ACSC/STAT 3720, Life Contingencies I
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Homework Sheet 4
Model Solutions

Basic Questions

1. *Using the lifetable in Table 1, calculate $\ddot{a}_{[42]+3}$ at interest rate $i = 0.05$. You are given that $A_{[42]+3} = 0.118065$.*

   We have that $\ddot{a}_{[42]+3} = \frac{1.05(1-A_{[42]+3})}{0.05} = 21 \times (1 - 0.118065) = 18.52064$.

2. *An individual aged 43 for whom Table 1 is appropriate, takes out a 5-year term-insurance policy. The annual premiums are \$12,000, payable at the begining of each year. If the current interest rate is $i = 0.04$, what is the expected present value of the premiums received?*

   We have

   $$\ddot{a}_{43:\overline{5}|} = 1+\frac{9936.94}{9942.98}(1.04)^{-1}+\frac{9930.38}{9942.98}(1.04)^{-2}+\frac{9923.26}{9942.98}(1.04)^{-3}+\frac{9915.52}{9942.98}(1.04)^{-4} = 4.624016$$

   so the EPV of premiums received is

   $$12000 \times 4.624016 = \$55,488.19$$

3. *An annuity pays out continuously at a rate of \$3,000 a year until the death of a select individual currently aged 64 to whom the lifetable in Table 1 applies. What is the expected present value of this annuity, using the uniform distribution of deaths assumption, and force of interest $\delta = 0.07$? You are given the following values of $A_{[64]}$ at various interest rates:*

i	$A_{[64]}$
0.06	0.2065638
0.06765865	0.1757489
0.07	0.1675631
0.072508	0.159354

   Force of interest $\delta = 0.07$ gives interest rate $i = e^{0.07} - 1 = 0.072508$, so we have $A_{[64]} = 0.159354$. Under the UDD assumption, this gives $\overline{A}_{[64]} = \frac{0.072508}{0.07} \times 0.159354 = 0.1650638$. We then calculate $\overline{a}_{[64]} = \frac{1-0.1650638}{0.07} = 11.92766$, so the EPV is $3000 \times 11.92766 = \$35,782.98$.

4. *A pension plan pays monthly benefits of \$4,000 to an individual aged 74. What is the expected present value of the benefit under the uniform distribution of deaths assumption, interest rate $i^{(12)} = 0.09$ and the lifetable in Table 1? [These allow us to calculate $A_{74} = 0.197024$.]*

   We calculate $i = 1.0075^{12} - 1 = 0.093807$, so $A^{(12)}_{74} = \frac{0.093807}{0.09} \times 0.197024 = 0.2053579$. We also calculate $d^{(12)} = 12(1 - 1.0075^{-1}) = 0.08933002$. This gives that $\ddot{a}^{(12)}_{74} = \frac{1-0.2053579}{0.08933002} = 8.895577$. The EPV of the payments is therefore $4000 \times 12 \times 8.895577 = \$426,987.72$.

Standard Questions

5. *A pension plan pays an annual benefit of \$24,000 to an individual aged 63, for whom the ultimate part of the lifetable in Table 1 applies. The interest rate is* $i = 0.06$*, which gives* $A_{63} = 0.199371$ *and* $A_{68} = 0.248787$*. The individual wants to change the policy to have guaranteed payments for the first 5 years, but keep the EPV of the benefits the same. What should the new annual payments be?*

   Under the current plan, we have $\ddot{a}_{63} = \frac{(1-0.199371)1.06}{0.06} = 14.14444567$. We also have $\ddot{a}_{68} = \frac{(1-248787)1.06}{0.06} = 13.27142967$. The value of the annuity with guaranteed payments is therefore $\ddot{a}_{\overline{5}|0.06} + {}_5p_{63}(1.06)^{-5}\ddot{a}_{68} = \frac{1.06-1.06^{-5}}{0.06} + \frac{943830}{9638.51} \times 13.27142967(1.06)^{-5} = 14.17629131$. The new annual payments are therefore given by $24000 \times \frac{14.14444567}{14.17629131} = \$23,946.09$.

