ACSC/STAT 3720, Life Contingencies I
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Homework Sheet 5
Model Solutions

Basic Questions

1. *An insurance company offers a 10-year term policy with benefit \$800,000 payable at the end of the year of death. The premium for this policy for a select individual aged 37 for whom the lifetable in Table 1 is appropriate, is \$400, payable at the start of each year. If the current interest rate is $i = 0.07$, what is the probability that the present value of future loss for this policy exceeds \$150,000?*

   If the individual dies in year n, then the present value of the benefits is $800000(1.07)^{-n}$, and the present value of the premiums paid is $400\frac{1.07-1.07^{1-n}}{0.07}$. The present value of future loss is therefore $800000(1.07)^{-n} - 400\frac{1.07-1.07^{1-n}}{0.07} = 806114.29(1.07)^{-n} - 6114.29$. We see that for $n \leqslant 10$, this is more than \$150,000, so the PVFL exceeds \$150,000 whenever the life dies within 10 years. The probability of this is $1 - \frac{9915.52}{9967.80} = 0.005244889$.

2. *An insurance company offers a 5-year term policy with death benefit \$400,000 payable at the end of the year of death. If the interest rate is $i = 0.06$, calculate the annual premium for this policy for a select individual aged 36, using the lifetable in Table 1 and the equivalence principle.*

   For a select individual aged 36, we use the standard recurrence to calculate

$$\begin{aligned}
A^{1}_{41:\overline{0}|} &= 0\\
A^{1}_{40:\overline{1}|} &= 0.000449983\\
A^{1}_{39:\overline{2}|} &= 0.000840022\\
A^{1}_{[36]+2:\overline{3}|} &= 0.001127282\\
A^{1}_{[36]+1:\overline{4}|} &= 0.001331923\\
A^{1}_{[36]:\overline{5}|} &= 0.001473842
\end{aligned}$$

   We also get $A_{[36]:\overline{5}|} = A^{1}_{[36]:\overline{5}|} + {}_5p_{[36]}(1.06)^{-5} = 0.7473973$, so $\ddot{a}_{[36]:\overline{5}|} = \frac{(1-0.7473973)(1.06)}{0.06} = 4.462648$. The premium is therefore $\frac{400000\times 0.001473842}{4.462648} = \$132.10$.

3. *The current interest rate is $i = 0.04$. An individual aged 42 to whom the ultimate part of the lifetable in Table 1 applies, wants to purchase a 15 year endowment insurance policy. The benefit of this policy should be \$500,000 either at the end of the year of death or at the end of 15 years. The initial costs to the insurance company are \$3,000 plus 10% of the first premium. Renewal costs are 4% of subsequent premiums. Calculate the Gross annual premiums for this policy. You calculate $A_{42} = 0.154332$ and $A_{57} = 0.261926$.*

We calculate $A_{42:\overline{15}|} = A_{42} + {}_{15}p_{42}(1.04)^{-15}(1-A_{57}) = 0.154332 + \frac{9788.18}{9948.55}(1-0.261926)(1.04)^{-15} = 0.5575519$. The EPV of benefits is therefore $500000 \times 0.5575519 = \$278,775.96$. We also calculate $\ddot{a}_{42:\overline{15}|} = \frac{(1-A_{42:\overline{15}|})1.04}{0.04} = 9.2914097$, so if the premium is P, the EPV of premiums minus expenses is $9.2914097(P(1-0.04)) - 0.06P - 3000 = 8.8597533P - 3000$. We therefore get

$$P = \frac{278775.96 + 3000}{8.8597533} = \$31,804.04$$

Standard Questions

4. *A select individual aged 55, to whom the lifetable in Table 1 applies, wants to purchase a whole life insurance policy. She can afford to pay annual premiums of \$4,100 from now until age 80 (so she pays the last premium at age 79). The interest rate is $i = 0.05$, which gives $A_{[55]} = 0.1801786$ and $A_{[77]+3} = 0.457434$. Using the equivalence principal to calculate net premiums, what is the largest death benefit that she can afford to purchase?*

