ACSC/STAT 3720, Life Contingencies I
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Sample Final Examination Model Solutions

1. *For a life aged 32, for whom Table 1 is appropriate, an insurance company provides a whole life insurance policy with annual premiums \$3,803 and death benefits of \$600,000. The interest rate is $i = 0.03$. What is the probability that the policy makes a profit?*

   If the life dies during the nth year, the present value of the premiums is $3803\ddot{a}_{\overline{n}|0.03} = 3803 \times 1.03\frac{1-1.03^{-n}}{0.03}$, while the present value of the benefit is $60000(1.03)^{-n}$. The policy therefore makes a profit if $3803 \times 1.03\frac{1-1.03^{-n}}{0.03} \geqslant 600000(1.03)^{-n}$ which gives

$$\begin{aligned}
130569.7 &\geqslant 730569.7(1.03)^{-n} \\
(1.03)^{-n} &\geqslant 0.1787231 \\
n &\geqslant -\frac{\log(0.1787231)}{\log(1.03)} = 58.25
\end{aligned}$$

   So the policy makes a profit if the life survives for 58 years. The probability of this is $\frac{6513.04}{9985.80} = 0.6522302$.

2. *An insurance company sells a 15-year endowment policy to a life aged 47. The policyholder can afford annual premiums of \$1,400. The interest rate is $i = 0.04$. Calculate the death benefit that matches this premium under the equivalence principle. [$A_{47:\overline{15}|} = 0.558772$.]*

   $A_{47:\overline{15}|} = 0.558772$, so we have $\ddot{a}_{47:\overline{15}|} = 26(1 - 0.558772) = 11.47193$. The benefit is therefore

$$\frac{11.47193 \times 1400}{0.558772} = \$28,742.85$$

3. *An insurance company sells a 10-year term insurance policy to a life aged 51. The death benefit is \$750,000. Calculate the net premium using the equivalence principle at an interest rate $i = 0.06$. You are given $A^1_{51:\overline{10}|} = 0.0128582$.*

   Since $A^1_{51:\overline{10}|} = 0.0128582$, we calculate $A_{51:\overline{10}|} = A^1_{51:\overline{10}|} + {}_{10}p_{51}(1.06)^{-10} = 0.0128582 + \frac{9697.28}{9877.13}(1.06)^{-10} = 0.5610853$. This gives $\ddot{a}_{51:\overline{10}|} = \frac{1.06}{0.06}(1 - 0.5610853) = 7.754159$. The premium is therefore given by $\frac{750000\times0.0128582}{7.754159} = \$1,243.67$.

4. *An insurance company sells a whole-life insurance policy to a life aged 39. The interest rate is $i = 0.05$. The death benefit is \$350,000, and premiums are payable in advance until age 80. Calculate the annual net premium using the equivalence principle. [You calculate that $A_{39} = 0.0905389$ and $A_{80} = 0.457434$.]*

   We have that $A_{39:\overline{41}|} = A_{39} + {}_{41}p_{39}(1.05)^{-41}(1 - A_{80}) = 0.0905389 + \frac{8423.00}{9962.82}(1.05)^{-41}(1 - 0.457434) = 0.1525938$. This gives $\ddot{a}_{39:\overline{41}|} = 21(1 - 0.1525938) = 17.79553$. The premium is therefore $350000\frac{0.0905389}{17.79553} = \$1,780.71$.

5. *An insurance company sells a deferred annuity to a life aged 45. The interest rate is $i = 0.05$. The policy pays an annual annuity of \$40,000, starting at age 65. It is purchased with annual premiums from age 45 to age 65. Calculate the net annual premiums required.*

   From the table, we have $A_{65} = 0.270592$ and $A_{45} = 0.118065$. This gives $A_{45:\overline{20}|} = A_{45} + {}_{20}p_{45}(1.05)^{-20}(1-A_{65}) = 0.118065+\frac{9568.61}{9930.38}(1.05)^{-20}(1-0.270592) = 0.3829562$. We therefore get $\ddot{a}_{45:\overline{20}|} = \frac{1-0.3829562}{d}$, while $\ddot{a}_{65} = \frac{1-0.270592}{d}$. The EPV of the benefits is ${}_{20}p_{45}(1.05)^{-20}\ddot{a}_{65} = \frac{9568.61}{9930.38}(1.05)^{-20}\left(\frac{1-0.270592}{d}\right)$. Therefore the premium is $\frac{\left(\frac{9568.61}{9930.38}(1.05)^{-20}(1-0.270592)\right)}{1-0.3829562} \times 40000 =$ \$17,171.63.

6. *An insurance company sells a 10-year term insurance policy to a life aged 42. The interest rate is $i = 0.06$. The death benefit is \$200,000. Initial expenses are \$200 plus 40% of the first premium. Renewal expenses are 2% of each subsequent premium. Calculate the annual premiums using the equivalence principle.*

   We have $A_{42} = 0.0714153$ and $A_{52} = 0.118287$. This gives $A_{42:\overline{10}|} = A_{42} + {}_{10}p_{42}(1.06)^{-10}(1-A_{52}) = 0.0714153 + \frac{9865.30}{9948.55}(1.06)^{-10} \times (1-0.118287) = 0.5596393$ and $A_{42:\overline{10}|} = A_{42} - {}_{10}p_{42}(1.06)^{-10}A_{52} = 0.0714153 - \frac{9865.30}{9948.55}(1.06)^{-10} \times 0.118287 = 0.005917174$. We get $\ddot{a}_{42:\overline{10}|} = \frac{1.06}{0.06}(1-0.5596393) = 7.779706$.

   Let P be the premium. The equivalence principle gives

$$7.779706P = 200000 \times 0.005917174 + 200 + 0.38P + 0.02 \times 7.779706P$$
$$P = \frac{200000 \times 0.005917174 + 200}{0.98 \times 7.779706 - 0.38} = 190.97$$

   so the premium is \$190.97.

7. *An insurance company sells a deferred annuity to a life aged 33. The interest rate is $i = 0.04$ and the annual annuity payment is \$82,000, starting from age 65. The annuity is purchased with annual premiums until age 65. The initial expenses are \$600 plus 50% of the first premium, and renewal expenses are 2% of each premium during the deferred period, and \$123 each year while the annuity is being payed. Calculate the premium using the equivalence principle.*

   The expected present cost of the benefits is $82123{}_{32}p_{33}(1.04)^{-32}\ddot{a}_{65}$. The expected present value of the premium less expenses is $0.98P\ddot{a}_{33:\overline{32}|} - 0.48P - 600$. We have $A_{65} = 0.340726$, and $A_{33} = 0.110866$. We calculate $\ddot{a}_{65} = \frac{1.04}{0.04}(1-0.340726) = 17.14111$, so the EPV of the benefit is $82123 \times \frac{9568.61}{9983.18}(1.04)^{-32} \times 17.14111 = 384606.78$. We also calculate $A_{33:\overline{32}|} = A_{33} + {}_{32}p_{33}(1.04)^{-32}(1-A_{65}) = 0.110866 + \frac{9568.61}{9983.18}(1.04)^{-32}(1-0.340726) = 0.290993$, and $\ddot{a}_{33:\overline{32}|} = \frac{1.04}{0.04}(1-0.290993) = 18.43418$. The premium is then the solution to

$$(0.98 \times 18.43418 - 0.48)P - 600 = 384606.78$$
$$P = \frac{385206.78}{0.98 \times 18.43418 - 0.48} = 21870.68$$

   so the premium is \$21,870.68.

8. *An insurance company issues a whole life insurance policy to a life aged 43. The interest rate is $i = 0.03$, and the death benefit is \$380,000. Net monthly premiums are payable until death. Calculate the monthly premium using*

   (a) The Uniform Distribution of Deaths assumption.

   We have $A_{43} = 0.243514$, We also have $i^{(12)} = 12((1.03)^{\frac{1}{12}} - 1) = 0.02959524$, so $A_{43}^{(12)} = \frac{0.03}{0.02959524} \times 0.243514 = 0.2468444$. This gives $\ddot{a}_{43}^{(12)} = \frac{1-0.2468444}{d^{(12)}}$. We have $d^{(12)} = 12\left(1 - \frac{1}{\left(1+\frac{i^{(12)}}{12}\right)}\right) = \frac{12i^{(12)}}{12+i^{(12)}} = 0.02952243$. We therefore get $\ddot{a}_{43}^{(12)} = 25.5113$. The monthly premium is $380000\frac{A_{43}^{(12)}}{12\ddot{a}_{43}^{(12)}} = \frac{380000\times 0.2468444}{12\times 25.5113} = \$306.40$.

   (b) Woolhouse's formula.

   We calculate $\ddot{a}_{43} = \frac{1.03}{0.03}(1 - 0.243514) = 25.97269$. We also approximate $\mu_{43} \approx \frac{1}{2}(q_{42} + q_{43}) = \frac{1}{2}\left(\frac{5.57}{9948.55} + \frac{6.04}{9942.98}\right) = 0.0005836722$, and $\delta = \log(1.03) = 0.0295588$. Woolhouse's formula gives

   $$\ddot{a}_{43}^{(12)} = \ddot{a}_{43} - \frac{11}{24} - \frac{143}{1728}(0.0295588 + 0.0005836722) = 25.51186$$

   We then get $A_{43}^{(12)} = 1 - d^{(12)}\ddot{a}_{43}^{(12)} = 1 - 0.02952243 \times 25.51186 = 0.246828$. The monthly premium is therefore $380000\frac{A_{43}^{(12)}}{12\ddot{a}_{43}^{(12)}} = \frac{380000\times 0.246828}{12\times 25.51186} = \$306.38$.

