ACSC/STAT 3720, Life Contingencies I
WINTER 2015
Toby Kenney
Sample Midterm Examination

This Sample examination has more questions than the actual midterm, in order to cover a wider range of questions. Estimated times are provided after each question to help your preparation. Assume lives are in the ultimate part of the model unless otherwise specified, and policies are annual unless otherwise specified.

1. (a) A life insurance company uses the Gompertz law $\mu_x = 0.0000036(1.12)^x$ to model mortality. Calculate $_{50}p_{22}$.

   (b) Calculate $_{25|30}q_{35}$.

2. An insurance company models lifetime as

$$F_0(t) = 1 - \left(1 - \frac{x}{125}\right)^{\frac{1}{3}}$$

   Calculate the complete expectation of future life for a life aged 38.

3. Using the lifetable in Table 1, calculate the curtate expected lifetime for a standard life aged 110.

4. Compute a lifetable using a Makeham model of mortality $\mu_x = A + BC^x$ with $A = 0.0000011$, $B = 0.00000194$ and $C = 1.09$, between ages 40 and 45, with radix 10,000. [You may use $\mu_{x+0.5}$ as an approximation for q_x.]

5. For the lifetable in Table 1, calculate $_3p_{28.25}$ using:

   (a) the uniform distribution of deaths assumption

   (b) the constant rate of mortality assumption.

6. Consider a select survival model based on Makeham's model $\mu_x = A + BC^x$ with $A = 0.0000014$, $B = 0.0000181$ and $C = 1.10$, and with selection period 2 years, and $\mu_{[x]+s} = 0.87^{2-s}\mu_{x+s}$. Construct a select lifetable between ages at selection 35 and 38, with radix 10,000. [You may use $\mu_{[x+0.5]+s}$ as an approximation for $q_{[x]+s}$.]

7. Using the lifetable in Table 1, compute the probability that an individual currently aged 36 who was select one year ago survives to age 59.

8. The lifetable in Table 1 applied last year. This year, an insurance company updates it using the following reduction factors by age:

Age	Reduction Factor
28	0.98
29	0.975
30	0.985
31	0.98
32	0.975
33	0.97

Calculate the expected benefit of a 5-year endowment insurance with benefit $150,000, sold to a standard life aged 28 if the current interest rate is $i = 0.04$.

9. The interest rate is $i = 0.06$. For a life aged 36 with mortality following the lifetable in Table 1, an insurance company calculates $A_{[35]+1} = 0.05192755$. However, the insurance company made a mistake with the select status, and the life should have been treated as a standard life, not a select life. Calculate the new value A_{36}.

10. For a standard life aged 53, for whom the lifetable in Table 1 is appropriate, at interest rate $i = 0.04$, calculate $A^1_{53:\overline{5}|}$

11. The interest rate is $i = 0.05$. For a life aged 63, following the lifetable in Table 1, you calculate $A_{63:10} = 0.620875$. Calculate $\overline{A}_{63:10}$ under a Uniform Distribution of deaths assumption.

12. For a standard life aged 37, for whom the lifetable in Table 1 is appropriate, at interest rate $i = 0.03$, you calculate $A_{37} = 0.205862$ and $A_{57} = 0.355623$. Calculate the expected present value of the benefit of 20-year term insurance for a life aged 37, with a death benefit of $250,000.

13. Calculate the expected present value and the variance of the present value of a 5-year endowment insurance, sold to a standard life aged 43 on Table 1, if the endowment benefit is $100,000 and the interest rate is $i = 0.08$.

14. The interest rate is $i = 0.04$. For a whole-life insurance policy sold to a life aged 46 on Table 1, you calculate $A_{46} = 0.178312$. The death benefit is $300,000. The company wants to change its policies so that the benefits are payable immediately on the death of the insured. How much does this increase the EPV of the policy?

15. The interest rate is $i = 0.04$. The current death benefit of a life insurance policy is $150,000. The benefit increases by 2% every year (so if the life dies in the first year, the benefit will be $153,000 at the end of the year). The policy is a 5-year term insurance policy, sold to a life aged 37, for whom Table 1 is appropriate. Calculate the EPV of this policy.

