ACSC/STAT 3720, Life Contingencies I

Winter 2017

Toby Kenney

Homework Sheet 5

Model Solutions

Basic Questions

1. *An insurance company offers a whole life insurance policy with benefit \$800,000 payable at the end of the year of death. The premium for this policy for a select individual aged 33 for whom the lifetable in Table 1 is appropriate, is \$1624, payable at the start of each year. If the current interest rate is $i = 0.07$, what is the probability that the present value of future loss for this policy exceeds \$250,000?*

   If the policyholder dies in the nth year, then the present value of premiums received is $1624\ddot{a}_{\overline{n}|0.07} = 1624\times 1.07\left(\frac{1-1.07^{-n}}{0.07}\right) = 24824(1-1.07^{-n})$. The present value of the insurance benefit is $800000(1.07)^{-n}$, so the present value of future loss is $824824(1.07)^{-n} - 24824$. This is more than \$250,000 whenever

$$\begin{aligned}
824824(1.07)^{-n} - 24824 &> 250000\\
824824(1.07)^{-n} &> 274824\\
\log(824824) - n\log(1.07) &> \log(274824)\\
n\log(1.07) &< \log(824824) - \log(274824)\\
n &< \frac{\log(824824) - \log(274824)}{\log(1.07)} = 16.24
\end{aligned}$$

   So the PVFL exceeds 250000 if the individual dies within the first 16 years. The probability of this is

$$_{16}q_{[33]} = 1 - {}_{16}p_{[33]} = 1 - \frac{9897.94}{9981.07} = 0.008328766$$

2. *An insurance company offers a 5-year endowment insurance policy with death benefit \$200,000 payable at the end of the year of death. If the interest rate is $i = 0.06$, calculate the annual premium for this policy for a select individual aged 47, using the lifetable in Table 1 and the equivalence principle.*

   We calculate that

$$A_{[47]:\overline{5}|} = \frac{5.53}{9909.11}(1.06)^{-1} + \frac{6.93}{9909.11}(1.06)^{-2} + \frac{8.67}{9909.11}(1.06)^{-3} + \frac{10.85}{9909.11}(1.06)^{-4} + \frac{9877.13}{9909.11}(1.06)^{-5} = 0.7475974$$

   . We then get $\ddot{a}_{[47]:\overline{5}|} = \frac{1.06}{0.06}(1 - A_{[47]:\overline{5}|}) = 4.459113$. The premium is therefore

$$\frac{200000 \times 0.7475974}{4.459113} = \$33,531.22$$

3. *The current interest rate is $i = 0.05$. An individual aged 44 to whom the ultimate part of the lifetable in Table 1 applies, wants to purchase a whole life insurance policy. Premiums are payable until age 80. The benefit of this policy should be \$1,500,000 at the end of the year of death. The initial costs to the insurance company are \$3,000 plus 10% of the first premium. Renewal costs are 4% of subsequent premiums. Calculate the Gross annual premiums for this policy. You calculate $A_{44} = 0.112997$ and $A_{80} = 0.457434$.*

   The EPV of the benefit is $1500000 \times 0.112997 = \$169,495.50$. If the gross premium is P, then the premiums less expenses are $P\left(0.96\ddot{a}_{44:\overline{36}|} - 0.06\right) - 3000$. We calculate

$$\ddot{a}_{44:\overline{36}|} = \ddot{a}_{44} - {}_{36}p_{44}(1.05)^{-36}\ddot{a}_{80} = \frac{1.05}{0.05}\left(1 - 0.112997 - (1 - 0.457434)\frac{8423.00}{9936.94}(1.05)^{-36}\right) = 16.95954$$

   We therefore get

$$\begin{aligned}(16.95954 \times 0.96 - 0.06)P - 3000 &= 169495.50\\ 16.22116P &= 172495.50\\ P &= \$10,633.98\end{aligned}$$

Standard Questions

4. *A select individual aged 49, to whom the lifetable in Table 1 applies, wants to purchase a whole life insurance policy. She can afford to pay annual premiums of \$9,300. The interest rate is $i = 0.05$, which gives $A_{[49]} = 0.1398689$. She wants to receive a death benefit of \$1,200,000. Using the equivalence principal to calculate net premiums, at what age can she stop paying premiums for the insurance?*