9. *An insurance company issues a 10-year endowment policy to a life aged 53. The interest rate is $i = 0.04$, and the benefit is \$220,000. Calculate the net monthly premium using*

   (a) The Uniform Distribution of Deaths assumption.

   We have $A_{53} = 0.228327$ and $A_{63} = 0.319569$, so Under the UDD, we have $i^{(12)} = 12((1.04)^{\frac{1}{12}} - 1) = 0.03928488$, so $A_{53}^{(12)} = \frac{0.04}{0.03928488} \times 0.228327 = 0.2324833$ and $A_{53}^{(12)} = \frac{0.04}{0.03928488} \times 0.319569 = 0.3253863$ so $A_{53:\overline{10|}}^{(12)} = A_{53}^{(12)} + {}_{10}p_{43}(1.04)^{-10}(1 - A_{63}^{(12)}) = 0.2324833 + \frac{9638.51}{9852.42}(1.04)^{-10}(1 - 0.3253863) = 0.6783333$. This gives $\ddot{a}_{53}^{(12)} = \frac{1-A_{43}^{(12)}}{d^{(12)}} = \frac{1-0.67833330.690364}{0.03915669} = 8.214859$. The monthly premium is therefore $\frac{220000\times 0.6783333}{12\times 8.214859} = \$1513.86$.

   (b) Woolhouse's formula.

   We have $\ddot{a}_{53:10} = \frac{1.04}{0.04}(1 - 0.6780216) = 8.371438$. We approximate $\mu_{53} \approx \frac{1}{2}(q_{52} + q_{53}) = \frac{1}{2}\left(\frac{12.88}{9865.30} + \frac{14.04}{9852.42}\right) = 0.001365308$ and $\mu_{63} \approx \frac{1}{2}(q_{62} + q_{63}) = \frac{1}{2}\left(\frac{30.56}{9669.17} + \frac{33.44}{9638.51}\right) = 0.003314988$. We have $\delta = \log(1.04) = 0.03922071$ and ${}_{10}p_{53}(1.04)^{-10} = \frac{9638.51}{9852.42}(1.04)^{-10} = 0.6608967$. Woolhouse's formula gives

   $$\begin{aligned}\ddot{a}_{53:\overline{10|}}^{(12)} &= \ddot{a}_{53:\overline{10|}} - \frac{11}{24}(1 - {}_{10}p_{53}(1.04)^{-10}) - \frac{143}{1728}(\mu_{53} + \delta - {}_{10}p_{53}(1.04)^{-10}(\mu_{63} + \delta)) \\ &= 8.371438 - \frac{11}{24} \times 0.3391033 - \frac{143}{1728}(0.001365308 + 0.03922071 - 0.6608967(0.003314988 + 0.03922071)) \\ &= 8.214983\end{aligned}$$

Alternatively, we can apply Woolhouse's formula separately to get $\ddot{a}_{53}^{(12)}$ and $\ddot{a}_{63}^{(12)}$:

$$\begin{aligned}
\ddot{a}_{53:\overline{10}|}^{(12)} &= \ddot{a}_{53:\overline{10}|} - \frac{11}{24} - \frac{143}{1728}(\mu_{53} + \delta) \\
&= 20.063494 - \frac{11}{24} - \frac{143}{1728}(0.001365308 + 0.03922071) \\
&= 19.58188
\end{aligned}$$

and

$$\begin{aligned}
\ddot{a}_{63:\overline{10}|}^{(12)} &= \ddot{a}_{63:\overline{10}|} - \frac{11}{24} - \frac{143}{1728}(\mu_{63} + \delta) \\
&= 17.691206 - \frac{11}{24} - \frac{143}{1728}(0.003314988 + 0.03922071) \\
&= 17.22935
\end{aligned}$$

then calculate

$$\begin{aligned}
\ddot{a}_{53:\overline{10}|}^{(12)} &= \ddot{a}_{53}^{(12)} - {}_{10}p_{53}(1.04)^{-10}\ddot{a}_{63}^{(12)} \\
&= 19.58188 - 0.6608967 \times 17.22935 \\
&= 8.214983
\end{aligned}$$

This gives $A_{53:\overline{10}|}^{(12)} = 1 - d^{(12)}\ddot{a}_{53:\overline{10}|}^{(12)} = 1 - 8.214983 \times 0.03915669 = 0.6783285$. This gives a monthly premium of

$$\frac{380000 \times 0.6783285}{12 \times 8.214983} = \$1,513.82$$

10. *An insurance company sells 5000 whole-life insurance policies to lives aged 49. The interest rate is $i = 0.03$. The policies have a death benefit of \$700,000. Use the portfolio percentile premium principle to calculate the net annual premium for these policies, so that the probability of a loss on the portfolio is 0.02. [You may use a normal approximation for aggregate losses. ${}^2A_{49} = 0.099137$.]*

If the life dies in the nth year of the policy, then the present value of the premiums collected is $P\frac{1.03}{0.03}(1 - (1.03)^{-n})$, while the present value of the benefits paid is $700000(1.03)^{-n}$, so the present value of future loss is $700000(1.03)^{-n} - P\frac{1.03}{0.03}(1-(1.03)^{-n}) = \left(700000 + \frac{103}{3}P\right)(1.03)^{-n} - \frac{103}{3}P$. The expected present value of this is $\left(700000 + \frac{103}{3}P\right)A_{49} - \frac{103}{3}P$, while the variance of this is $\left(700000 + \frac{103}{3}P\right)^2 \mathrm{Var}((1.03)^{-T_x})$. We have $\mathrm{Var}((1.03)^{-T_x}) = {}^2A_{49} - (A_{49})^2 = 0.099137 - 0.287221^2 = 0.0166411$. The probability of a profit over a portfolio of 5000 policies is therefore

$$\Phi\left(\frac{\frac{103}{3}P - 0.287221\left(700000 + \frac{103}{3}P\right)}{\sqrt{\frac{0.0166411}{5000}\left(700000 + \frac{103}{3}P\right)^2}}\right)$$

Setting this equal to 0.98 gives

$$\begin{aligned}
\Phi\left(\frac{\frac{103}{3}P - 0.287221\left(700000 + \frac{103}{3}P\right)}{\sqrt{\frac{0.0166411}{5000}\left(700000 + \frac{103}{3}P\right)^2}}\right) &= 0.98\\
\frac{\frac{103}{3}P - 0.287221\left(700000 + \frac{103}{3}P\right)}{\sqrt{5000 \times 0.0166411\left(700000 + \frac{103}{3}P\right)^2}} &= 2.053749\\
\frac{103}{3}P - 0.287221\left(700000 + \frac{103}{3}P\right) &= 2.053749\sqrt{\frac{0.0166411}{5000}}\left(700000 + \frac{103}{3}P\right)\\
\frac{103}{3}P &= 0.2909677\left(700000 + \frac{103}{3}P\right)\\
0.7090323\frac{103}{3}P &= 0.2909677 \times 700000\\
P &= \frac{0.2909677 \times 700000}{0.7090323\frac{103}{3}} = \$8,366.82
\end{aligned}$$

11. *An insurance company sells 3000 whole-life insurance policies to lives aged 55. The interest rate is $i = 0.05$. The policies have a death benefit of \$200,000. Use the portfolio percentile premium principle to calculate the net annual premium for these policies, so that the probability of a loss on the portfolio is 0.02. [You may use a normal approximation for aggregate losses. ${}^2A_{55} = 0.0520745$.]*

If the life dies in the nth year of the policy, then the present value of the premiums collected is $P\frac{1.05}{0.05}(1-(1.05)^{-n})$, while the present value of the benefits paid is $200000(1.05)^{-n}$, so the present value of future loss is $200000(1.05)^{-n} - 21P(1-(1.05)^{-n}) = (200000 + 21P)(1.05)^{-n} - 21P$. The expected present value of this is $(200000 + 21P)A_{55} - 21P$, while the variance of this is $(200000 + 21P)^2\operatorname{Var}((1.05)^{-T_x})$. We have $\operatorname{Var}((1.05)^{-T_x}) =^2 A_{55} - (A_{55})^2 = 0.0520745 - 0.181136^2 = 0.01926425$. The probability of a profit over a portfolio of 5000 policies is therefore

$$\Phi\left(\frac{21P - 0.181136(200000 + 21P)}{\sqrt{\frac{0.01926425}{3000}(200000 + 21P)^2}}\right)$$

