0.03506029$. The appropriate premium is therefore given by $\frac{250000\times0.03506029}{4.475743}=\$1,958.35$.

3. *An insurance company has a whole life insurance policy for a select individual aged 39. The death benefit of this policy is \$800,000, and the interest rate is $i=0.07$. Premiums are payable until age 80. The insurance company calculates $A_{[39]}=0.0423697$, and $A_{[39]+41}=0.353734$. Therefore, the net annual premium for the policy is \$2,361.64. What is the policy value if the life survives to age 75? [Use the lifetable in Table 1. $A_{[39]+36}=0.286183$.]*

   If the life survives to age 75, the EPV of the benefits is $800000A_{75}=228946.40$. The EPV of the remaining premiums is $2361.64a_{75:\overline{5}|}$. Now $A_{75:\overline{5}|}=A_{75}+{}_5p_{75}(1.07)^{-5}(1-A_{80})=0.286183+\frac{8423.00}{8967.97}\times(1-0.353734)(1.07)^{-5}=0.718961$, so $\ddot{a}_{75:\overline{5}|}=\frac{1-0.718961}{d}=4.295883$. The expected value of the remaining premiums is therefore $4.295883\times2361.64=10145.33$, so the policy value is $228946.40-10145.33=\$218,801.07$.

Standard Questions

4. *For the individual in Question 3, what is the policy value at age 75 if the individual is found to be select at age 75? [All other assumptions remain the same.]*

If the individual is select at age 75, then we calculate $A_{[75]} = 0.281373$. This gives $A_{[75]:\overline{5}|} = A_{[75]} +_5 p_{[75]}(1.07)^{-5}(1 - A_{80}) = 0.281373 + \frac{8423.00}{8897.04} \times (1 - 0.353734)(1.07)^{-5} = 0.717601$, so $\ddot{a}_{[75]:\overline{5}|} = \frac{1.07(1-0.717601)}{0.07} = 4.316672$. The EPV of the benefit is therefore $800000 \times 0.281373 = 225098.22$ and the EPV of the remaining premiums is $2361.64 \times 4.316672 = 10194.42$, so the policy value is $225098.22 - 10194.42 = \$214,903.79$.

5. *For the individual in Question 3, what is the policy value at age 75 if the interest rate has increased to $i = 0.09$? [All other assumptions remain the same as in Question 3. At this interest rate, we have $A_{80} = 0.281157$ and $A_{75} = 0.218262$.]*

   The EPV of the benefits is $800000 \times 0.218262 = 174609.600000$. We also have $A_{75:\overline{5}|} = A_{75} + _5p_{75}(1.09)^{-5}(1 - A_{80}) = 0.218262 + \frac{8423.00}{8967.97} \times (1 - 0.281157)(1.09)^{-5} = 0.657070$ This gives $\ddot{a}_{75:\overline{5}|} = \frac{1.09(1-0.657070)}{0.09} = 4.153267$. The EPV of premiums is therefore $4.153267 \times 2361.64 = 9808.52$, so the policy value is $174609.60 - 9808.52 = \$164,801.08$.

6. *A select life aged 48 takes out a 5-year endowment insurance with benefit \$600,000. The initial cost of this insurance is \$1000 plus 30% of the first annual premium. The renewal cost is 2% of each subsequent premium. The interest rate is $i = 0.04$. Using the lifetable in Table 1, we can calculate $A_{[48]:\overline{5}|} = 0.8221934$.*

   (a) Calculate the gross premium for this policy.

   The EPV of benefits is $0.8221934 \times 600000 = \$493,316.04$. If the premium is P, the EPV of premiums minus renewal costs is $26(1 - 0.8221934) \times 0.98P = 4.530512P$. The initial costs are $1000 + 0.28P$ (we counted $0.02P$ as a renewal cost so that renewal costs are applied to all premiums). We therefore need to solve

   $$\begin{aligned} 4.530512P &= 493316.04 + 1000 + 0.28P \\ 4.250512P &= 494316.06 \\ P &= \$116,295.64 \end{aligned}$$

   (b) Calculate the gross policy value after 2 years.

   After 2 years, we have $A_{[48]+2:\overline{3}|} = 0.8891082$, so $\ddot{a}_{[48]+2:\overline{3}|} = 26(1 - 0.8891082) = 2.883187$. The EPV of benefits is therefore $600000 \times 0.8891082 = 533464.90$ and the EPV of premiums minus renewal expenses is $0.98 \times 2.883187 \times 116295.64 = 328596.09$. Therefore, the policy value is $533464.90 - 328596.09 = \$204,868.81$.

Bonus Questions

7. *An insurance company wants to develop a new policy with a variable death benefit, designed so that if the policy basis does not change, then the net policy value is 0 at all future times. Assuming the policy has a constant annual premium, what should the death benefit be if the policyholder dies in the nth year of the policy?*

   If the annual premium is P and the death benefit at age x is D_x, then the recurrence relation gives $_xV = (q_x D_x + p_{x\,x+1}V)(1 + i)^{-1} - P$. Since we are given that $_xV =_{x+1} V = 0$, this becomes $P(1 + i) = q_x D_x$, so we get $D_x = \frac{P(1+i)}{q_x}$.