   You may use the following values of $f(x) = (1 - A_x)_{x-49}p_{[49]}(1.05)^{-(x-49)}$:

x	$f(x)$	x	$f(x)$	x	$f(x)$	x	$f(x)$
70	0.2281648	81	0.09288482	92	0.02711120	103	0.003734326
71	0.2120393	82	0.08451214	93	0.02358338	104	0.002915033
72	0.1967820	83	0.07668011	94	0.02038711	105	0.002238788
73	0.1823552	84	0.06936650	95	0.01750544	106	0.001688852
74	0.1687230	85	0.06255000	96	0.01492144	107	0.001248978
75	0.1558514	86	0.05621010	97	0.01261817	108	0.0009035773
76	0.1437080	87	0.05032701	98	0.01057860	109	0.0006378986
77	0.1322619	88	0.04488158	99	0.008785621	110	0.0004382116
78	0.1214839	89	0.03985522	100	0.007222023	111	0.0002919749
79	0.1113459	90	0.03522981	101	0.005870512	112	0.0001879673
80	0.1018213	91	0.03098761	102	0.004713731	113	0.0001164021

   The EPV of the benefits is $1200000 \times 0.1398689 = \$167,842.68$. We therefore want to choose the number of premiums n so that

$$\begin{aligned}9300\ddot{a}_{[49]:\overline{n}|} &\geqslant 167842.68\\ \ddot{a}_{[49]:\overline{n}|} &\geqslant 18.04753\\ \ddot{a}_{[49]} - {}_{n}p_{[49]}(1.05)^{-n}\ddot{a}_{[49]+n} &= 18.04753\end{aligned}$$

We have that $\ddot{a}_{[49]} = \frac{1.05}{0.05}(1 - A_{[49]}) = 21 \times (1 - 0.1398689) = 18.06275$. The above equation therefore becomes

$$\begin{aligned}
{}_np_{[49]}(1.05)^{-n}\ddot{a}_{[49]+n} &= 18.06275 - 18.04753 \\
{}_np_{[49]}(1.05)^{-n}\left(\frac{1.05}{0.05}\right)(1 - A_{[49]+n}) &= 0.01522 \\
{}_np_{[49]}(1.05)^{-n}(1 - A_{[49]+n}) &= 0.0007247619
\end{aligned}$$

from the table given, we see that the solution to this is when $49 + n = 109$, so she can stop paying premiums at age 109.

5. *An individual aged 45 is paying premiums of \$200 a month for a 10-year endowment insurance policy which pays benefits at the end of the month of death. The individual's mortality follows the ultimate part of Table 1, and the interest rate is* $i^{(12)} = 0.09$, *so that* $A_{45} = 0.0313219$ *and* $A_{55} = 0.0617432$, *calculate the equivalent annual premiums (for a policy which pays benefits at the end of the year of death, and has the same death benefit) using:*

   (a) Uniform distribution of deaths

   We have $A_{45} = 0.0313219$, so under UDD, we get $A_{45}^{(12)} = \frac{i}{i^{(12)}}A_{45} = \frac{0.0938069}{0.09} \times 0.0313219 = 0.03264678$ and similarly $A_{55}^{(12)} = 0.06435487$. This gives that $A_{45:\overline{10}|}^{(12)} = A_{45}^{(12)} + {}_{10}p_{45}(1.0075)^{-120}(1 - A_{55}^{(12)}) = 0.4102071$. This gives $\ddot{a}_{45:\overline{10}|}^{(12)} = \frac{1-0.4102071}{d^{(12)}}$. We have that $d^{(12)} = \frac{0.09}{1.0075} = 0.08933002$, so that $\ddot{a}_{45:\overline{10}|}^{(12)} = 6.813033$.

   The benefit is $\frac{200\times12\times6.813033}{0.4102071} = \$39,861.03$. For annual premiums we have $A_{45:\overline{10}|} = A_{45} + {}_{10}p_{45}(1.0075)^{-120}(1 - A_{55}) = 0.4099362$ and $\ddot{a}_{45:\overline{10}|} = (1 - 0.4099362)\frac{1.0075^{12}}{1.0075^{12}-1} = 6.880260$.

   The annual premium is therefore $\frac{39861.03\times0.4099362}{6.88026} = \$2,374.98$.

   (b) Woolhouse's formula

   We have $\ddot{a}_{45} = (1 - A_{45})\frac{1.0075^{12}}{1.0075^{12}-1} = 11.29498$ and $\ddot{a}_{55} = 10.94026$. From the lifetable, we get $q_{45} = 1 - \frac{9923.26}{9930.38}$ and $q_{44} = 1 - \frac{9930.38}{9936.94}$, so we approximate $\mu_{45} = 0.0006885773$. Similarly, we approximate $\mu_{55} = 0.001627097$. Force of interest is $\delta = 12\log(1.0075) = 0.08966418$ Now Woolhouse's formula gives

$$\begin{aligned}
\ddot{a}_{45}^{(12)} &= \ddot{a}_{45} - \frac{11}{24} + \frac{143}{1728}(0.08966418 + 0.0006885773) = 10.84412 \\
\ddot{a}_{55}^{(12)} &= \ddot{a}_{55} - \frac{11}{24} + \frac{143}{1728}(0.08966418 + 0.0006885773) = 10.48948
\end{aligned}$$