Setting this equal to 0.98 gives

$$\begin{aligned}
\Phi\left(\frac{21P - 0.181136\,(200000 + 21P)}{\sqrt{\frac{0.01926425}{3000}\,(200000 + 21P)^2}}\right) &= 0.98 \\
\frac{21P - 0.181136\,(200000 + 21P)}{\sqrt{3000 \times 0.01926425\,(200000 + 21P)^2}} &= 2.053749 \\
21P - 0.181136\,(200000 + 21P) &= 2.053749\sqrt{\frac{0.01926425}{3000}}\,(200000 + 21P) \\
21P &= 0.1863403\,(200000 + 21P) \\
0.8136597 \times 21P &= 0.1863403 \times 200000 \\
P &= \frac{0.1863403 \times 200000}{21 \times 0.8136597} = \$2,181.10
\end{aligned}$$

12. *An insurance company sells 4000 10-year term insurance policies to lives aged 38. The interest rate is $i = 0.05$. The policies have a death benefit of \$100,000. Use the portfolio percentile premium principle to calculate the net annual premium for these policies, so that the probability of a loss on the portfolio is 0.02. [You may use a normal approximation for aggregate losses. ${}^2A^{1}_{38:\overline{10}|} = 0.00341782$.]*

We compute

$$A^{1}_{38:\overline{10}|} = A_{38} - {}_{10}p_{38}(1.05)^{-10}A_{48} = 0.086580 - \frac{9907.10}{9966.88}(1.05)^{-10}0.134533 = 0.004483781$$

and

$$\ddot{a}_{38:\overline{10}|} = \frac{1.05}{0.05}(1 - A_{38} - {}_{10}p_{38}(1.05)^{-10}(1 - A_{48})) = 8.090982$$

If the life dies in the nth year, the present value of profit is $\frac{1.05}{0.05}P(1-(1.05)^{-n})-100000(1.05)^{-n}$, while if the life survives to the end of 10 years, the present value of profit is $\frac{1.05}{0.05}P(1-(1.05)^{-10})$. The expected present value of profit is therefore $P\ddot{a}_{38:\overline{10}|} - 100000A^{1}_{38:\overline{10}|}$. The variance of profit can be obtained by conditioning on whether the life survives to the end of the 10 years. Let S be the present value of profit on the policy, and let $X = 1$ if the life survives to the end of 10 years, and $X = 0$ if the life dies within 10 years. We then have

$$\mathrm{Var}(S) = \mathbb{E}(\mathrm{Var}(S|X)) + \mathrm{Var}(\mathbb{E}(S|X))$$

We clearly have $\mathrm{Var}(S|X = 1) = 0$, since if the life survives, the profit is exactly $\frac{1.05}{0.05}P(1 - (1.05)^{-10})$. On the other hand, if $X = 0$, then the profit is $21P - (21P + 100000)(1.05)^{-n}$, so the variance of the profit is $(21P+100000)^2\,\mathrm{Var}((1.05)^{-n}|X = 0)$. We have $\mathbb{E}(((1.05)^{-n})^2|X = 0) = \frac{{}^2A^{1}_{38:\overline{10}|}}{{}_{10}q_{38}}$ so

$$\begin{aligned}
\mathrm{Var}((1.05)^{-n}|X = 0) &= \frac{{}^2A^{1}_{38:\overline{10}|}}{{}_{10}q_{38}} - \left(\frac{A^{1}_{38:\overline{10}|}}{{}_{10}q_{38}}\right)^2 \\
&= \frac{0.00341782 \times 9966.88}{59.78} - \left(\frac{0.004483781 \times 9966.88}{59.78}\right)^2 = 0.01098923
\end{aligned}$$

We have $\mathbb{E}(S|X=1) = 21(1-1.05^{-10})P$, and $\mathbb{E}(S|X=0) = 21P - (100000+21P)\frac{A^{1}_{38:\overline{10}|}}{{}_{10}q_{38}}$, so

$$\begin{aligned}
\mathrm{Var}(\mathbb{E}(S|X)) &= {}_{10}\,p_{38\,10}q_{38}\left(21(1-1.05^{-10})P - \left(21P - (100000+21P)\frac{A^{1}_{38:\overline{10}|}}{{}_{10}q_{38}}\right)\right)^2\\
&= {}_{10}\,p_{38\,10}q_{38}\left((100000+21P)\frac{A^{1}_{38:\overline{10}|}}{{}_{10}q_{38}} - 21(1.05)^{-10}P\right)^2
\end{aligned}$$

$$\begin{aligned}
\mathrm{Var}(S) &= \mathbb{E}(\mathrm{Var}(S|X)) + \mathrm{Var}(\mathbb{E}(S|X))\\
&= 0.01098923\,{}_{10}q_{38}(21P+100000)^2 + 0.005961891\left(0.7475629(100000+21P) - 21(1.05)^{-10}P\right)^2\\
&= 0.00006591192(21P+100000)^2 + 0.005961891\left(2.806642P + 74756.29\right)^2
\end{aligned}$$

The premium is calculated by solving

$$\begin{aligned}
\Phi\left(\frac{8.090982P - 448.3781}{\sqrt{0.00006591192(21P+100000)^2 + 0.005961891\left(2.806642P + 74756.29\right)^2}}\right) &= 0.98\\
\frac{8.090982P - 448.3781}{\sqrt{0.00006591192(21P+100000)^2 + 0.005961891\left(2.806642P + 74756.29\right)^2}} &= 2.053749
\end{aligned}$$

$$\begin{aligned}
8.090982P - 448.3781 &= 2.053749\sqrt{0.00006591192(21P+100000)^2 + 0.005961891\left(2.806642P + 74756.29\right)^2}\\
(8.090982P - 448.3781)^2 &= 4.217885(0.00006591192(21P+100000)^2 + 0.005961891\left(2.806642P + 74756.29\right)^2)
\end{aligned}$$

$$\begin{aligned}
65.1433P^2 - 18975.48P - 143110729 = 0&\\
P &= \frac{18975.48 \pm \sqrt{18975.48^2 + 4 \times 65.1433 \times 143110729}}{2 \times 65.1433}\\
&= 1634.96
\end{aligned}$$

13. *An insurance company sells a 10-year term insurance policy to a life aged 42. The interest rate is $i = 0.04$. The life enjoys a number of dangerous hobbies, and so has mortality rate increased by 0.009569451. The death benefit of this policy is $350,000. Calculate the net annual premium for this policy.*

The annuity function for the mortality rate increased by 0.009569451 is the same as the annuity function for the force of interest increased by 0.009569451. This gives $i = 1.04e^{0.009569451} = 1.05$. At this interest rate, we have

$$A_{42:\overline{10}|} = A_{42} + {}_{10}p_{42}(1.05)^{-10}(1-A_{52}) = 0.103456 + \frac{9865.30}{9948.55}(1.05)^{-10}(1-0.159677) = 0.6150245$$

so

$$\ddot{a}_{42:\overline{10}|} = \frac{1.05}{0.05}(1 - 0.6150245) = 8.084486$$

To calculate the actual value of $A_{42:\overline{10}|}$, we use the actual interest rate $i = 0.04$, so we get

$$A_{42:\overline{10}|} = 1 - \frac{0.04}{1.04}8.084486 = 0.6890582$$

Finally, ${}_{10}p_{42} = \frac{9865.30}{9948.55}e^{-0.09569451} = 0.9011372$, so

$$A^{1}_{42:\overline{10}|} = A_{42:\overline{10}|} - {}_{10}p_{42}(1.04)^{-10} = 0.6890582 - 0.9011372(1.04)^{-10} = 0.08028223$$

The premium therefore is $\frac{350000A^{1}_{42:\overline{10}|}}{\ddot{a}_{42:\overline{10}|}} = \frac{350000\times 0.08028223}{8.084486} = \$3,475.64$.

14. *An insurance company sells a 15-year endowment policy to a life aged 47. The death benefit is \$240,000. The interest rate is* $i = 0.04$. *The net annual premium is \$11,689.86. Calculate the net policy value of this policy after 4 years, using a basis with interest rate* $i = 0.03$.

    Net policy value is based on the net premium for the policy basis, so we need to recalculate the premium. We calculate

    $$A_{47:\overline{15}|} = A_{47} + {}_{15}p_{47}(1.03)^{-15}(1 - A_{62}) = 0.271948 + \frac{9669.17}{9915.52}(1.03)^{-15}(1 - 0.404336) = 0.644783$$

    This gives

    $$\ddot{a}_{47:\overline{15}|} = \frac{1.03}{0.03}(1 - 0.644783) = 12.19578$$

    So the premium is $P = \frac{240000\times 0.644783}{12.19578} = \$12,688.64$. We now calculate

    $$A_{51:\overline{11}|} = A_{51} + {}_{11}p_{51}(1.03)^{-11}(1 - A_{62}) = 0.303217 + \frac{9669.17}{9877.13}(1.03)^{-11}(1 - 0.404336) = 0.7244771$$

    This gives

    $$\ddot{a}_{51:\overline{11}|} = \frac{1.03}{0.03}(1 - 0.7244771) = 9.45962$$

    so the policy value is $240000 \times 0.7244771 - 12688.64 \times 9.45962 = \$53,844.79$.

