ACSC/STAT 3720, Life Contingencies I
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Homework Sheet 7
Model Solutions

Basic Questions

1. *A woman aged 36, for whom the ultimate part of the lifetable in Table 1 is appropriate, buys a 10-year term insurance policy with a death benefit of \$800,000. (The policy uses a net annual premium.) Five years later, she wants to surrender the policy. The interest rate is $i = 0.04$. If the insurance company pays a cash surrender value of 80% of the net policy value, how much does she receive?*

   Using the standard recurrence, we have:

$$
\begin{array}{lll}
A^1_{46:\overline{0}|} = 0 & A^1_{45:\overline{1}|} = 0.000689415 & A^1_{44:\overline{2}|} = 0.00129723 \\
A^1_{43:\overline{3}|} = 0.00183068 & A^1_{42:\overline{4}|} = 0.00229763 & A^1_{41:\overline{5}|} = 0.00270465 \\
A^1_{40:\overline{6}|} = 0.00305802 & A^1_{39:\overline{7}|} = 0.0033628 & A^1_{38:\overline{8}|} = 0.00362383 \\
A^1_{37:\overline{9}|} = 0.00384574 & A^1_{36:\overline{10}|} = 0.00403298 &
\end{array}
$$

   This gives

$$\ddot{a}_{36:\overline{10}} = \frac{1.04(1 - A^1_{36:\overline{10}|} - {}_{10}p_{36}(1.04)^{-10})}{0.04} = 26\left(1 - 0.00403298 - \frac{9923.26}{9974.13}(1.04)^{-10}\right) = 8.42005735$$

   so the annual net premium is $800000 \times 0.004032988.42005735 = \$383.18$.

   After 5 years, we calculate

$$\ddot{a}_{41:\overline{5}} = \frac{1.04(1 - A^1_{41:\overline{5}|} - {}_{5}p_{41}(1.04)^{-5})}{0.04} = 26\left(1 - 0.00270465 - \frac{9923.26}{9953.69}(1.04)^{-5}\right) = 4.624906105$$

   so the policy value is $800000 \times 0.00270465 - 383.18 \times 4.624906105 = \$391.56$.

   The surrender value is 80% of the policy value, which is $0.80 \times 391.56 = \$313.24$.

2. *An insurance company sells a 15-year term insurance policy to a life aged 29 to whom the ultimate part of the lifetable in Table 1 applies. The death benefit is \$180,000 in the first two years, \$160,000 in the second to fifth year and \$140,000 for the remaining 10 years. The premiums are \$96.85 for the first three years, and \$26.64 for the remaining twelve years. The interest rate is $i = 0.05$ for the first 4 years, and $i = 0.07$ for the remaining 11 years. Calculate the retrospective policy value after 2 years.*

   At the end of year 2, the expected accumulated value of premiums received is

$$96.85\left((1.05)^2 + \frac{9990.52}{9992.66}(1.05)\right) = \$208.45$$

The expected accumulated value of death benefits paid is

$$180000\left(\frac{2.14}{9992.66}(1.05)+\frac{2.28}{9992.66}\right)=180000\times\frac{4.527}{9992.66}=\$81.55$$

The retrospective policy value per policy purchased is therefore $208.45-81.55=\$126.90$. The retrospective policy value is therefore $\frac{9992.66}{9988.24}\times 126.90=\$126.96$.

3. *A man aged 61, who is a select life on Table 1 buys a 10-year term insurance with a benefit of \$700,000. The interest rate is $i=0.04$, which gives $A_{[61]}=0.2979703$, $A_{[61]+1}=0.3085693$, $A_{64}=0.330027$, $A_{65}=0.340726$ and $A_{71}=0.409741$. Using a Full preliminary term of 1 year, calculate the policy value after 4 years.*