   This gives

$$\ddot{a}_{45:\overline{10}|}^{(12)} = 10.84412 -_{10} p_{45}(1.0075)^{-120} \times 10.48948 = 6.611306$$

   Hence we get

$$A_{45:\overline{10}|}^{(12)} = 1 - 0.08933002 \times 6.611306 = 0.4094119$$

   This gives a benefit of

$$\frac{200 \times 12 \times 6.611306}{0.4094119} = \$38,755.92$$

The premium for an annual policy with this benefit is therefore

$$\frac{38755.92 \times 0.4099362}{6.88026} = \$2,309.14$$

6. *An insurance company provides a regular annual premium annuity contract to a select individual aged 44, using the lifetable in Table 1. The interest rate is $i = 0.06$. This gives that $A_{[62]+3} = 0.218135$, $A_{[67]+3} = 0.270910$ and $A_{[44]} = 0.07872046$. The individual will pay annual net premiums until age 65 (so the last premium will be at age 64). From age 65, they will receive an annuity of \$30,000 at the start of each year. Premiums are calculated using the equivalence principle. The annuity is guaranteed for 5 years (regardless of whether the individual survives to age 65). What is the probability that the insurance company makes a net profit on this policy?*

   The benefits are a 5-year annuity certain from age 65 to 70, which has present value $30000 \times (1.06)^{-21}\frac{1.06-(1.06)^{-4}}{0.06} = 39403.05$ and a deferred life annuity starting from age 70, which has EPV $30000 \times (1.06)^{-26}_{26}p_{[44]}\ddot{a}_{70} = 30000 \times (1.06)^{-26}\frac{9330.85}{9931.96} \times (1-0.270910) \times \frac{1.06}{0.06} = \$79797.77$. The total EPV of benefits is therefore \$119,200.82.

   We calculate $A_{[44]:\overline{21}|} = A_{[44]} + (1 - A_{65})_{21}p_{[44]}(1.06)^{-21} = 0.3002963$ and so $\ddot{a}_{[44]:\overline{21}|} = \frac{1.06}{0.06}(1 - 0.3002963) = 12.36143$. The premium is therefore $\frac{119200.82}{12.36143} = \$9,642.96$.

   To make a profit, the present value of premiums received must exceed the guaranteed annuity value of \$39,403.05. That is, they must pay at least n premiums such that

$$\begin{aligned}
9642.96\ddot{a}_{\overline{n}|} &> 39403.05 \\
\frac{1.06}{0.06}(1 - 1.06^{-n}) &> \frac{39403.05}{9642.96} = 4.086199 \\
1 - 1.06^{-n} &> 0.2312943 \\
1.06^{-n} &< 0.76870570.2312943 \\
n &> -\frac{\log(0.7687057)}{\log(1.06)} = 4.514364
\end{aligned}$$

   so they must make at least 5 payments, which happens if they survive 4 years. If they survive to age 65, the accumulated value at age 65 of the premiums received is $9642.96\left(\frac{1.06^{21}-1.06^{1}}{0.06}\right) = 376005.30$. To make a profit, the present value at age 65 of the annuity paid out must be less than this, that is, the number of payments m in the annuity must satisfy:

$$\begin{aligned}
30000 \times \frac{1.06}{0.06}(1 - 1.06^{-m}) &< 376005.30 \\
(1 - 1.06^{-m}) &< 0.709444 \\
1.06^{-m} &> 0.290556 \\
m &< -\frac{\log(0.290556)}{\log(1.06)} = 21.211
\end{aligned}$$

So they must die before the 22nd payment of the annuity is due at age 87. The policy makes a net profit if the age at death is between 48 and 87. The probability of this is $\frac{9907.10-6999.51}{9931.96} = 0.2927509$.