15. *An insurance company sells a whole-life insurance policy to a life aged 39. The interest rate is* $i = 0.05$. *The death benefit is \$350,000, and premiums are payable in advance until age 80. The annual net premium is therefore \$1,780.71. Calculate the policy value after 14 years, using the same policy value basis as the premium basis.*

    After 14 years, we have $A_{53} = 0.166573$, while $A_{53:\overline{27}|} = A_{53} + {}_{27}p_{53}(1.05)^{-27}(1 - A_{80}) = 0.166573 + \frac{8423.00}{9852.42}(1.05)^{-27}(1 - 0.457434) = 0.2908141$, so $\ddot{a}_{53:\overline{27}|} = \frac{1.05}{0.05}(1 - 0.2908141) = 14.89290$. The policy value is $350000 \times 0.166573 - 1780.71 \times 14.89290 = \$31,780.61$.

16. *An insurance company sells a deferred annuity to a life aged 33. The interest rate is $i = 0.04$ and the annual annuity payment is \$82,000, starting from age 65. The annuity is purchased with annual premiums of \$21,870.68 until age 65. The initial expenses are \$600 plus 50% of the first premium, and renewal expenses are 2% of each premium during the deferred period, and \$123 each year while the annuity is being payed. Calculate the policy value after 7 years using the same basis as the premium basis.*

    The EPV of the benefits plus expenses during the annuity is ${}_{25}p_{40}(1.04)^{-25}82123\ddot{a}_{65} = \frac{9568.61}{9958.44}(1.04)^{-25}82123\frac{1.04}{0.04}(1-0.340726) = \$507,373.95$. The EPV of the premiums minus expenses during the payment period is $0.98 \times 21870.68\ddot{a}_{40:\overline{25}|} = 557264.93(1 - A_{40:\overline{25}|}) = 557264.93(1 - A_{40} - {}_{25}p_{40}(1.04)^{-25}(1 - A_{65})) = 557264.93(1 - 0.143482 - \frac{9568.61}{9958.44}(1.04)^{-25}(1 - 0.340726)) = \$344,888.01$. The policy value is $507373.95 - 344888.01 = \$162,485.94$.

17. *An insurance company sells 1000 10-year term insurance policies to lives aged 42. The interest rate is $i = 0.06$. The death benefit is \$200,000. Initial expenses are \$200 plus 40% of the first premium. Renewal expenses are 2% of each subsequent premium. The gross annual premium is calculated as \$190.97. In the first 3 years of the policy:*

    - *The interest rate is $i = 0.07$ in the first year of the policy; $i = 0.05$ in the second year of the policy; $i = 0.08$ in the third and fourth years of the policy; and $i = 0.06$ in the fifth year.*
    - *One policyholder dies in the first year. One policyholder dies in the third year. No other policyholders die.*
    - *The total initial expenses for the policies are \$240,000.*
    - *The total renewal expenses for the policies are calculated as \$2,600 at the start of each of years 2, 3, 4 and 5.*

    Calculate the total profit on these policies at the end of the first 5 years

    The policy value after 5 years is $200000A^{1}_{47:\overline{5}|} - 0.98 \times 190.97\ddot{a}_{47:\overline{5}|}$. We have $A^{1}_{47:\overline{5}|} = A_{47} - {}_5p_{47}(1.06)^{-5}A_{52} = 0.0921683 - \frac{9865.30}{9915.52}(1.06)^{-5}0.118287 = 0.004225054$ and $A_{47:\overline{5}|} = A_{47} + {}_5p_{47}(1.06)^{-5}(1 - A_{52}) = 0.0921683 + \frac{9865.30}{9915.52}(1.06)^{-5}(1 - 0.118287) = 0.7476985$, so $\ddot{a}_{47:\overline{5}|} = \frac{1.06}{0.06}(1 - 0.7476985) = 4.457326$. This gives a policy value of $200000 \times 0.004225054 - 0.98 \times 190.97 \times 4.457326 = 10.81956$

    We therefore have the following cashflows from the policies:

    - 190970 in premiums at the start of the first year, 190779.03 at the start of years 2 and 3, and 190588.06 at the start of years 4 and 5.
    - Benefits of 200000 at the end of the first and third years.
    - Initial expenses of 240000
    - Renewal expenses of 2600 at the start of years 2, 3, 4, and 5.

    The accumulated value of these cashflows is $(190970 - 240000)(1.07)(1.05)(1.08)^2(1.06) + (190779.03 - 200000 - 2600)(1.05)(1.08)^2(1.06) + (190779.03 - 2600)(1.08)^2(1.06) + (190588.06 - 200000 - 2600)(1.08)(1.06) + (190588.06 - 2600)(1.06) = \$334,725.13$. However, the company also holds 998 policies with policy value 10.81956, so the value of these is $998 \times 10.81956 = 10797.92$. The total profit is therefore $334725.13 - 10797.92 = \$323,927.21$.

18. *An insurance company sells 1000 10-year term insurance policies to lives aged 42. The interest rate is $i = 0.06$. The death benefit is \$200,000. Initial expenses are \$200 plus 40% of the first premium. Renewal expenses are 2% of each subsequent premium. The gross annual premium is calculated as \$190.97. In the first 3 years of the policy:*

    - *The interest rate is $i = 0.07$ in the first year of the policy; $i = 0.07$ in the second year of the policy; $i = 0.05$ in the third year of the policy;*
    - *One policyholders dies in the second year. No policyholders die in the first or third years.*
    - *The total initial expenses for the policies are \$260,000.*
    - *The total renewal expenses for the policies are calculated as \$2,200 at the start of each of years 2 and 3.*

    Calculate the asset share of the remaining policies at the end of the first 3 years.

    The company receives premiums of 190970 at the start of the first two years and 190588.06 at the start of the third year. It pays expenses of 260000 at the start of the first year, 2200 at the start of the second and third year, and death benefits of 400000 at the end of the second year. The accumulated value of these cashflows is

$$(190970-260000)(1.07)^2(1.05)+(190970-2200)(1.07)(1.05)+(190779.06-200000-2200)(1.05) = \$117,107.01$$

    This is divided between the remaining 999 policies, so the asset share of each policy is $\frac{117107.01}{999} = \$117.22$.

19. *An insurance company issues a whole life insurance policy to a life aged 43. The interest rate is $i = 0.03$, and the death benefit is \$380,000. Net monthly premiums are payable until death. Using Woolhouse's formula, you calculate the monthly premium is \$306.38.*

    (a) Calculate the policy value after 8 years 4 months.

    We have $A_{51} = 0.303217$, so $\ddot{a}_{51} = \frac{1.03}{0.03}(1 - 0.303217) = 23.92288$. This gives

$$a_{51}^{(12)} = a_{51} - \frac{11}{24} - \frac{143}{1728}(\delta + \mu_{51}) = 23.92288 - \frac{11}{24} - \frac{143}{1728}\left(\log(1.03) + \frac{1}{2}\left(\frac{10.85}{9887.98} + \frac{11.83}{9877.13}\right)\right) = 23.46201$$

    We also have

$$\ddot{a}_{51}^{(12)} = \frac{1}{12}\left(1 + {}_{\frac{1}{12}}p_{51}(1.03)^{-\frac{1}{12}} + {}_{\frac{2}{12}}p_{51}(1.03)^{-\frac{2}{12}} + {}_{\frac{3}{12}}p_{51}(1.03)^{-\frac{3}{12}}\right) + {}_{\frac{4}{12}}p_{51}(1.03)^{-\frac{4}{13}}\ddot{a}_{51.33333333}^{(12)}$$

    which gives

$$\ddot{a}_{51.33333333}^{(12)} = \frac{23.46201(1.03)^{\frac{4}{12}} - \frac{1}{12}\left(\frac{9876.1441666667}{9877.13} \times (1.03)^{\frac{3}{12}} + \frac{9875.1583333333}{9877.13} \times (1.03)^{\frac{2}{12}} + \frac{9874.1725}{9877.13} \times (1.03)^{\frac{1}{12}}\right)}{\left(\frac{9873.186666666}{9877.13}\right)}$$

$$= 23.52617$$

    [We used the UDD assumption to estimate ${}_{\frac{1}{12}}p_{51}$.]

This gives

$$A^{(12)}_{51.33333333} = 1 - \frac{23.52617}{12\left(1-(1.03)^{-\frac{1}{12}}\right)} = 0.3054504$$

and therefore, the policy value is

$$380000 \times 0.3054504 - 12 \times 306.38 \times 23.52617 = \$29,575.78$$

(b) Calculate the policy value after 8 years 3.4 months.

After 8 years 3.4 months, the policy value is

$${}_{0.05}p_{51.28333333}(1.03)^{-0.05}29575.78 + {}_{0.05}q_{51.28333333}(1.03)^{-0.05} \times 380000$$

We have ${}_{0.05}p_{51.283333333} = \frac{9873.186666666}{9873.7781667}$, so the policy value is

$$\frac{9873.186666667}{9873.7781667}(1.03)^{-0.05} \times 29575.78 + \frac{0.5915}{9873.7781667}(1.03)^{-0.05} \times 380000 = \$29,553.06$$

20. *An insurance company issues a 10-year endowment policy to a life aged 53. The interest rate is $i = 0.04$, and the benefit is \$220,000. Using the Uniform Distribution of Deaths assumption, you calculate the net monthly premium is \$1,513.86.*

(a) Calculate the policy value after 4 years 7 months.