16. The interest rate is $i = 0.05$. A man aged 34 buys a house with a mortgage of $220,000, amortised over 25 years, with annual payments of $15609.54, which perfectly pay off the mortgage with no adjustment to the final payment. He buys mortgage insurance, which pays off the outstanding balance of the mortgage in the event of his death. The same interest rate is used by the mortgage company and the insurance company. You calculate that $A_{34:\overline{25}|} = 0.378751$, $A_{34:\overline{26}|} = 0.364661$ and $A^1_{34:\overline{25}|} = 0.0124184$. [You only need one of these, but different methods may use different values.] Calculate the EPV of the mortgage insurance.

17. A man aged 67 has saved up $370,000 for his retirement. He wishes to purchase an annual life annuity with this, with EPV equal to $370,000. The interest rate is $i = 0.04$. From the lifetable in Table 1, you calculate $A_{67} = 0.362830$.

    (a) Calculate the annual payments for this annuity

    (b) If he wants to convert to a monthly annuity, what should the monthly payments be, using the UDD assumption.

18. A man aged 44 buys a deferred annuity, which will begin paying an annual life annuity when he reaches 65. The lifetable is Table 1, and the interest rate is $i = 0.06$. From that table, you calculate $A_{65} = 0.218135$ and $A_{44} = 0.079134$. If he wants the annuity to pay $26,000 at the start of each year:

    (a) What is the EPV of the deferred annuity?

(b) If he instead pays for the annuity with annual payments starting now and ending after his 64th birthday, what should the annual payments be to match the EPV of the deferred annuity?

19. A woman aged 68 is currently receiving a pension of \$30,000 at the start of each year. She wants her pension to be guaranteed for the first 5 years. The current interest rate is $i = 0.04$, and the lifetable is Table 1. From this you calculate $\ddot{a}_{68:\overline{5}|} = 4.577015$. By how much does the 5-year guarantee increase the EPV of the annuity?

20. A man aged 62 is retiring, and has saved up \$420,000 for his retirement. He wishes to purchase a life annuity with payments increasing by 2% every year. If the current interest rate is $i = 0.05$, and the lifetable is in Table 1, what How what should the payments be?

You are given the value of A_{62} at various interest rates:

i	A_{62}
0.0200	0.537587
0.0294	0.410974
0.0306	0.397701
0.0421	0.293070
0.0500	0.240730

21. A man aged 68 buys a whole life annuity with monthly payments of \$2,000. Using the lifetable in Table 1, and interest rate $i = 0.05$, you calculate $\ddot{a}_{68} = 14.63488$. Use Woolhouse's formula to calculate the expected present value of this annuity.

22. The interest rate is $i = 0.04$. For a standard life aged 59, from the lifetable in Table 1, you calculate $A_{59} = 0.280165$ and $\ddot{a}_{59} = 18.71571$. Calculate the expectated present value of an annuity which pays \$10,000 annually to a select life aged 56.

