ACSC/STAT 3720, Life Contingencies I
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Homework Sheet 3
Model Solutions

Basic Questions

1. *Calculate the expected benefit of a whole life insurance sold to an individual aged 100, if the death benefit is \$1,200,000 at the end of the year of death, the lifetable is Table 1, and the interest rate is* $i = 0.07$.

   We use the recurrence relation $A_x = (p_x A_{x+1} + q_x)(1.07)^{-1}$, starting from $A_{125} = 1$ (or $A_{125} = 0$) to get:

$$
\begin{aligned}
A_{124} &= (1.07)^{-1} = 0.934579 \\
A_{123} &= \left(0.934579 \times \frac{0.01}{0.09} + \frac{0.08}{0.09}\right)(1.07)^{-1} = 0.927786 \\
A_{122} &= 0.918163 \\
A_{121} &= 0.911192 \\
A_{120} &= 0.904147 \\
A_{119} &= 0.896728 \\
A_{118} &= 0.88895 \\
A_{117} &= 0.880699 \\
A_{116} &= 0.872013 \\
A_{115} &= 0.862864 \\
A_{114} &= 0.853237 \\
A_{113} &= 0.843138 \\
A_{112} &= 0.832569 \\
A_{111} &= 0.821528 \\
A_{110} &= 0.81002 \\
A_{109} &= 0.79805 \\
A_{108} &= 0.785629 \\
A_{107} &= 0.772769 \\
A_{106} &= 0.759486 \\
A_{105} &= 0.745796 \\
A_{104} &= 0.73172 \\
A_{103} &= 0.71728 \\
A_{102} &= 0.702499 \\
A_{101} &= 0.687404 \\
A_{100} &= 0.672024
\end{aligned}
$$

The expected benefit is therefore $1200000 \times 0.672024 = \$806,428.80$.

2. *Calculate the expected benefit, and the variance of the benefit of a 5-year term policy with benefit $200,000 at the end of year of death of the policyholder. The lifetable for this policy is Table 1, and the interest rate is $i = 0.06$. The policy is sold to an individual aged 41 [who is not select].*

   We use the recurrence relation $A_x = (p_x A_{x+1} + q_x)(1.06)^{-1}$, starting from $A^1_{46:\overline{0}|} = 0$ to get:

$$
\begin{aligned}
A^{1}_{45:\overline{1}|} &= \left(0 \times \frac{9923.26}{9930.38} + \frac{7.12}{9930.38}\right)(1.06)^{-1} = 0.000676407 \\
A^{1}_{44:\overline{2}|} &= 0.00126049 \\
A^{1}_{43:\overline{3}|} &= 0.0017615 \\
A^{1}_{42:\overline{4}|} &= 0.00218905 \\
A^{1}_{41:\overline{5}|} &= 0.00255124
\end{aligned}
$$

The EPV of benefit is then $200000 \times 0.00255124 = \$510.25$.

To get the variance of the benefit we find that doubling the force of interest corresponds to $i = 1.06^2 - 1 = 0.1236$. Using this interest rate we calculate

$$
\begin{aligned}
{}^2A^{1}_{45:\overline{1}|} &= \left(0 \times \frac{9923.26}{9930.38} + \frac{7.12}{9930.38}\right)(1.06)^{-2} = 0.00063812 \\
{}^2A^{1}_{44:\overline{2}|} &= 0.00115509 \\
{}^2A^{1}_{43:\overline{3}|} &= 0.00156804 \\
{}^2A^{1}_{42:\overline{4}|} &= 0.00189306 \\
{}^2A^{1}_{41:\overline{5}|} &= 0.00214354
\end{aligned}
$$

The variance of PV of a benefit of \$1 is therefore $0.00214354 - 0.00255124^2 = 0.002137031$. The variance of PV of benefit for the policy is then $200000^2 \times 0.002137031 = 85,481,240$.

3. *A select individual aged 45 purchases a 5-year term insurance policy with a benefit of \$300,000 payable immediately upon the death of the individual. Force of interest is $\delta = 0.046$. The expected benefit from a 5-year term policy for this inidividual with payment at the end of year of death would be \$959.24. Using a uniform distribution of deaths assumption, calculate the expected benefit from the policy with payment immediately upon death.*

   Using uniform distribution of deaths, we have

   $$\overline{A}_{[45]:\overline{5}|} = \frac{i}{\delta} A_{[45]:\overline{5}|} = \frac{e0.46 - 1}{0.46} \times \frac{959.24}{300000}$$

   so the EPV of the continuous policy is

   $$\frac{e^{0.46} - 1}{0.46} \times 959.24 = \$981.64$$

4. *An individual aged 39 wants to purchase whole life insurance that pays a benefit at the end of the year of death. The interest rate is $i = 0.07$. The individual has a number of dangerous hobbies and uses the special lifetable:*

x	l_x	d_x
39	*10000.00*	*4.80*
40	*9995.20*	*4.86*
41	*9990.34*	*4.93*
42	*9985.41*	*5.01*
43	*9980.40*	*5.09*
44	*9975.31*	*5.18*
45	*9970.13*	*5.29*