Under the UDD assumption we have $A^{(12)}_{58} = \frac{0.04}{12((1.04)^{\frac{1}{12}}-1)}A_{58} = 0.2758558$ and $A^{(12)}_{63} = \frac{0.04}{12((1.04)^{\frac{1}{12}}-1)}A_{63} = 0.3253863$. This gives $A^{(12)}_{58:\overline{5}|} = A^{(12)}_{58} + {}_5p_{58}(1.04)^{-5}(1-A^{(12)}_{63}) = 0.2758558 + \frac{9638.51}{9768.33}(1.04)^{-5}(1-0.3253863) = 0.8229701$. Using the recurrence, we get

$$\begin{aligned}
{}_{\frac{1}{12}}p_{57\frac{7}{12}} &= \frac{9774.746667}{9776.400833} \\
{}_{\frac{1}{12}}p_{57\frac{8}{12}} &= \frac{9773.0925}{9774.746667} \\
{}_{\frac{1}{12}}p_{57\frac{9}{12}} &= \frac{9771.638333}{9773.0925} \\
{}_{\frac{1}{12}}p_{57\frac{10}{12}} &= \frac{9769.984167}{9771.638333} \\
{}_{\frac{1}{12}}p_{57\frac{11}{12}} &= \frac{9768.33}{9769.984167}
\end{aligned}$$

$$A^{(12)}_{57\frac{11}{12}:\overline{5\frac{1}{12}}|} = ({}_{\frac{1}{12}}q_{57\frac{11}{12}} + {}_{\frac{1}{12}}\,p_{57\frac{11}{12}}A^{(12)}_{58:\overline{5}|})(1.04)^{-\frac{1}{12}} = 0.8203139$$
$$A^{(12)}_{57\frac{10}{12}:\overline{5\frac{2}{12}}|} = ({}_{\frac{1}{12}}q_{57\frac{10}{12}} + {}_{\frac{1}{12}}\,p_{57\frac{10}{12}}A^{(12)}_{58\frac{11}{12}:\overline{5\frac{1}{12}}|})(1.04)^{-\frac{1}{12}} = 0.8176668$$
$$A^{(12)}_{57\frac{9}{12}:\overline{5\frac{3}{12}}|} = ({}_{\frac{1}{12}}q_{57\frac{9}{12}} + {}_{\frac{1}{12}}\,p_{57\frac{9}{12}}A^{(12)}_{58\frac{10}{12}:\overline{5\frac{2}{12}}|})(1.04)^{-\frac{1}{12}} = 0.8150287$$
$$A^{(12)}_{57\frac{8}{12}:\overline{5\frac{4}{12}}|} = ({}_{\frac{1}{12}}q_{57\frac{8}{12}} + {}_{\frac{1}{12}}\,p_{57\frac{8}{12}}A^{(12)}_{58\frac{9}{12}:\overline{5\frac{3}{12}}|})(1.04)^{-\frac{1}{12}} = 0.8123997$$
$$A^{(12)}_{57\frac{7}{12}:\overline{5\frac{5}{12}}|} = ({}_{\frac{1}{12}}q_{57\frac{7}{12}} + {}_{\frac{1}{12}}\,p_{57\frac{7}{12}}A^{(12)}_{58\frac{8}{12}:\overline{5\frac{4}{12}}|})(1.04)^{-\frac{1}{12}} = 0.8097797$$

This gives

$$\ddot{a}^{(12)}_{57\frac{7}{12}:\overline{5\frac{5}{12}}|} = \frac{(1-0.8097797)(1.04)^{\frac{1}{12}}}{12((1.04)^{\frac{1}{12}}-1)} = 4.857926$$

The policy value is therefore given by $220000 \times -4.857926 \times 12 \times 1513.86 = \$89,900.89$

(b) Calculate the policy value after 4 years 6.8 months.

We calculate ${}_{\frac{0.2}{12}}p_{57+\frac{6.8}{12}} = \frac{9776.400833}{9776.73566667}$. The policy value after 4 years 6.8 months is

$$({}_{\frac{0.2}{12}}p_{57+\frac{6.8}{12}} \times 89900.89 + 220000\,{}_{\frac{0.2}{12}}q_{57+\frac{6.8}{12}})(1.04)^{-\frac{0.2}{12}} = \$89,846.50$$

21. *An insurance company wants to design a policy with continuous premiums so that the policy value is given by ${}_tV = 100t(t-8)(t-15)$. The death benefits are \$200,000. The policy is sold to a life aged 46, with mortality given by $\mu_x = 0.0000012(1.097)^x$. Calculate the premiums as a function of time if force of interest is $\delta = 0.024$.*

Using Thiele's differential equation, we get

$$\frac{d}{dt}{}_tV = \delta_tV + P_t - \mu_{x+t}(S - {}_tV)$$

Substituting the values given, we get

$$100(3t^2 - 46t + 120) = 0.024ct(t-8)(t-15) + P_t - 0.0000012(1.097)^{46+t}(200000 - 100t(t-8)(t-15))$$
$$P_t = 100(120 - 48.88t + 3.552t^2 - 0.024t^3 + 0.00012(1.097)^{46+t}(t(t-8)(t-15))) - 0.24(1.097)^{46+t}$$

22. *An insurance company sells a 10-year endowment insurance policy with continuous premiums at a rate of \$1700 per year. The benefits are \$200,000. The policy is sold to a life aged 46, with mortality given by $\mu_x = 0.0000012(1.097)^x$. The company finds that the policy value is given by ${}_tV = 200000\frac{(1-e^{-0.03t})}{1-e^{-0.3}}$. What force of interest are they using (as a function of time)?*

Using Thiele's differential equation, we get

$$\frac{d}{dt}{}_tV = \delta_t{}_tV + P_t - \mu_{x+t}(S - {}_tV)$$

Substituting the values given, we get

$$\begin{aligned}
\frac{6000e^{-0.03t}}{1-e^{-0.3}} &= 200000\delta_t\frac{1-e^{-0.03t}}{1-e^{-0.3}} + 1700 - 200000\left(1 - \frac{(1-e^{-0.03t})}{1-e^{-0.3}}\right)0.0000012(1.097)^{46+t}\\
6000e^{-0.03t} &= 200000\delta_t(1-e^{-0.03t}) + 1700(1-e^{-0.3}) - 200000\left(e^{-0.03t} - e^{-0.3}\right)0.0000012(1.097)^{46+t}\\
6000e^{-0.03t} &= 200000\delta_t(1-e^{-0.03t}) + 440.609 - 16.97134e^{0.09257918t}\left(e^{-0.03t} - e^{-0.3}\right)\\
200000\delta_t(1-e^{-0.03t}) &= 6000e^{-0.03t} - 440.609 + 16.97134e^{0.09257918t}\left(e^{-0.03t} - e^{-0.3}\right)\\
\delta_t &= \frac{6000e^{-0.03t} - 440.609 + 16.97134e^{0.09257918t}\left(e^{-0.03t} - e^{-0.3}\right)}{200000(1-e^{-0.03t})}
\end{aligned}$$

23. *An insurance company offers a 10-year term policy with death benefit \$600,000 payable at the end of the year of death. If the interest rate is* $i = 0.05$. *For a select individual aged 36, the annual premium for this policy is \$264.88. The policy pays a cash surrender value of 85% of the policy value. After 4 years, the policyholder wants to reduce the annual premiums to \$100 for the remainder of the policy. Calculate the new death benefits for the policy.*

    We have $A^{1}_{40:\overline{6}|} = A_{40} - {}_6p_{40}(1.05)^{-6}A_{46} = 0.0946669 - \frac{9923.26}{9958.44}(1.05)^{-6}0.123339 = 0.002954578$ and $A^{1}_{40:\overline{6}|} = A_{40} + {}_6p_{40}(1.05)^{-6}(1 - A_{46}) = 0.0946669 + \frac{9923.26}{9958.44}(1.05)^{-6}(1 - 0.123339) = 0.7465338$, so $\ddot{a}_{40:\overline{6}|} = \frac{1.05}{0.05}(1 - 0.7465338) = 5.32279$. The policy value after 4 years is therefore ${}_4V = 600000 \times 0.002954578 - 264.88 \times 5.32279 = 362.85$, so the cash surrender value is $0.85 \times 362.85 = \$308.42$. With new premiums of \$100, the EPV of the premiums is $100 \times 5.32279 = \$532.28$. With the cash surrender value, this makes a total of $532.279 + 308.419 = 840.698$, so this should be the EPV of the new death benefit. The new death benefit should be $\frac{840.698}{0.002954578} = \$284,541$.

24. *An insurance company sells a whole-life insurance policy to a life aged 44. The policy has a death benefit of \$250,000. The interest rate is* $i = 0.06$. *The annual premium for this policy is \$1,216.05. The policy has a cash surrender value of 80% of the policy value. After 8 years, the man asks to change the terms of the policy so that he pays no premiums for the next two years, but pays increased premiums after that time, so that the benefits for the policy remain the same. Is this change permissable, and if so, what should the new premiums be after the two year break?*

    We have $A_{52} = 0.118287$, so $\ddot{a}_{52} = \frac{1.06}{0.06}(1 - 0.118287) = 15.57693$. This means the policy value after 8 years is $250000 \times 0.118287 - 1216.05 \times 15.57693 = \$10,629.42$, and the cash surrender value is $0.8 \times 10629.42 = 8503.539$. The EPV of the benefits over the break period is $250000(q_{52}(1.06)^{-1} + 1|q_{52}(1.06)^{-2}) = 250000\frac{12.88(1.06)^{-1}+14.04(1.06)^{-2}}{9865.30} = 624.5754$, which is less than the surrender value, so the surrender value is sufficient to fund these benefits without the policy value becoming negative. Therefore, the change should be permissible. The EPV of the new benefits is $250000A_{52} = 29571.75$.