   With death benefit D, the EPV of death benefits is $DA_{[55]} = 0.1801786D$. We also calculate $A_{[55]:\overline{25}|} = A_{[55]} + {}_{25}\,p_{[55]}(1.05)^{-25}(1-A_{80}) = 0.1801786 + \frac{8423.00}{9810.34}(1-0.457434)(1.05)^{-25} = 0.31944198$, so $\ddot{a}_{[55]:\overline{25}|} = \frac{(1-0.31944198)1.05}{0.05} = 14.291718$. With a premium of \$4,100, the EPV of premiums is $4100 \times 14.291718 = \$58,596.05$. The death benefit should therefore be $\frac{58596.05}{0.1801786} = \$325,210.90$.

5. *An individual aged 51 is paying premiums of \$200 a month for a whole life insurance policy which pays benefits at the end of the month of death. If the individual's mortality follows the ultimate part of Table 1, and the interest rate is $i^{(12)} = 0.06$, so that $A_{51} = 0.112588$, calculate the equivalent annual premiums (for a policy which pays benefits at the end of the year of death) using:*

   (a) Uniform distribution of deaths

   Using UDD, we calculate $i = 1.005^{12} - 1 = 0.06167781$, so $A^{(12)}_{51} = \frac{i}{i^{(12)}}A_{51} = \frac{0.06167781}{0.06} \times 0.112588 = 0.115736358$. We also calculate $d^{(12)} = 12(1-(1.005)^{-1}) = 0.05970149254$. From this, we get $\ddot{a}_{51} = \frac{1-0.115736358}{0.05970149254} = 14.811416$, so the EPV of premiums is therefore $12 \times 200 \times 14.811416 = \$35,547.40$, so the death benefit should be $\frac{35547.40}{0.115736358} = \$307,141.15$. For the annual policy, the EPV of this death benefit is $307141.15 \times 0.112588 = \$34,580.41$. We also get $\ddot{a}_{51} = \frac{(1-0.112588)1.06167781}{0.06167781} = 15.2752765$. Therefore, the premium is $\frac{34580.41}{15.2752765} = \$2,263.82$.

   (b) Woolhouse's formula

   As in part (a), we have $\ddot{a}_{51} = 15.2752765$. Woolhouse's formula therefore gives

$$\ddot{a}^{(12)}_{51} = 15.2752765 - \frac{11}{24} - \frac{143}{1728}(\mu_{51} + \delta)$$

   We approximate

$$\mu_{51} = \frac{1}{2}(q_{50} + q_{51}) = \frac{1}{2}\left(\frac{10.85}{9887.98} + \frac{11.83}{9877.13}\right) = 0.001147504102$$

   and calculate $\delta = 12\log(1.005) = 0.05985049813$, so Woolhouse's formula gives

$$\ddot{a}^{(12)}_{51} = 15.2752765 - \frac{11}{24} - \frac{143}{1728}(0.001147504102 + 0.05985049813) = 14.8118953$$

We then calculate

$$A_{51}^{(12)} = 1 - d^{(12)}\ddot{a}_{51}^{(12)} = 1 - 14.8118953 \times 12(1 - 1.005^{-1}) = 0.1157077433$$

This gives that the death benefit is $\frac{12\times200\times14.8118953}{0.1157077433} = \$307,227.05$. The annual premium for this annual death benefit is therefore $\frac{307227.05\times0.112588}{15.2752765} = \$2,264.45$.

6. *An insurance company provides a regular annual premium annuity contract to a select individual aged 44, using the lifetable in Table 1. The interest rate is $i = 0.06$. This gives that $A_{[62]+3} = 0.218135$, $A_{[72]+3} = 0.332028$ and $A_{[44]} = 0.07872046$. The individual will pay annual net premiums until age 65 (so the last premium will be at age 64). From age 65, they will receive an annuity of \$30,000 at the start of each year. This annuity is guaranteed for 10 years (regardless of whether the individual survives to age 65). What is the probability that the insurance company makes a net profit on this policy?*