   Since $A_{[61]+1}=0.3085693$, we have $\ddot{a}_{[61]+1}=26(1-0.3085693)=17.9772$. We also have

$$A_{[61]+1:\overline{9}|}=A_{[61]+1}+(1.04)^{-9}_{\ 9}p_{[61]+1}(1-A_{71})=0.3085693+(1.04)^{-9}\frac{9269.88}{9657.33}(1-0.409741)=0.7066394$$

   and

$$A^{1}_{[61]+1:\overline{9}|}=A_{[61]+1}-(1.04)^{-9}_{\ 9}p_{[61]+1}A_{71}=0.3085693-0.409741(1.04)^{-9}\frac{9269.88}{9657.33}=0.03224034$$

   This gives $\ddot{a}_{[61]+1:\overline{9}|}=26(1-0.7066394)=7.627376$. This means the premium for the last 9 years of the policy is $\frac{700000\times 0.03224034}{7.627376}=\$2,958.85$.
   4 years from the start of the policy, we have

$$A_{65:\overline{6}|}=A_{65}+(1.04)^{-6}_{\ 6}p_{65}(1-A_{71})=0.340726+(1.04)^{-6}\frac{9269.88}{9568.61}(1-0.409741)=0.7926525$$

   and

$$A^{1}_{65:\overline{6}|}=A_{65}-A_{71}(1.04)^{-6}_{\ 6}p_{65}=0.340726-0.409741(1.04)^{-6}\frac{9269.88}{9568.61}=0.02701146$$

   and therefore

$$\ddot{a}_{65:\overline{6}|}=26(1-0.7926525)=5.391035$$

   so the policy value is $700000\times 0.02701146-2,958.85\times 5.391035=\$2,956.77$.

Standard Questions

4. *An insurance company is valuing its policies. It finds that the total value of a large group of 200 policies was \$1,100,000. The total annual premium for all these policies is \$96,000. The interest rate is $i=0.06$. All of the policies have a mortality rate $q_x=0.00029$. 130 of the policies have death benefit \$900,000; 50 have death benefit \$1,500,000; and the remaining 20 have death benefit \$1,300,000. There are no expenses associated with the policies, and during the following year none of the policy holders dies. What is the total value of all the remaining policies the following year?*

   After the premiums, the company should have \$1,196,000. At the end of the year, the company should have $1196000\times 1.06=\$1,267,760$. The expected death benefit payments are $(900000\times 130+1500000\times 50+1300000\times 20)\times 0.00029=\$63,220$. Therefore, the expected policy value of all the policies is $1267760-63220=\$1,204,540$. The expected number of policies is $200\times 0.99971=199.942$, but the actual number is 200, so the total policy value is $\frac{200}{199.942}\times 1204540=\$1,204,889.42$

5. *A man aged 38, who is a select life on Table 1 buys a 10-year annual term insurance policy with a death benefit of $500,000. The interest rate is* $i = 0.06$*, so* $A^{1}_{[38]:\overline{10}|} = 0.00396899$*. The insurance company pays a cash surrender value of 85% of the policy value. If he is still a select life at age 45, would he save money by surrendering his current policy and buying a new 3-year policy for the same coverage?*

   We calculate

$$\begin{aligned} A_{[38]:\overline{10}|} &= A^{1}_{[38]:\overline{10}|} + {}_{10}p_{[38]}(1.06)^{-10} = 0.00396899 + \frac{9907.10}{9963.81}(1.06)^{-10} = 0.5591856 \\ \ddot{a}_{[38]:\overline{10}|} &= \frac{1.06}{0.06}(1 - 0.5591856) = 7.787721 \end{aligned}$$

   The premium for the policy is therefore $\frac{500000\times 0.00396899}{7.787721} = \$254.82$.

   After 7 years, we calculate the policy value is

$$\begin{aligned} 500000\left(\frac{7.12}{9930.38}(1.06)^{-1} + \frac{7.74}{9930.38}(1.06)^{-2} + \frac{8.42}{9930.38}(1.06)^{-3}\right) \\ -254.82\left(1 + \frac{9923.26}{9930.38}(1.06)^{-1} + \frac{9915.52}{9930.38}(1.06)^{-2}\right) = \$319.50 \end{aligned}$$

   The surrender value is therefore $319.50 \times 0.85 = 271.58$.