If the new premium is P, then the EPV of future premiums is

$$P(\ddot{a}_{52} - 1 - p_{52}(1.06)^{-1}) = P\left(15.57693 - 1 - \frac{9852.42}{9865.30}(1.06)^{-1}\right) = 13.63477P$$

[We could instead calculate it as ${}_2p_{52}(1.06)^{-2}\ddot{a}_{52}P$, but since we already calculated $\ddot{a}_{52}$, the way above is slightly easier.]

This means P must satisfy

$$\begin{aligned} 13.63477P + 8503.539 &= 29571.75 \\ P &= \frac{29571.75 - 8503.539}{13.63477} = \$1,545.18 \end{aligned}$$

25. *An insurance company offers a 20-year endowment insurance policy with benefit \$400,000 to a life aged 45. The interest rate is $i = 0.04$. The annual premium for this policy is \$13,208.28. The policy pays a cash surrender value of 90% of the policy value. After 12 years, the policyholder wants to change the policy to a whole-life insurance policy with the same premiums and an increased death benefit. Calculate the new death benefit.*

    After 12 years, we have $A_{57:\overline{8}} = A_{57} + {}_8p_{57}(1.04)^{-8}(1 - A_{65}) = 0.261926 + \frac{9568.61}{9788.18}(1.04)^{-8}(1 - 0.340726) = 0.7328449$, and $\ddot{a}_{57:\overline{8}} = \frac{1.04}{0.04}(1 - 0.7328449) = 6.946032$. This means the policy value is $0.7328449 \times 400000 - 6.946032 \times 13208.28 = 201392.83$. The cash surrender value is $0.9 \times 201392.83 = \$181,253.55$. The EPV of the new premiums is therefore $13208.28\ddot{a}_{57} = 13208.28\frac{1.04}{0.04}(1 - 0.261926) = 253465.89$, so the EPV of the benefits should be $253465.89 + 181253.55 = 434719.44$. The new death benefits should therefore be $\frac{434719.44}{0.261926} = \$1,659,703.27$.

26. *An insurance company sells a whole-life insurance policy to a life aged 44. The policy has a death benefit of \$450,000. The interest rate is $i = 0.07$. The annual premium for this policy is \$ 1,779.91. The policy has a cash surrender value of 85% of the policy value. After 11 years, the woman asks to convert the policy to a paid-up term policy with the same death benefits. Calculate the term of the policy after this modification.*

    (i) 13 years

    (ii) 22 years

    (iii) 26 years

    (iv) 31 years

    The policy value after 11 years is $450000A_{55} - 1779.91\ddot{a}_{55} = 450000 \times 0.105686 - 1779.91 \times \frac{1.07}{0.07}(1 - 0.105686) = \$23,226.92$ so the cash surrender value is $0.85 \times 23226.92 = \$19,742.89$. The woman wants to convert the policy to a term policy with term n years where n satisfies

$$\begin{aligned} 450000A^{1}_{55:\overline{n}|} &= 19742.89 \\ A_{55} - {}_np_{55}(1.07)^{-n}A_{55+n} &= \frac{19742.89}{450000} \\ {}_np_{55}(1.07)^{-n}A_{55+n} &= 0.105686 - \frac{19742.89}{450000} = 0.06181292 \end{aligned}$$

We try the values given:

n	${}_np_{55}(1.07)^{-n}A_{55+n}$
13	0.08247194
22	0.06294674
26	0.05348709
31	0.04130942

So the new term is (ii) 22 years.

27. *An insurance company offers a whole-life insurance policy with annual premiums, in which the death benefits for a given year are equal to \$100,000 plus the policy value at the start of the year. The policy is sold to a life aged 32. The interest rate is $i = 0.06$. The annual premiums for this policy are \$881.11. Calculate the policy value after 3 years.*

The policy value follows the recurrence relation:

$$\begin{aligned} {}_tV &= 1.06^{-1}(p_{32+t\,t+1}V + q_{32+t}(100000 + {}_tV)) - P \\ (1 - 1.06^{-1}q_{32+t})_tV &= 1.06^{-1}(p_{32+t\,t+1}V + 100000q_{32+t}) - P \\ {}_{t+1}V &= \frac{(1.06 - q_{32+t})_tV - 100000q_{32+t} + 1.06P}{p_{32+t}} \end{aligned}$$

This gives

$$\begin{aligned} {}_1V &= \frac{\left(1.06 \times 881.11 - 100000\frac{2.62}{9985.80}\right)}{\left(\frac{9983.18}{9985.80}\right)} = \frac{933.9766 \times 9985.80 - 262000}{9983.18} = 907.9776 \\ {}_2V &= \frac{933.9766 \times 9983.18 + (1.06 \times 9983.18 - 2.80)907.9776 - 280000}{9980.38} = 1868.655 \\ {}_2V &= \frac{933.9766 \times 9980.38 + (1.06 \times 9980.38 - 3.01)1868.655 - 301000}{9977.37} = 2884.898 \end{aligned}$$

So the policy value after 3 years is \$2884.90.

28. *An insurance company sells a 15-year term insurance policy to a life aged 29. The death benefit is \$180,000 in the first two years, \$160,000 in the second to fifth year and \$140,000 for the remaining 10 years. The premiums are \$96.85 for the first three years, and \$26.64 for the remaining twelve years. The interest rate is $i = 0.05$ for the first 4 years, and $i = 0.07$ for the remaining 11 years. Calculate the retrospective policy value after 2 years.*

The expected accumulated value of the premiums after two years is $96.85\left((1.05)^2 + \frac{9990.52}{9992.66}1.05\right) = 208.4478$. The expected accumulated value of the benefits is $180000\left(\frac{2.14}{9992.66}(1.05) + \frac{2.28}{9992.66}\right) = 81.54585$, so the expected cashflow per policy issued is $208.4478 - 81.54585 = 126.9020$. The retrospective policy value is the expected cashflow per policy *still in force*, which is $\frac{9992.66}{9988.24} \times 126.9020 = \$126.96$.

29. *A man aged 42 buys a whole-life insurance policy with a death benefit of \$400,000. The interest rate is* $i = 0.06$. *The annual premium for this policy is therefore \$1741.31. Using a full preliminary term of 1 year, calculate the policy value of this policy after 4 years.*

    Using the full preliminary term, the policy value is obtained by taking the premium for the policy starting from age 43. That is, we have $A_{43} = 0.0751824$, so $\ddot{a}_{43} = \frac{1.06}{0.06}(1-0.0751824) = 16.33844$. The premium for a whole-life insurance policy sold to a life aged 43 is therefore $\frac{400000A_{43}}{\ddot{a}_{43}} = 1840.63$. At age 46 we have $A_{46} = 0.0876193$ so $\ddot{a}_{46} = \frac{1.06}{0.06}(1 - 0.0876193) = 16.11873$. The policy value is therefore given by $400000 \times 0.0876193 - 1840.63 \times 16.11873 = \$5,379.11$.

30. *A man aged 34 buys a 25-year endowment insurance policy with a benefit of \$500,000. The interest rate is* $i = 0.05$. *The annual premium for this policy is therefore \$10,162.60. Using a full preliminary term of 2 years, calculate the policy value of this policy after 13 years.*

    After 2 years, we have $A_{36:\overline{23}|} = A_{36} + {}_{23}p_{36}(1.05)^{-23}(1-A_{59}) = 0.079149 + \frac{9746.67}{9974.13}(1.05)^{-23}(1-0.213496) = 0.3293726$ so $\ddot{a}_{36:\overline{23}|} = \frac{1.05}{0.05}(1 - 0.3293726) = 14.08318$. Therefore the premium is $\frac{500000\times0.3293726}{14.08318} = 11693.83$

    After 13 years, we have

    $A_{47:\overline{12}|} = A_{47} + {}_{12}p_{47}(1.05)^{-12}(1-A_{59}) = 0.128827 + \frac{9746.67}{9915.52}(1.05)^{-12}(1-0.213496) = 0.559324$ so $\ddot{a}_{36:\overline{23}|} = \frac{1.05}{0.05}(1 - 0.559324) = 9.254196$.

    The policy value is therefore $500000 \times 0.559324 - 11693.83 \times 9.254196 = \$171445.01$.