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66
26	9997.00	9995.83	9994.40	9992.66
27	9995.14	9993.90	9992.38	9990.52
28	9993.16	9991.84	9990.22	9988.24
29	9991.05	9989.65	9987.92	9985.80
30	9988.81	9987.30	9985.46	9983.18
31	9986.40	9984.80	9982.82	9980.38
32	9983.83	9982.11	9979.99	9977.37
33	9981.07	9979.23	9976.95	9974.13
34	9978.11	9976.13	9973.68	9970.64
35	9974.93	9972.79	9970.16	9966.88
36	9971.50	9969.20	9966.36	9962.82
37	9967.80	9965.33	9962.25	9958.44
38	9963.81	9961.14	9957.82	9953.69
39	9959.50	9956.61	9953.02	9948.55
40	9954.84	9951.71	9947.82	9942.98
41	9949.79	9946.41	9942.19	9936.94
42	9944.32	9940.66	9936.08	9930.38
43	9938.39	9934.41	9929.45	9923.26
44	9931.96	9927.64	9922.25	9915.52
45	9924.97	9920.28	9914.42	9907.10
46	9917.37	9912.28	9905.91	9897.94
47	9909.11	9903.58	9896.65	9887.98
48	9900.13	9894.11	9886.57	9877.13
49	9890.36	9883.80	9875.59	9865.30
50	9879.71	9872.57	9863.63	9852.42
51	9868.12	9860.34	9850.59	9838.38
52	9855.48	9847.01	9836.39	9823.08
53	9841.72	9832.48	9820.90	9806.39
54	9826.71	9816.64	9804.02	9788.18
55	9810.34	9799.37	9785.60	9768.33
56	9792.49	9780.52	9765.51	9746.67
57	9773.03	9759.97	9743.60	9723.05
58	9751.79	9737.56	9719.69	9697.28
59	9728.63	9713.10	9693.62	9669.17
60	9703.36	9686.43	9665.17	9638.51
61	9675.80	9657.33	9634.15	9605.07
62	9645.73	9625.59	9600.31	9568.61
63	9612.94	9590.98	9563.42	9528.85
64	9577.18	9553.24	9523.19	9485.52
65	9538.19	9512.09	9479.35	9438.30
66	9495.69	9467.25	9431.58	9386.86
67	9449.37	9418.39	9379.54	9330.85
68	9398.90	9365.17	9322.87	9269.88
69	9343.95	9307.23	9261.20	9203.55
70	9284.12	9244.18	9194.11	9131.43
71	9219.03	9175.59	9121.17	9053.07
72	9148.24	9101.03	9041.91	8967.97
73	9071.30	9020.03	8955.85	8875.63
74	8987.73	8932.10	8862.49	8775.52
75	8897.04	8836.71	8761.27	8667.10
76	8798.69	8733.34	8651.66	8549.78
77	8692.13	8621.41	8533.09	8423.00
78	8576.81	8500.36	8404.95	8286.16
79	8452.13	8369.60	8266.68	8138.66
80	8317.52	8228.53	8117.67	7979.93
81	8172.36	8076.57	7957.35	7809.41
82	8016.08	7913.13	7785.15	7626.56
83	7848.11	7737.67	7600.54	7430.89
84	7667.89	7549.66	7403.05	7221.99
85	7474.92	7348.64	7192.27	6999.51
86	7268.77	7134.21	6967.86	6763.22
87	7049.07	6906.07	6729.62	6513.04
88	6815.55	6664.05	6477.46	6249.02
89	6568.09	6408.10	6211.48	5971.42
90	6306.70	6138.35	5931.96	5680.73
91	6031.59	5855.15	5639.41	5377.67
92	5743.19	5559.08	5334.61	5063.27
93	5442.15	5250.97	5018.61	4738.86
94	5129.44	4931.97	4692.79	4406.12
95	4806.33	4603.54	4358.89	4067.08
96	4474.39	4267.51	4018.96	3724.10
97	4135.60	3926.04	3675.44	3379.91
98	3792.25	3581.66	3331.11	3037.57
99	3447.02	3237.23	2989.05	2700.39
100	3102.90	2895.94	2652.63	2371.88
101	2763.19	2561.21	2325.37	2055.64
102	2431.39	2236.61	2010.90	1755.27
103	2111.15	1925.80	1712.81	1474.18
104	1806.12	1632.34	1434.48	1215.44
105	1519.82	1359.55	1178.94	981.65
106	1255.46	1110.36	948.70	774.71
107	1015.81	887.14	745.58	595.71
108	802.96	691.49	570.56	444.87
109	618.23	524.17	423.71	321.41
110	462.04	385.00	304.13	223.65
111	333.80	272.80	210.00	149.10
112	231.99	185.53	138.71	94.62
113	154.19	120.34	87.07	56.74
114	97.30	73.90	51.50	31.84
115	57.78	42.55	28.41	16.52
116	31.92	22.69	14.43	7.81
117	16.15	11.04	6.63	3.30
118	7.34	4.79	2.69	1.21
119	2.90	1.79	0.93	0.37
120	0.95	0.55	0.26	0.09
121	0.23	0.13	0.05	0.01
122	0.03	0.02	0.01	0.00