   As a select life at age 45, we calculate

$$\begin{aligned} A^{1}_{[45]:\overline{3}|} &= \frac{4.69(1.06)^{-1} + 5.86(1.06)^{-2} + 7.32(1.06)^{-3}}{9924.97} = 0.001590526 \\ \ddot{a}_{[45]:\overline{3}|} &= 1 + \frac{9920.28}{9924.97}(1.06)^{-1} + \frac{9914.42}{9924.97}(1.06)^{-2} = 2.832001 \end{aligned}$$

   So with an initial payment of $271.58, the new premium is

$$\frac{500000 \times 0.001590526 - 271.58}{2.832001} = \$184.92$$

   Since this is lower than the original premium, it would be worthwhile for him to surrender and repurchase.

6. *A man bought a whole life insurance policy 6 years ago. At the time, his age was 42, and his mortality followed the ultimate part of the lifetable in Table 1. The benefit of the policy was $800,000. The interest rate is* $i = 0.06$*. He now wants to convert the policy to a paid-up term policy with a term of 5 years. The insurance company offers a cash surrender value of 70% of the policy value. What is the death benefit of the new insurance contract? [*$A_{42} = 0.0714153$*,* $A_{48} = 0.0969315$ *and* $A_{53} = 0.124241$*.]*

   We calculate

$$\ddot{a}_{42} = \frac{1.06(1 - 0.0714153)}{0.06} = 16.405$$

The premium is therefore $\frac{800000\times 0.0714153}{16.405} = \$3,482.61$. After 6 years, we calculate

$$\ddot{a}_{48} = \frac{1.06(1-0.0969315)}{0.06} = 15.95421$$

so the policy value is $800000 \times 0.0969315 - 15.95421 \times 3482.61 = \$21,982.89$. The surrender value is $21982.89 \times 0.70 = \$15,388.02$. We also calculate

$$\begin{aligned}
\ddot{a}_{53} &= \frac{1.06(1-0.124241)}{0.06} = 15.47174 \\
\ddot{a}_{48:\overline{5}|} &= \ddot{a}_{48} - {}_5p_{48}(1.06)^{-5}\ddot{a}_{53} = 15.95421 - 15.47174 \times \frac{9852.42}{9907.10}(1.06)^{-5} = 4.456635 \\
A^1_{48:\overline{5}|} &= A_{48} - {}_5p_{48}(1.06)^{-5}A_{53} = 0.0969315 - 0.124241 \times \frac{9852.42}{9907.10}(1.06)^{-5} = 0.004603807
\end{aligned}$$

The EPV of premiums plus the surrender value is $4.456635 \times 3,482.61 + 15388.02 = \$30,908.75$. The new death benefit is therefore given by $\frac{30908.75}{0.004603807} = \$6,713,736$.