Table 1: Select lifetable to be used for questions on this practice final

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66
26	9997.00	9995.83	9994.40	9992.66
27	9995.14	9993.90	9992.38	9990.52
28	9993.16	9991.84	9990.22	9988.24
29	9991.05	9989.65	9987.92	9985.80
30	9988.81	9987.30	9985.46	9983.18
31	9986.40	9984.80	9982.82	9980.38
32	9983.83	9982.11	9979.99	9977.37
33	9981.07	9979.23	9976.95	9974.13
34	9978.11	9976.13	9973.68	9970.64
35	9974.93	9972.79	9970.16	9966.88
36	9971.50	9969.20	9966.36	9962.82
37	9967.80	9965.33	9962.25	9958.44
38	9963.81	9961.14	9957.82	9953.69
39	9959.50	9956.61	9953.02	9948.55
40	9954.84	9951.71	9947.82	9942.98
41	9949.79	9946.41	9942.19	9936.94
42	9944.32	9940.66	9936.08	9930.38
43	9938.39	9934.41	9929.45	9923.26
44	9931.96	9927.64	9922.25	9915.52
45	9924.97	9920.28	9914.42	9907.10
46	9917.37	9912.28	9905.91	9897.94
47	9909.11	9903.58	9896.65	9887.98
48	9900.13	9894.11	9886.57	9877.13
49	9890.36	9883.80	9875.59	9865.30
50	9879.71	9872.57	9863.63	9852.42
51	9868.12	9860.34	9850.59	9838.38
52	9855.48	9847.01	9836.39	9823.08
53	9841.72	9832.48	9820.90	9806.39
54	9826.71	9816.64	9804.02	9788.18
55	9810.34	9799.37	9785.60	9768.33
56	9792.49	9780.52	9765.51	9746.67
57	9773.03	9759.97	9743.60	9723.05
58	9751.79	9737.56	9719.69	9697.28
59	9728.63	9713.10	9693.62	9669.17
60	9703.36	9686.43	9665.17	9638.51
61	9675.80	9657.33	9634.15	9605.07
62	9645.73	9625.59	9600.31	9568.61
63	9612.94	9590.98	9563.42	9528.85
64	9577.18	9553.24	9523.19	9485.52
65	9538.19	9512.09	9479.35	9438.30
66	9495.69	9467.25	9431.58	9386.86
67	9449.37	9418.39	9379.54	9330.85
68	9398.90	9365.17	9322.87	9269.88
69	9343.95	9307.23	9261.20	9203.55
70	9284.12	9244.18	9194.11	9131.43
71	9219.03	9175.59	9121.17	9053.07
72	9148.24	9101.03	9041.91	8967.97
73	9071.30	9020.03	8955.85	8875.63

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
74	8987.73	8932.10	8862.49	8775.52
75	8897.04	8836.71	8761.27	8667.10
76	8798.69	8733.34	8651.66	8549.78
77	8692.13	8621.41	8533.09	8423.00
78	8576.81	8500.36	8404.95	8286.16
79	8452.13	8369.60	8266.68	8138.66
80	8317.52	8228.53	8117.67	7979.93
81	8172.36	8076.57	7957.35	7809.41
82	8016.08	7913.13	7785.15	7626.56
83	7848.11	7737.67	7600.54	7430.89
84	7667.89	7549.66	7403.05	7221.99
85	7474.92	7348.64	7192.27	6999.51
86	7268.77	7134.21	6967.86	6763.22
87	7049.07	6906.07	6729.62	6513.04
88	6815.55	6664.05	6477.46	6249.02
89	6568.09	6408.10	6211.48	5971.42
90	6306.70	6138.35	5931.96	5680.73
91	6031.59	5855.15	5639.41	5377.67
92	5743.19	5559.08	5334.61	5063.27
93	5442.15	5250.97	5018.61	4738.86
94	5129.44	4931.97	4692.79	4406.12
95	4806.33	4603.54	4358.89	4067.08
96	4474.39	4267.51	4018.96	3724.10
97	4135.60	3926.04	3675.44	3379.91
98	3792.25	3581.66	3331.11	3037.57
99	3447.02	3237.23	2989.05	2700.39
100	3102.90	2895.94	2652.63	2371.88
101	2763.19	2561.21	2325.37	2055.64
102	2431.39	2236.61	2010.90	1755.27
103	2111.15	1925.80	1712.81	1474.18
104	1806.12	1632.34	1434.48	1215.44
105	1519.82	1359.55	1178.94	981.65
106	1255.46	1110.36	948.70	774.71
107	1015.81	887.14	745.58	595.71
108	802.96	691.49	570.56	444.87
109	618.23	524.17	423.71	321.41
110	462.04	385.00	304.13	223.65
111	333.80	272.80	210.00	149.10
112	231.99	185.53	138.71	94.62
113	154.19	120.34	87.07	56.74
114	97.30	73.90	51.50	31.84
115	57.78	42.55	28.41	16.52
116	31.92	22.69	14.43	7.81
117	16.15	11.04	6.63	3.30
118	7.34	4.79	2.69	1.21
119	2.90	1.79	0.93	0.37
120	0.95	0.55	0.26	0.09
121	0.23	0.13	0.05	0.01
122	0.03	0.02	0.01	0.00

Table 2: Values of A_x at various interest rates

x	A_x				
	$i = 0.03$	$i = 0.04$	$i = 0.05$	$i = 0.06$	$i = 0.07$
28	0.159448	0.0920362	0.0550919	0.0343021	0.0222699
29	0.164063	0.0955357	0.0576569	0.0361664	0.0236325
30	0.168807	0.0991642	0.0603386	0.0381304	0.025078
31	0.173682	0.102926	0.0631417	0.0401992	0.0266113
32	0.178692	0.106825	0.0660706	0.0423772	0.0282367
33	0.18384	0.110866	0.0691308	0.0446701	0.0299597
34	0.189128	0.115052	0.0723272	0.0470831	0.0317853
35	0.194558	0.119388	0.0756648	0.0496214	0.0337189
36	0.200135	0.123879	0.079149	0.052291	0.0357661
37	0.205862	0.12853	0.0827855	0.0550978	0.0379331
38	0.21174	0.133344	0.0865803	0.0580484	0.0402265
39	0.217774	0.138327	0.0905389	0.0611489	0.0426523
40	0.223965	0.143482	0.0946669	0.0644056	0.0452173
41	0.230317	0.148816	0.0989705	0.0678253	0.0479284
42	0.236832	0.154332	0.103456	0.0714153	0.0507932
43	0.243514	0.160035	0.108129	0.0751824	0.053819
44	0.250364	0.165929	0.112997	0.079134	0.0570135
45	0.257384	0.17202	0.118065	0.0832768	0.0603841
46	0.264579	0.178312	0.123339	0.0876193	0.0639399
47	0.271948	0.184808	0.128827	0.0921683	0.0676885
48	0.279495	0.191514	0.134533	0.0969315	0.0716383
49	0.287221	0.198434	0.140465	0.101917	0.0757985
50	0.295128	0.205571	0.14663	0.107134	0.0801788
51	0.303217	0.212931	0.153032	0.112588	0.0847871
52	0.311489	0.220514	0.159677	0.118287	0.0896318
53	0.319946	0.228327	0.166573	0.124241	0.0947241
54	0.328587	0.236372	0.173724	0.130456	0.100072
55	0.337414	0.244652	0.181136	0.136941	0.105686
56	0.346426	0.253169	0.188814	0.143702	0.111574
57	0.355623	0.261926	0.196764	0.150748	0.117747
58	0.365004	0.270924	0.20499	0.158086	0.124213
59	0.374567	0.280165	0.213496	0.165721	0.130981
60	0.384312	0.28965	0.222286	0.173662	0.138061
61	0.394236	0.299379	0.231363	0.181913	0.145461
62	0.404336	0.309353	0.24073	0.190481	0.153188
63	0.41461	0.319569	0.25039	0.199371	0.161252
64	0.425054	0.330027	0.260343	0.208588	0.169658
65	0.435663	0.340726	0.270592	0.218135	0.178416
66	0.446433	0.351661	0.281134	0.228016	0.187529
67	0.457358	0.36283	0.291971	0.238233	0.197005
68	0.468433	0.374228	0.303101	0.248787	0.206847
69	0.47965	0.38585	0.31452	0.259679	0.217059
70	0.491002	0.39769	0.326225	0.27091	0.227644
71	0.502481	0.409741	0.338212	0.282476	0.238604
72	0.514079	0.421995	0.350475	0.294375	0.249939
73	0.525785	0.434443	0.363008	0.306604	0.261649
74	0.53759	0.447075	0.375801	0.319157	0.273732
75	0.549483	0.459881	0.388846	0.332028	0.286183
76	0.561452	0.472848	0.402132	0.345207	0.298998