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66	74	8987.73	8932.10	8862.49	8775.52
26	9997.00	9995.83	9994.40	9992.66	75	8897.04	8836.71	8761.27	8667.10
27	9995.14	9993.90	9992.38	9990.52	76	8798.69	8733.34	8651.66	8549.78
28	9993.16	9991.84	9990.22	9988.24	77	8692.13	8621.41	8533.09	8423.00
29	9991.05	9989.65	9987.92	9985.80	78	8576.81	8500.36	8404.95	8286.16
30	9988.81	9987.30	9985.46	9983.18	79	8452.13	8369.60	8266.68	8138.66
31	9986.40	9984.80	9982.82	9980.38	80	8317.52	8228.53	8117.67	7979.93
32	9983.83	9982.11	9979.99	9977.37	81	8172.36	8076.57	7957.35	7809.41
33	9981.07	9979.23	9976.95	9974.13	82	8016.08	7913.13	7785.15	7626.56
34	9978.11	9976.13	9973.68	9970.64	83	7848.11	7737.67	7600.54	7430.89
35	9974.93	9972.79	9970.16	9966.88	84	7667.89	7549.66	7403.05	7221.99
36	9971.50	9969.20	9966.36	9962.82	85	7474.92	7348.64	7192.27	6999.51
37	9967.80	9965.33	9962.25	9958.44	86	7268.77	7134.21	6967.86	6763.22
38	9963.81	9961.14	9957.82	9953.69	87	7049.07	6906.07	6729.62	6513.04
39	9959.50	9956.61	9953.02	9948.55	88	6815.55	6664.05	6477.46	6249.02
40	9954.84	9951.71	9947.82	9942.98	89	6568.09	6408.10	6211.48	5971.42
41	9949.79	9946.41	9942.19	9936.94	90	6306.70	6138.35	5931.96	5680.73
42	9944.32	9940.66	9936.08	9930.38	91	6031.59	5855.15	5639.41	5377.67
43	9938.39	9934.41	9929.45	9923.26	92	5743.19	5559.08	5334.61	5063.27
44	9931.96	9927.64	9922.25	9915.52	93	5442.15	5250.97	5018.61	4738.86
45	9924.97	9920.28	9914.42	9907.10	94	5129.44	4931.97	4692.79	4406.12
46	9917.37	9912.28	9905.91	9897.94	95	4806.33	4603.54	4358.89	4067.08
47	9909.11	9903.58	9896.65	9887.98	96	4474.39	4267.51	4018.96	3724.10
48	9900.13	9894.11	9886.57	9877.13	97	4135.60	3926.04	3675.44	3379.91
49	9890.36	9883.80	9875.59	9865.30	98	3792.25	3581.66	3331.11	3037.57
50	9879.71	9872.57	9863.63	9852.42	99	3447.02	3237.23	2989.05	2700.39
51	9868.12	9860.34	9850.59	9838.38	100	3102.90	2895.94	2652.63	2371.88
52	9855.48	9847.01	9836.39	9823.08	101	2763.19	2561.21	2325.37	2055.64
53	9841.72	9832.48	9820.90	9806.39	102	2431.39	2236.61	2010.90	1755.27
54	9826.71	9816.64	9804.02	9788.18	103	2111.15	1925.80	1712.81	1474.18
55	9810.34	9799.37	9785.60	9768.33	104	1806.12	1632.34	1434.48	1215.44
56	9792.49	9780.52	9765.51	9746.67	105	1519.82	1359.55	1178.94	981.65
57	9773.03	9759.97	9743.60	9723.05	106	1255.46	1110.36	948.70	774.71
58	9751.79	9737.56	9719.69	9697.28	107	1015.81	887.14	745.58	595.71
59	9728.63	9713.10	9693.62	9669.17	108	802.96	691.49	570.56	444.87
60	9703.36	9686.43	9665.17	9638.51	109	618.23	524.17	423.71	321.41
61	9675.80	9657.33	9634.15	9605.07	110	462.04	385.00	304.13	223.65
62	9645.73	9625.59	9600.31	9568.61	111	333.80	272.80	210.00	149.10
63	9612.94	9590.98	9563.42	9528.85	112	231.99	185.53	138.71	94.62
64	9577.18	9553.24	9523.19	9485.52	113	154.19	120.34	87.07	56.74
65	9538.19	9512.09	9479.35	9438.30	114	97.30	73.90	51.50	31.84
66	9495.69	9467.25	9431.58	9386.86	115	57.78	42.55	28.41	16.52
67	9449.37	9418.39	9379.54	9330.85	116	31.92	22.69	14.43	7.81
68	9398.90	9365.17	9322.87	9269.88	117	16.15	11.04	6.63	3.30
69	9343.95	9307.23	9261.20	9203.55	118	7.34	4.79	2.69	1.21
70	9284.12	9244.18	9194.11	9131.43	119	2.90	1.79	0.93	0.37
71	9219.03	9175.59	9121.17	9053.07	120	0.95	0.55	0.26	0.09
72	9148.24	9101.03	9041.91	8967.97	121	0.23	0.13	0.05	0.01
73	9071.30	9020.03	8955.85	8875.63	122	0.03	0.02	0.01	0.00