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66	74	8987.73	8932.10	8862.49	8775.52
26	9997.00	9995.83	9994.40	9992.66	75	8897.04	8836.71	8761.27	8667.10
27	9995.14	9993.90	9992.38	9990.52	76	8798.69	8733.34	8651.66	8549.78
28	9993.16	9991.84	9990.22	9988.24	77	8692.13	8621.41	8533.09	8423.00
29	9991.05	9989.65	9987.92	9985.80	78	8576.81	8500.36	8404.95	8286.16
30	9988.81	9987.30	9985.46	9983.18	79	8452.13	8369.60	8266.68	8138.66
31	9986.40	9984.80	9982.82	9980.38	80	8317.52	8228.53	8117.67	7979.93
32	9983.83	9982.11	9979.99	9977.37	81	8172.36	8076.57	7957.35	7809.41
33	9981.07	9979.23	9976.95	9974.13	82	8016.08	7913.13	7785.15	7626.56
34	9978.11	9976.13	9973.68	9970.64	83	7848.11	7737.67	7600.54	7430.89
35	9974.93	9972.79	9970.16	9966.88	84	7667.89	7549.66	7403.05	7221.99
36	9971.50	9969.20	9966.36	9962.82	85	7474.92	7348.64	7192.27	6999.51
37	9967.80	9965.33	9962.25	9958.44	86	7268.77	7134.21	6967.86	6763.22
38	9963.81	9961.14	9957.82	9953.69	87	7049.07	6906.07	6729.62	6513.04
39	9959.50	9956.61	9953.02	9948.55	88	6815.55	6664.05	6477.46	6249.02
40	9954.84	9951.71	9947.82	9942.98	89	6568.09	6408.10	6211.48	5971.42
41	9949.79	9946.41	9942.19	9936.94	90	6306.70	6138.35	5931.96	5680.73
42	9944.32	9940.66	9936.08	9930.38	91	6031.59	5855.15	5639.41	5377.67
43	9938.39	9934.41	9929.45	9923.26	92	5743.19	5559.08	5334.61	5063.27
44	9931.96	9927.64	9922.25	9915.52	93	5442.15	5250.97	5018.61	4738.86
45	9924.97	9920.28	9914.42	9907.10	94	5129.44	4931.97	4692.79	4406.12
46	9917.37	9912.28	9905.91	9897.94	95	4806.33	4603.54	4358.89	4067.08
47	9909.11	9903.58	9896.65	9887.98	96	4474.39	4267.51	4018.96	3724.10
48	9900.13	9894.11	9886.57	9877.13	97	4135.60	3926.04	3675.44	3379.91
49	9890.36	9883.80	9875.59	9865.30	98	3792.25	3581.66	3331.11	3037.57
50	9879.71	9872.57	9863.63	9852.42	99	3447.02	3237.23	2989.05	2700.39
51	9868.12	9860.34	9850.59	9838.38	100	3102.90	2895.94	2652.63	2371.88
52	9855.48	9847.01	9836.39	9823.08	101	2763.19	2561.21	2325.37	2055.64
53	9841.72	9832.48	9820.90	9806.39	102	2431.39	2236.61	2010.90	1755.27
54	9826.71	9816.64	9804.02	9788.18	103	2111.15	1925.80	1712.81	1474.18
55	9810.34	9799.37	9785.60	9768.33	104	1806.12	1632.34	1434.48	1215.44
56	9792.49	9780.52	9765.51	9746.67	105	1519.82	1359.55	1178.94	981.65
57	9773.03	9759.97	9743.60	9723.05	106	1255.46	1110.36	948.70	774.71
58	9751.79	9737.56	9719.69	9697.28	107	1015.81	887.14	745.58	595.71
59	9728.63	9713.10	9693.62	9669.17	108	802.96	691.49	570.56	444.87
60	9703.36	9686.43	9665.17	9638.51	109	618.23	524.17	423.71	321.41
61	9675.80	9657.33	9634.15	9605.07	110	462.04	385.00	304.13	223.65
62	9645.73	9625.59	9600.31	9568.61	111	333.80	272.80	210.00	149.10
63	9612.94	9590.98	9563.42	9528.85	112	231.99	185.53	138.71	94.62
64	9577.18	9553.24	9523.19	9485.52	113	154.19	120.34	87.07	56.74
65	9538.19	9512.09	9479.35	9438.30	114	97.30	73.90	51.50	31.84
66	9495.69	9467.25	9431.58	9386.86	115	57.78	42.55	28.41	16.52
67	9449.37	9418.39	9379.54	9330.85	116	31.92	22.69	14.43	7.81
68	9398.90	9365.17	9322.87	9269.88	117	16.15	11.04	6.63	3.30
69	9343.95	9307.23	9261.20	9203.55	118	7.34	4.79	2.69	1.21
70	9284.12	9244.18	9194.11	9131.43	119	2.90	1.79	0.93	0.37
71	9219.03	9175.59	9121.17	9053.07	120	0.95	0.55	0.26	0.09
72	9148.24	9101.03	9041.91	8967.97	121	0.23	0.13	0.05	0.01
73	9071.30	9020.03	8955.85	8875.63	122	0.03	0.02	0.01	0.00