6. *A man aged 98, to whom the ultimate part of the lifetable in Table 1 applies, wants a pension which will pay \$50,000 in a year's time, and thereafter will provide annual pensions increasing by 6% every year (so the second payment when the man turns 100 will be \$53,000). What is the expected present value of the benefits of this pension if the current interest rate is* $i = 0.07$*?*

   The current interest rate is $i = 0.07$. With inflation of 6%, the "real" rate of interest is $\frac{0.07-0.06}{1.06} = 0.009433962264$. At this rate of interest, we use the standard recurrence to calculate:

$A_{125} = 1$	$A_{124} = 0.990654$	$A_{123} = 0.989625$
$A_{122} = 0.988154$	$A_{121} = 0.987066$	$A_{120} = 0.985956$
$A_{119} = 0.984776$	$A_{118} = 0.983524$	$A_{117} = 0.98218$
$A_{116} = 0.980748$	$A_{115} = 0.97922$	$A_{114} = 0.977589$
$A_{113} = 0.975853$	$A_{112} = 0.974009$	$A_{111} = 0.972052$
$A_{110} = 0.969978$	$A_{109} = 0.967784$	$A_{108} = 0.965467$
$A_{107} = 0.963025$	$A_{106} = 0.960453$	$A_{105} = 0.957751$
$A_{104} = 0.954916$	$A_{103} = 0.951946$	$A_{102} = 0.948841$
$A_{101} = 0.945599$	$A_{100} = 0.94222$	$A_{99} = 0.938704$

   This gives $\ddot{a}_{99} = \frac{(1-0.938704)1.009433962264}{0.009433962264} = 6.558672$. Therefore, if the life survives to age 99, the EPV of this pension at that time will be $50000 \times 6.558672 = \$327,933.60$. The current EPV is therefore $327933.60 \times \frac{3724.10}{4067.08}(1.07)^{-1} = \$280,634.30$.

7. *A woman aged 62 is receiving a pension of \$30,000 at the start of each year. She wants to change this to a monthly pension. If the appropriate life table is in Table 1 and the interest rate is* $i = 0.05$*, then we can calculate* $A_{62} = 0.24073$*. Use Woolhouse's formula to calculate the monthly pension that has the same expected present value.*

   We calculate $\ddot{a}_{62} = \frac{(1-0.24073)1.05}{0.05} = 15.94467$. Woolhouse's formula gives

   $$\ddot{a}_{62}^{(12)} = 15.94467 - \frac{11}{24} - \frac{143}{1728}(\delta + \mu_{62})$$

   We approximate

   $$\mu_{62} = \frac{1}{2}(q_{61} + q_{62}) = 1 - \frac{1}{2}\left(\frac{9669.17}{9697.28} + \frac{9638.51}{9669.17}\right) = 0.003034826987$$

and we have $\delta = \log(1.05) = 0.04879016417$, so Woolhouse's formula gives

$$\ddot{a}_{62}^{(12)} = 15.94467 - \frac{11}{24} - \frac{143}{1728}(0.04879016417 + 0.003034826987) = 15.48204791$$

The current EPV of the pension payments is 30000×15.94467, therefore, the annual rate of payments that give the same EPV is $\frac{30000 \times 15.94467}{15.48204791} = \$30,896.44$. Since there are 12 payments per year, the monthly payments are $\frac{30896.44}{12} = \$2,574.70$.