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66	74	8987.73	8932.10	8862.49	8775.52
26	9997.00	9995.83	9994.40	9992.66	75	8897.04	8836.71	8761.27	8667.10
27	9995.14	9993.90	9992.38	9990.52	76	8798.69	8733.34	8651.66	8549.78
28	9993.16	9991.84	9990.22	9988.24	77	8692.13	8621.41	8533.09	8423.00
29	9991.05	9989.65	9987.92	9985.80	78	8576.81	8500.36	8404.95	8286.16
30	9988.81	9987.30	9985.46	9983.18	79	8452.13	8369.60	8266.68	8138.66
31	9986.40	9984.80	9982.82	9980.38	80	8317.52	8228.53	8117.67	7979.93
32	9983.83	9982.11	9979.99	9977.37	81	8172.36	8076.57	7957.35	7809.41
33	9981.07	9979.23	9976.95	9974.13	82	8016.08	7913.13	7785.15	7626.56
34	9978.11	9976.13	9973.68	9970.64	83	7848.11	7737.67	7600.54	7430.89
35	9974.93	9972.79	9970.16	9966.88	84	7667.89	7549.66	7403.05	7221.99
36	9971.50	9969.20	9966.36	9962.82	85	7474.92	7348.64	7192.27	6999.51
37	9967.80	9965.33	9962.25	9958.44	86	7268.77	7134.21	6967.86	6763.22
38	9963.81	9961.14	9957.82	9953.69	87	7049.07	6906.07	6729.62	6513.04
39	9959.50	9956.61	9953.02	9948.55	88	6815.55	6664.05	6477.46	6249.02
40	9954.84	9951.71	9947.82	9942.98	89	6568.09	6408.10	6211.48	5971.42
41	9949.79	9946.41	9942.19	9936.94	90	6306.70	6138.35	5931.96	5680.73
42	9944.32	9940.66	9936.08	9930.38	91	6031.59	5855.15	5639.41	5377.67
43	9938.39	9934.41	9929.45	9923.26	92	5743.19	5559.08	5334.61	5063.27
44	9931.96	9927.64	9922.25	9915.52	93	5442.15	5250.97	5018.61	4738.86
45	9924.97	9920.28	9914.42	9907.10	94	5129.44	4931.97	4692.79	4406.12
46	9917.37	9912.28	9905.91	9897.94	95	4806.33	4603.54	4358.89	4067.08
47	9909.11	9903.58	9896.65	9887.98	96	4474.39	4267.51	4018.96	3724.10
48	9900.13	9894.11	9886.57	9877.13	97	4135.60	3926.04	3675.44	3379.91
49	9890.36	9883.80	9875.59	9865.30	98	3792.25	3581.66	3331.11	3037.57
50	9879.71	9872.57	9863.63	9852.42	99	3447.02	3237.23	2989.05	2700.39
51	9868.12	9860.34	9850.59	9838.38	100	3102.90	2895.94	2652.63	2371.88
52	9855.48	9847.01	9836.39	9823.08	101	2763.19	2561.21	2325.37	2055.64
53	9841.72	9832.48	9820.90	9806.39	102	2431.39	2236.61	2010.90	1755.27
54	9826.71	9816.64	9804.02	9788.18	103	2111.15	1925.80	1712.81	1474.18
55	9810.34	9799.37	9785.60	9768.33	104	1806.12	1632.34	1434.48	1215.44
56	9792.49	9780.52	9765.51	9746.67	105	1519.82	1359.55	1178.94	981.65
57	9773.03	9759.97	9743.60	9723.05	106	1255.46	1110.36	948.70	774.71
58	9751.79	9737.56	9719.69	9697.28	107	1015.81	887.14	745.58	595.71
59	9728.63	9713.10	9693.62	9669.17	108	802.96	691.49	570.56	444.87
60	9703.36	9686.43	9665.17	9638.51	109	618.23	524.17	423.71	321.41
61	9675.80	9657.33	9634.15	9605.07	110	462.04	385.00	304.13	223.65
62	9645.73	9625.59	9600.31	9568.61	111	333.80	272.80	210.00	149.10
63	9612.94	9590.98	9563.42	9528.85	112	231.99	185.53	138.71	94.62
64	9577.18	9553.24	9523.19	9485.52	113	154.19	120.34	87.07	56.74
65	9538.19	9512.09	9479.35	9438.30	114	97.30	73.90	51.50	31.84
66	9495.69	9467.25	9431.58	9386.86	115	57.78	42.55	28.41	16.52
67	9449.37	9418.39	9379.54	9330.85	116	31.92	22.69	14.43	7.81
68	9398.90	9365.17	9322.87	9269.88	117	16.15	11.04	6.63	3.30
69	9343.95	9307.23	9261.20	9203.55	118	7.34	4.79	2.69	1.21
70	9284.12	9244.18	9194.11	9131.43	119	2.90	1.79	0.93	0.37
71	9219.03	9175.59	9121.17	9053.07	120	0.95	0.55	0.26	0.09
72	9148.24	9101.03	9041.91	8967.97	121	0.23	0.13	0.05	0.01
73	9071.30	9020.03	8955.85	8875.63	122	0.03	0.02	0.01	0.00