After age 45, the individual will be too old to participate in these hobbies and will use a standard lifetable, which will give the value $A_{45} = 0.06038$*. Calculate the EPV of the benefit for this individual from a whole-life policy which has a death benefit of $500,000.*

We can use our recurrence $A_x = (p_x A_{x+1} + q_x)(1+i)^{-1}$, starting from $A_{45} = 0.06038$ to get:

$$
\begin{aligned}
A_{44} &= \left(\frac{9970.13}{9975.31} \times 0.06038 + \frac{5.18}{9975.31}\right)(1.07)^{-1} = 0.0568859 \\
A_{43} &= 0.0536139 \\
A_{42} &= 0.0505502 \\
A_{41} &= 0.0476811 \\
A_{40} &= 0.0449945 \\
A_{39} &= 0.0424794
\end{aligned}
$$

The EPV of the death benefit is therefore $500000 \times 0.0424794 = \$21,239.70$.

Standard Questions

5. *A select individual aged 42 has a 15-year term insurance with a benefit of $800,000 payable at the end of the year of death provided death occurs within 15 years. The individual wants to convert this to a whole life insurance policy. If the current interest rate is* $i = 0.06$*, what benefit for the whole life policy would have the same EPV as the term insurance policy? [The company has already calculated that* $A_{[54]+3} = 0.150748$ *and* $A_{[42]} = 0.07106117$*.]*

   We calculate

   $$A^{1}_{[42]:\overline{15}|} = A_{[42]} - {}_{15}p_{[42]}(1.06)^{-15}A_{57} = 0.07106117 - \frac{9788.18}{9944.32}(1.06)^{-15} \times 0.150748 = 0.009146946$$

   So the EPV of the term insurance policy is

   $$800000 \times 0.009146946 = \$7,317.56$$

   We want to find the benefit for a whole-life insurance policy with the same EPV. That is, we want to solve

   $$
   \begin{aligned}
   0.07106117B &= 7317.56 \\
   B &= \$102,975.46
   \end{aligned}
   $$

6. *A woman aged 30 buys a house with a mortgage of \$600,000. She amortises this amount with annual payments over a period of 30 years at* $i = 0.06$*. She takes out mortgage insurance, which pays off the outstanding balance (principle plus interest) of the mortgage at the end of the year in which she dies. [Assume that the mortgage company does not charge a penalty for early repayment in this case.] If the insurance company uses an interest rate* $i = 0.03$ *and the ultimate part of the life table from Table 1, calculate the expected present value of the benefit on this policy. You are given the following values, some of which may be useful:*

i	$A_{30:\overline{31}\vert}$
-0.03	*2.55429*
-0.02830189	*2.42021*
-0.02	*1.86207*
-0.01941748	*1.8283*
-0.01904762	*1.80718*
0.01904762	*0.560387*
0.01941748	*0.55419*
0.02	*0.544574*
0.02830189	*0.4249*
0.03	*0.403993*
0.06	*0.168561*

We first want to calculate the outstanding value at the end of each year.

The annual payments on the mortgage are given by

$$\frac{600000}{a_{\overline{30}|0.03}} = \frac{600000 \times 0.06}{1 - (1.06)^{-30}} = \$43,589.35$$

The outstanding balance after n years is given by

$$43589.35\ddot{a}_{\overline{31-n}|0.06} = 43589.35 \times 1.06 \times \frac{1 - (1.06)^{-(31-n)}}{0.06} = 770078.46 - 126489.11(1.06)^n$$

The EPV of benefits is therefore

$$770078.46A_{30:\overline{31}|} - 126489.11A_{30:\overline{31}|i^*}$$

where i^* is the "real" rate of interest given by

$$i^* = \frac{0.03 - 0.06}{1.06} = -0.02830189$$

[Technically, we should use term benefits here instead of endowment benefits. We justify the use of endowment benefits because the outstanding value at the end of 31 years is 0, so the expected benefit is the same whether a payment of 0 is made or not. Since the mortgage payments are rounded, so the final payment is not exactly zero, this may make a difference of several cents to the final answer.]