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66
26	9997.00	9995.83	9994.40	9992.66
27	9995.14	9993.90	9992.38	9990.52
28	9993.16	9991.84	9990.22	9988.24
29	9991.05	9989.65	9987.92	9985.80
30	9988.81	9987.30	9985.46	9983.18
31	9986.40	9984.80	9982.82	9980.38
32	9983.83	9982.11	9979.99	9977.37
33	9981.07	9979.23	9976.95	9974.13
34	9978.11	9976.13	9973.68	9970.64
35	9974.93	9972.79	9970.16	9966.88
36	9971.50	9969.20	9966.36	9962.82
37	9967.80	9965.33	9962.25	9958.44
38	9963.81	9961.14	9957.82	9953.69
39	9959.50	9956.61	9953.02	9948.55
40	9954.84	9951.71	9947.82	9942.98
41	9949.79	9946.41	9942.19	9936.94
42	9944.32	9940.66	9936.08	9930.38
43	9938.39	9934.41	9929.45	9923.26
44	9931.96	9927.64	9922.25	9915.52
45	9924.97	9920.28	9914.42	9907.10
46	9917.37	9912.28	9905.91	9897.94
47	9909.11	9903.58	9896.65	9887.98
48	9900.13	9894.11	9886.57	9877.13
49	9890.36	9883.80	9875.59	9865.30
50	9879.71	9872.57	9863.63	9852.42
51	9868.12	9860.34	9850.59	9838.38
52	9855.48	9847.01	9836.39	9823.08
53	9841.72	9832.48	9820.90	9806.39
54	9826.71	9816.64	9804.02	9788.18
55	9810.34	9799.37	9785.60	9768.33
56	9792.49	9780.52	9765.51	9746.67
57	9773.03	9759.97	9743.60	9723.05
58	9751.79	9737.56	9719.69	9697.28
59	9728.63	9713.10	9693.62	9669.17
60	9703.36	9686.43	9665.17	9638.51
61	9675.80	9657.33	9634.15	9605.07
62	9645.73	9625.59	9600.31	9568.61
63	9612.94	9590.98	9563.42	9528.85
64	9577.18	9553.24	9523.19	9485.52
65	9538.19	9512.09	9479.35	9438.30
66	9495.69	9467.25	9431.58	9386.86
67	9449.37	9418.39	9379.54	9330.85
68	9398.90	9365.17	9322.87	9269.88
69	9343.95	9307.23	9261.20	9203.55
70	9284.12	9244.18	9194.11	9131.43
71	9219.03	9175.59	9121.17	9053.07
72	9148.24	9101.03	9041.91	8967.97
73	9071.30	9020.03	8955.85	8875.63

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
74	8987.73	8932.10	8862.49	8775.52
75	8897.04	8836.71	8761.27	8667.10
76	8798.69	8733.34	8651.66	8549.78
77	8692.13	8621.41	8533.09	8423.00
78	8576.81	8500.36	8404.95	8286.16
79	8452.13	8369.60	8266.68	8138.66
80	8317.52	8228.53	8117.67	7979.93
81	8172.36	8076.57	7957.35	7809.41
82	8016.08	7913.13	7785.15	7626.56
83	7848.11	7737.67	7600.54	7430.89
84	7667.89	7549.66	7403.05	7221.99
85	7474.92	7348.64	7192.27	6999.51
86	7268.77	7134.21	6967.86	6763.22
87	7049.07	6906.07	6729.62	6513.04
88	6815.55	6664.05	6477.46	6249.02
89	6568.09	6408.10	6211.48	5971.42
90	6306.70	6138.35	5931.96	5680.73
91	6031.59	5855.15	5639.41	5377.67
92	5743.19	5559.08	5334.61	5063.27
93	5442.15	5250.97	5018.61	4738.86
94	5129.44	4931.97	4692.79	4406.12
95	4806.33	4603.54	4358.89	4067.08
96	4474.39	4267.51	4018.96	3724.10
97	4135.60	3926.04	3675.44	3379.91
98	3792.25	3581.66	3331.11	3037.57
99	3447.02	3237.23	2989.05	2700.39
100	3102.90	2895.94	2652.63	2371.88
101	2763.19	2561.21	2325.37	2055.64
102	2431.39	2236.61	2010.90	1755.27
103	2111.15	1925.80	1712.81	1474.18
104	1806.12	1632.34	1434.48	1215.44
105	1519.82	1359.55	1178.94	981.65
106	1255.46	1110.36	948.70	774.71
107	1015.81	887.14	745.58	595.71
108	802.96	691.49	570.56	444.87
109	618.23	524.17	423.71	321.41
110	462.04	385.00	304.13	223.65
111	333.80	272.80	210.00	149.10
112	231.99	185.53	138.71	94.62
113	154.19	120.34	87.07	56.74
114	97.30	73.90	51.50	31.84
115	57.78	42.55	28.41	16.52
116	31.92	22.69	14.43	7.81
117	16.15	11.04	6.63	3.30
118	7.34	4.79	2.69	1.21
119	2.90	1.79	0.93	0.37
120	0.95	0.55	0.26	0.09
121	0.23	0.13	0.05	0.01
122	0.03	0.02	0.01	0.00