x	A_x				
	$i = 0.03$	$i = 0.04$	$i = 0.05$	$i = 0.06$	$i = 0.07$
77	0.573485	0.485964	0.415648	0.358686	0.31217
78	0.585568	0.499215	0.42938	0.372454	0.32569
79	0.59769	0.512586	0.443314	0.386497	0.339549
80	0.609835	0.526062	0.457434	0.400802	0.353734
81	0.621989	0.539625	0.471723	0.415351	0.368232
82	0.634137	0.553258	0.486163	0.430129	0.383026
83	0.646262	0.566943	0.500734	0.445115	0.398099
84	0.658349	0.58066	0.515416	0.460289	0.413432
85	0.670382	0.594389	0.530187	0.475628	0.429002
86	0.682343	0.60811	0.545023	0.49111	0.444788
87	0.694217	0.621802	0.559901	0.506708	0.460763
88	0.705986	0.635444	0.574798	0.522398	0.476902
89	0.717634	0.649013	0.589687	0.538151	0.493176
90	0.729143	0.662489	0.604543	0.55394	0.509557
91	0.740498	0.675848	0.619339	0.569735	0.526011
92	0.751682	0.68907	0.634049	0.585506	0.542509
93	0.76268	0.702132	0.648647	0.601223	0.559017
94	0.773475	0.715013	0.663107	0.616856	0.575501
95	0.784054	0.727694	0.677402	0.632374	0.591929
96	0.794403	0.740153	0.691506	0.647747	0.608265
97	0.804509	0.752371	0.705396	0.662946	0.624476
98	0.814359	0.764332	0.719046	0.67794	0.640528
99	0.823943	0.776016	0.732435	0.692702	0.656388
100	0.83325	0.787409	0.745539	0.707204	0.672024
101	0.842272	0.798496	0.758339	0.72142	0.687404
102	0.851	0.809264	0.770816	0.735325	0.702499
103	0.85943	0.8197	0.782952	0.748897	0.71728
104	0.867554	0.829795	0.794732	0.762114	0.73172
105	0.875369	0.83954	0.806142	0.774958	0.745796
106	0.882873	0.848929	0.81717	0.787411	0.759486
107	0.890065	0.857957	0.827808	0.799459	0.772769
108	0.896944	0.866619	0.838046	0.81109	0.785629
109	0.903512	0.874915	0.84788	0.822293	0.79805
110	0.909771	0.882844	0.857306	0.833061	0.81002
111	0.915723	0.890405	0.86632	0.843386	0.821528
112	0.921375	0.897605	0.874926	0.853268	0.832569
113	0.92673	0.904445	0.883122	0.862704	0.843138
114	0.931798	0.910935	0.890918	0.8717	0.853237
115	0.936585	0.917079	0.898316	0.880256	0.862864
116	0.941093	0.922879	0.905316	0.88837	0.872013
117	0.945338	0.928352	0.911935	0.896059	0.880699
118	0.949337	0.93352	0.918198	0.903348	0.88895
119	0.953079	0.938364	0.924079	0.910206	0.896728
120	0.956622	0.942959	0.929669	0.916736	0.904147
121	0.959964	0.947302	0.934961	0.922928	0.911192
122	0.963253	0.951582	0.940182	0.929046	0.918163
123	0.967732	0.957429	0.947342	0.937463	0.927786
124	0.970874	0.961538	0.952381	0.943396	0.934579
125	1	1	1	1	1

Table 3: Values of a_x at various interest rates

x	a_x				
	$i = 0.03$	$i = 0.04$	$i = 0.05$	$i = 0.06$	$i = 0.07$
28	28.858952	23.607059	19.843070	17.060663	14.805627
29	28.700504	23.516072	19.789205	17.027727	14.784994
30	28.537626	23.421731	19.732889	16.993030	14.763105
31	28.370251	23.323924	19.674024	16.956481	14.739886
32	28.198241	23.222550	19.612517	16.918003	14.715273
33	28.021493	23.117484	19.548253	16.877495	14.689182
34	27.839939	23.008648	19.481129	16.834865	14.661537
35	27.653509	22.895912	19.411039	16.790022	14.632257
36	27.462032	22.779146	19.337871	16.742859	14.601256
37	27.265405	22.658220	19.261505	16.693272	14.568442
38	27.063593	22.533056	19.181814	16.641145	14.533713
39	26.856426	22.403498	19.098683	16.586369	14.496979
40	26.643868	22.269468	19.011995	16.528834	14.458138
41	26.425783	22.130784	18.921620	16.468420	14.417084
42	26.202101	21.987368	18.827424	16.404996	14.373703
43	25.972686	21.839090	18.729291	16.338444	14.327884
44	25.737503	21.685846	18.627063	16.268633	14.279510
45	25.496483	21.527480	18.520635	16.195443	14.228469
46	25.249454	21.363888	18.409881	16.118726	14.174624
47	24.996452	21.194992	18.294633	16.038360	14.117860
48	24.737338	21.020636	18.174807	15.954210	14.058049
49	24.472079	20.840716	18.050235	15.866133	13.995051
50	24.200605	20.655154	17.920770	15.773966	13.928721
51	23.922883	20.463794	17.786328	15.677612	13.858938
52	23.638878	20.266636	17.646783	15.576930	13.785576
53	23.348521	20.063498	17.501967	15.471742	13.708464
54	23.051846	19.854328	17.351796	15.361944	13.627481
55	22.748786	19.639048	17.196144	15.247376	13.542469
56	22.439374	19.417606	17.034906	15.127931	13.453308
57	22.123610	19.189924	16.867956	15.003452	13.359831
58	21.801529	18.955976	16.695210	14.873814	13.261917
59	21.473200	18.715710	16.516584	14.738929	13.159431
60	21.138621	18.469100	16.331994	14.598638	13.052219
61	20.797897	18.216146	16.141377	14.452870	12.940162
62	20.451131	17.956822	15.944670	14.301502	12.823153
63	20.098390	17.691206	15.741810	14.144446	12.701041
64	19.739813	17.419298	15.532797	13.981612	12.573750
65	19.375570	17.141124	15.317568	13.812948	12.441129
66	19.005800	16.856814	15.096186	13.638384	12.303132
67	18.630709	16.566420	14.868609	13.457884	12.159639
68	18.250467	16.270072	14.634879	13.271430	12.010603
69	17.865350	15.967900	14.395080	13.079004	11.855964
70	17.475598	15.660060	14.149275	12.880590	11.695677
71	17.081486	15.346734	13.897548	12.676257	11.529711
72	16.683288	15.028130	13.640025	12.466042	11.358067
73	16.281382	14.704482	13.376832	12.249996	11.180744
74	15.876077	14.376050	13.108179	12.028226	10.997773
75	15.467750	14.043094	12.834234	11.800839	10.809229
76	15.056815	13.705952	12.555228	11.568010	10.615173
77	14.643682	13.364936	12.271392	11.329881	10.415711
78	14.228832	13.020410	11.983020	11.086646	10.210980
79	13.812643	12.672764	11.690406	10.838553	10.001115
80	13.395665	12.322388	11.393886	10.585831	9.786314
81	12.978378	11.969750	11.093817	10.328799	9.566773
82	12.561296	11.615292	10.790577	10.067721	9.342749
83	12.145005	11.259482	10.484586	9.802968	9.114501
84	11.730018	10.902840	10.176264	9.534894	8.882315
85	11.316885	10.545886	9.866073	9.263905	8.646541
86	10.906224	10.189140	9.554517	8.990390	8.407496
87	10.498550	9.833148	9.242079	8.714825	8.165589
88	10.094481	9.478456	8.929242	8.437635	7.921198
89	9.694566	9.125662	8.616573	8.159332	7.674763
90	9.299424	8.775286	8.304597	7.880393	7.426708
91	8.909569	8.427952	7.993881	7.601348	7.177548
92	8.525585	8.084180	7.684971	7.322727	6.927721
93	8.147987	7.744568	7.378413	7.045060	6.677743
94	7.777358	7.409662	7.074753	6.768877	6.428128
95	7.414146	7.079956	6.774558	6.494726	6.179361
96	7.058830	6.756022	6.478374	6.223136	5.931987
97	6.711858	6.438354	6.186684	5.954621	5.686506
98	6.373674	6.127368	5.900034	5.689727	5.443433
99	6.044624	5.823584	5.618865	5.428931	5.203267
100	5.725083	5.527366	5.343681	5.172729	4.966494
101	5.415328	5.239104	5.074881	4.921580	4.733597
102	5.115667	4.959136	4.812864	4.675925	4.505015
103	4.826237	4.687800	4.558008	4.436153	4.281189
104	4.547313	4.425330	4.310628	4.202653	4.062526
105	4.278998	4.171960	4.071018	3.975742	3.849375
106	4.021360	3.927846	3.839430	3.755739	3.642069
107	3.774435	3.693118	3.616032	3.542891	3.440927
108	3.538256	3.467906	3.401034	3.337410	3.246189
109	3.312755	3.252210	3.194520	3.139490	3.058100
110	3.097862	3.046056	2.996574	2.949256	2.876840
111	2.893510	2.849470	2.807280	2.766847	2.702576
112	2.699458	2.662270	2.626554	2.592265	2.535384
113	2.515603	2.484430	2.454438	2.425563	2.375339
114	2.341602	2.315690	2.290722	2.266633	2.222411
115	2.177248	2.155946	2.135364	2.115477	2.076631
116	2.022474	2.005146	1.988364	1.972130	1.938089
117	1.876729	1.862848	1.849365	1.836291	1.806558
118	1.739430	1.728480	1.717842	1.707519	1.681614
119	1.610954	1.602536	1.594341	1.586361	1.563833
120	1.489311	1.483066	1.476951	1.470997	1.451488
121	1.374569	1.370148	1.365819	1.361605	1.344807
122	1.261647	1.258868	1.256178	1.253521	1.239246
123	1.107868	1.106846	1.105818	1.104820	1.093526
124	1.000000	1.000000	1.000000	1.000000	1.000000
125	0	0	0	0	0