   The present value of the guaranteed annuity payments is

$$30000a_{\overline{10}|0.06}(1.06)^{-20} = 30000 \times \frac{1 - 1.06^{-10}}{0.06}(1.06)^{-20} = \$68,847.31$$

   [It's slightly easier to use the immediate annuity here, instead of the annuity-due, but either can be used.] The EPV of the annuity payments after the guaranteed annuity is

$$30000\ddot{a}_{75}(1.06)^{-31}_{31}p_{[44]} = 30000\frac{(1 - 0.332028)(1.06)}{0.06}(1.06)^{-31} \times \frac{8967.97}{9931.96} = \$52,506.30$$

   We also calculate

$$A_{[44]:\overline{26}|} = A_{[44]}+{}_{26}p_{[44]}(1.06)^{-26}(1-A_{65}) = 0.07872046+\frac{9568.61}{9931.96}(1.06)^{-26}(1-0.218135) = 0.3002963$$

   so $\ddot{a}_{[44]:\overline{26}|} = \frac{1.06(1-0.3002963)}{0.06} = 12.3614313.35079096$. The required premiums are therefore $\frac{52506.30+68847.30}{12.36143} = \$9,817.12$.

   In order for the insurance company to make a profit, the present value of premiums received must be at least the present value of the guaranteed annuity. That is, if N is the number of premiums received, then we need

$$\begin{aligned}
9817.12\ddot{a}_{\overline{N}|0.06} &> 68,847.30 \\
9817.12(1.06)\frac{1 - 1.06^{-N}}{0.06} &> 68,847.30 \\
1 - 1.06^{-N} &> 0.3969613 \\
1.06^{-N} &< 0.6030387 \\
N &> -\frac{\log(0.6030387)}{\log(1.06)} = 8.679997
\end{aligned}$$

   Therefore, the insurance company must receive at least 9 premiums, so the life must survive for 8 years.

On the other hand, if the life survives to age 64, the value at age 64 of all premiums received is

$$9817.12 s_{\overline{21}|0.06} = 9817.12 \frac{1.06^{21} - 1}{0.06} = \$382,796.28$$

If the company makes M payments in the annuity, the present value of these payments at age 64 is $30000 a_{\overline{M}|0.06} = 300000 \frac{1-1.06^{-M}}{0.06}$. We therefore need to solve

$$\begin{aligned} 30000 \frac{1 - 1.06^{-M}}{0.06} &> 382796.28 \\ 1 - 1.06^{-M} &> 0.7655926 \\ 1.06^{-M} &< 0.2344074 \\ M &> -\frac{\log(0.2344074)}{\log(1.06)} = 24.89654 \end{aligned}$$

So the insurance company makes a loss if it pays 25 or more payments in the annuity, which happens if the life survives to age 89. Therefore, for the insurance company to make a profit, the life must die between ages 52 and 89. The probability of this is $\frac{9865.30-6763.22}{9931.96} = 0.3123331$.