From the table given, we have that $A_{30:\overline{31}|i^*} = 2.42021$. Substituting this into the formula, we get that the EPV of benefits is

$$0.403993 \times 770078.46 - 2.42021 \times 126489.11 = \$4,976.10$$

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66
26	9997.00	9995.83	9994.40	9992.66
27	9995.14	9993.90	9992.38	9990.52
28	9993.16	9991.84	9990.22	9988.24
29	9991.05	9989.65	9987.92	9985.80
30	9988.81	9987.30	9985.46	9983.18
31	9986.40	9984.80	9982.82	9980.38
32	9983.83	9982.11	9979.99	9977.37
33	9981.07	9979.23	9976.95	9974.13
34	9978.11	9976.13	9973.68	9970.64
35	9974.93	9972.79	9970.16	9966.88
36	9971.50	9969.20	9966.36	9962.82
37	9967.80	9965.33	9962.25	9958.44
38	9963.81	9961.14	9957.82	9953.69
39	9959.50	9956.61	9953.02	9948.55
40	9954.84	9951.71	9947.82	9942.98
41	9949.79	9946.41	9942.19	9936.94
42	9944.32	9940.66	9936.08	9930.38
43	9938.39	9934.41	9929.45	9923.26
44	9931.96	9927.64	9922.25	9915.52
45	9924.97	9920.28	9914.42	9907.10
46	9917.37	9912.28	9905.91	9897.94
47	9909.11	9903.58	9896.65	9887.98
48	9900.13	9894.11	9886.57	9877.13
49	9890.36	9883.80	9875.59	9865.30
50	9879.71	9872.57	9863.63	9852.42
51	9868.12	9860.34	9850.59	9838.38
52	9855.48	9847.01	9836.39	9823.08
53	9841.72	9832.48	9820.90	9806.39
54	9826.71	9816.64	9804.02	9788.18
55	9810.34	9799.37	9785.60	9768.33
56	9792.49	9780.52	9765.51	9746.67
57	9773.03	9759.97	9743.60	9723.05
58	9751.79	9737.56	9719.69	9697.28
59	9728.63	9713.10	9693.62	9669.17
60	9703.36	9686.43	9665.17	9638.51
61	9675.80	9657.33	9634.15	9605.07
62	9645.73	9625.59	9600.31	9568.61
63	9612.94	9590.98	9563.42	9528.85
64	9577.18	9553.24	9523.19	9485.52
65	9538.19	9512.09	9479.35	9438.30
66	9495.69	9467.25	9431.58	9386.86
67	9449.37	9418.39	9379.54	9330.85
68	9398.90	9365.17	9322.87	9269.88
69	9343.95	9307.23	9261.20	9203.55
70	9284.12	9244.18	9194.11	9131.43
71	9219.03	9175.59	9121.17	9053.07
72	9148.24	9101.03	9041.91	8967.97
73	9071.30	9020.03	8955.85	8875.63

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
74	8987.73	8932.10	8862.49	8775.52
75	8897.04	8836.71	8761.27	8667.10
76	8798.69	8733.34	8651.66	8549.78
77	8692.13	8621.41	8533.09	8423.00
78	8576.81	8500.36	8404.95	8286.16
79	8452.13	8369.60	8266.68	8138.66
80	8317.52	8228.53	8117.67	7979.93
81	8172.36	8076.57	7957.35	7809.41
82	8016.08	7913.13	7785.15	7626.56
83	7848.11	7737.67	7600.54	7430.89
84	7667.89	7549.66	7403.05	7221.99
85	7474.92	7348.64	7192.27	6999.51
86	7268.77	7134.21	6967.86	6763.22
87	7049.07	6906.07	6729.62	6513.04
88	6815.55	6664.05	6477.46	6249.02
89	6568.09	6408.10	6211.48	5971.42
90	6306.70	6138.35	5931.96	5680.73
91	6031.59	5855.15	5639.41	5377.67
92	5743.19	5559.08	5334.61	5063.27
93	5442.15	5250.97	5018.61	4738.86
94	5129.44	4931.97	4692.79	4406.12
95	4806.33	4603.54	4358.89	4067.08
96	4474.39	4267.51	4018.96	3724.10
97	4135.60	3926.04	3675.44	3379.91
98	3792.25	3581.66	3331.11	3037.57
99	3447.02	3237.23	2989.05	2700.39
100	3102.90	2895.94	2652.63	2371.88
101	2763.19	2561.21	2325.37	2055.64
102	2431.39	2236.61	2010.90	1755.27
103	2111.15	1925.80	1712.81	1474.18
104	1806.12	1632.34	1434.48	1215.44
105	1519.82	1359.55	1178.94	981.65
106	1255.46	1110.36	948.70	774.71
107	1015.81	887.14	745.58	595.71
108	802.96	691.49	570.56	444.87
109	618.23	524.17	423.71	321.41
110	462.04	385.00	304.13	223.65
111	333.80	272.80	210.00	149.10
112	231.99	185.53	138.71	94.62
113	154.19	120.34	87.07	56.74
114	97.30	73.90	51.50	31.84
115	57.78	42.55	28.41	16.52
116	31.92	22.69	14.43	7.81
117	16.15	11.04	6.63	3.30
118	7.34	4.79	2.69	1.21
119	2.90	1.79	0.93	0.37
120	0.95	0.55	0.26	0.09
121	0.23	0.13	0.05	0.01
122	0.03	0.02	0.01	0.00