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66	74	8987.73	8932.10	8862.49	8775.52
26	9997.00	9995.83	9994.40	9992.66	75	8897.04	8836.71	8761.27	8667.10
27	9995.14	9993.90	9992.38	9990.52	76	8798.69	8733.34	8651.66	8549.78
28	9993.16	9991.84	9990.22	9988.24	77	8692.13	8621.41	8533.09	8423.00
29	9991.05	9989.65	9987.92	9985.80	78	8576.81	8500.36	8404.95	8286.16
30	9988.81	9987.30	9985.46	9983.18	79	8452.13	8369.60	8266.68	8138.66
31	9986.40	9984.80	9982.82	9980.38	80	8317.52	8228.53	8117.67	7979.93
32	9983.83	9982.11	9979.99	9977.37	81	8172.36	8076.57	7957.35	7809.41
33	9981.07	9979.23	9976.95	9974.13	82	8016.08	7913.13	7785.15	7626.56
34	9978.11	9976.13	9973.68	9970.64	83	7848.11	7737.67	7600.54	7430.89
35	9974.93	9972.79	9970.16	9966.88	84	7667.89	7549.66	7403.05	7221.99
36	9971.50	9969.20	9966.36	9962.82	85	7474.92	7348.64	7192.27	6999.51
37	9967.80	9965.33	9962.25	9958.44	86	7268.77	7134.21	6967.86	6763.22
38	9963.81	9961.14	9957.82	9953.69	87	7049.07	6906.07	6729.62	6513.04
39	9959.50	9956.61	9953.02	9948.55	88	6815.55	6664.05	6477.46	6249.02
40	9954.84	9951.71	9947.82	9942.98	89	6568.09	6408.10	6211.48	5971.42
41	9949.79	9946.41	9942.19	9936.94	90	6306.70	6138.35	5931.96	5680.73
42	9944.32	9940.66	9936.08	9930.38	91	6031.59	5855.15	5639.41	5377.67
43	9938.39	9934.41	9929.45	9923.26	92	5743.19	5559.08	5334.61	5063.27
44	9931.96	9927.64	9922.25	9915.52	93	5442.15	5250.97	5018.61	4738.86
45	9924.97	9920.28	9914.42	9907.10	94	5129.44	4931.97	4692.79	4406.12
46	9917.37	9912.28	9905.91	9897.94	95	4806.33	4603.54	4358.89	4067.08
47	9909.11	9903.58	9896.65	9887.98	96	4474.39	4267.51	4018.96	3724.10
48	9900.13	9894.11	9886.57	9877.13	97	4135.60	3926.04	3675.44	3379.91
49	9890.36	9883.80	9875.59	9865.30	98	3792.25	3581.66	3331.11	3037.57
50	9879.71	9872.57	9863.63	9852.42	99	3447.02	3237.23	2989.05	2700.39
51	9868.12	9860.34	9850.59	9838.38	100	3102.90	2895.94	2652.63	2371.88
52	9855.48	9847.01	9836.39	9823.08	101	2763.19	2561.21	2325.37	2055.64
53	9841.72	9832.48	9820.90	9806.39	102	2431.39	2236.61	2010.90	1755.27
54	9826.71	9816.64	9804.02	9788.18	103	2111.15	1925.80	1712.81	1474.18
55	9810.34	9799.37	9785.60	9768.33	104	1806.12	1632.34	1434.48	1215.44
56	9792.49	9780.52	9765.51	9746.67	105	1519.82	1359.55	1178.94	981.65
57	9773.03	9759.97	9743.60	9723.05	106	1255.46	1110.36	948.70	774.71
58	9751.79	9737.56	9719.69	9697.28	107	1015.81	887.14	745.58	595.71
59	9728.63	9713.10	9693.62	9669.17	108	802.96	691.49	570.56	444.87
60	9703.36	9686.43	9665.17	9638.51	109	618.23	524.17	423.71	321.41
61	9675.80	9657.33	9634.15	9605.07	110	462.04	385.00	304.13	223.65
62	9645.73	9625.59	9600.31	9568.61	111	333.80	272.80	210.00	149.10
63	9612.94	9590.98	9563.42	9528.85	112	231.99	185.53	138.71	94.62
64	9577.18	9553.24	9523.19	9485.52	113	154.19	120.34	87.07	56.74
65	9538.19	9512.09	9479.35	9438.30	114	97.30	73.90	51.50	31.84
66	9495.69	9467.25	9431.58	9386.86	115	57.78	42.55	28.41	16.52
67	9449.37	9418.39	9379.54	9330.85	116	31.92	22.69	14.43	7.81
68	9398.90	9365.17	9322.87	9269.88	117	16.15	11.04	6.63	3.30
69	9343.95	9307.23	9261.20	9203.55	118	7.34	4.79	2.69	1.21
70	9284.12	9244.18	9194.11	9131.43	119	2.90	1.79	0.93	0.37
71	9219.03	9175.59	9121.17	9053.07	120	0.95	0.55	0.26	0.09
72	9148.24	9101.03	9041.91	8967.97	121	0.23	0.13	0.05	0.01
73	9071.30	9020.03	8955.85	8875.63	122	0.03	0.02	0.01	0.00