ACSC/STAT 3720, Life Contingencies I
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Homework Sheet 6
Model Solutions

Basic Questions

1. *An insurer issues 3,000 whole life insurance policies to standard lives aged 45. The appropriate interest rate is $i = 0.06$. The company calculates $A_{45} = 0.0832768$ and ${}^2A_{45} = 0.0167276$. If the death benefit is \$900,000, what annual premium should the company charge using the portfolio percentile method with a 95% probability of making a profit?*

   For a single policy, if the premium is P, then the EPV of the profit is $\ddot{a}_{45}P - 900000A_{45} = 16.1954432P - 74949.12$.

   If the life dies in year N, then the present value of profit is $P\frac{1.06-(1.06)^{1-n}}{0.06} - 900000(1.06)^{-N} = 17.66666667P - (900000 + 17.666666667P)(1.06)^{-N}$. The variance of 1.06^{-N} is ${}^2A_{45} - (A_{45})^2 = 0.0167276 - 0.0832768^2 = 0.009792574582$. Therefore the variance of profit on a single policy is $0.009792574582(900000+17.666666667P)^2$. The probability of a loss on the portfolio is therefore

$$\Phi\left(\frac{(16.1954432P - 74949.12)\sqrt{3000}}{\sqrt{0.009792574582(900000 + 17.666666667P)^2}}\right)$$

   Setting this equal to 0.95 gives:

$$\begin{aligned}
\Phi\left(\frac{(16.1954432P - 74949.12)\sqrt{3000}}{\sqrt{0.009792574582(900000 + 17.666666667P)^2}}\right) &= 0.95\\
\frac{(16.1954432P - 74949.12)\sqrt{3000}}{\sqrt{0.009792574582(900000 + 17.666666667P)^2}} &= 1.644853627\\
(16.1954432P - 74949.12)\sqrt{\frac{3000}{0.009792574582}} &= 1.644853627(900000 + 17.666666667P)\\
16.1954432P - 74949.12 &= 2674.592256 + 0.0525012554P\\
P &= \frac{77623.71}{16.14294194} = \$4,808.52
\end{aligned}$$

2. *Using the lifetable in Table 1, and interest rate $i = 0.05$, calculate the net annual premium for a 5-year endowment insurance policy with benefit \$350,000, sold to a standard life aged 44, if:*

   (a) The life works in a hazardous environment, and has mortality 0.012 higher than normal.

   For this higher than normal mortality, we have that $\ddot{a}_{44:\overline{5}|}$ is the value of $\ddot{a}_{44:\overline{5}|}$ calculated at a force of interest which is 0.012 higher than the true force of interest. That is at $i = e^{(\log(1.05)+0.012)} - 1 = 1.05e^{0.012} - 1 = 0.0626759$. Using the standard recurrence, we get

$A_{44:\overline{5}|} = 0.738251$, so $\ddot{a}_{44:\overline{5}|} = \frac{0.0626759(1-0.738251)}{0.0626759} = 4.437979$. Now to find the value of $A_{44:\overline{5}|}$, for this life, we calculate it from $\ddot{a}_{44:\overline{5}|}$ at the actual interest rate, to get

$$A_{44:\overline{5}|} = 1 - \frac{0.05}{1.05}\ddot{a}_{44:\overline{5}|} = 0.7886677$$

The premium is therefore $\frac{0.7886677\times 350000}{4.437979} = \$62,198.06$.

(b) The life is an impaired life, and is treated as a life five years older than its actual age.

Using the usual recurrence, we calculate $A_{49:\overline{5}|} = 0.7382510.783978$, and therefore $\ddot{a}_{49:\overline{5}} = \frac{1.05}{0.05}(1-0.783978) = 4.536462$. This gives the premium as $\frac{0.783978\times 350000}{4.536462} = \$60,485.97$.

3. *An insurance company has a whole life insurance policy for an individual aged 52. The death benefit of this policy is \$800,000, and the interest rate is $i = 0.06$. Premiums are payable until age 80. The insurance company calculates $A_{52} = 0.118287$, and $A_{80} = 0.400802$. Therefore, the net annual premium for the policy is \$6,852.85. What is the policy value if the life survives to age 60? [Use the lifetable in Table 1. $A_{65} = 0.218135$.]*

   (a) Using the same basis as the premium basis, which gives $A_{60} = 0.173662$.

   We calculate $\ddot{a}_{60} = \frac{1.06(1-0.173662)}{0.06} = 14.59864$, and $\ddot{a}_{80} = \frac{1.06(1-0.400802)}{0.06} = 10.58583$. This gives $\ddot{a}_{60:\overline{20}|} = 14.59864 - \frac{8423.00}{9723.05}10.58583(1.06)^{-20} = 11.73926$. This allows us to calculate

$$_{13}V = 800000\times 0.173662 - 6852.85\times 11.73926 = \$58,482.21$$

   (b) Using the reserve basis $i = 0.04$, which gives $A_{80} = 0.526062$, $A_{60} = 0.28965$, $A_{52} = 0.220514$, $\ddot{a}_{80} = 12.32239$, $\ddot{a}_{60} = 18.46910$ and $\ddot{a}_{52} = 20.26664$.

   Using the reserve basis, we calculate $\ddot{a}_{52:\overline{28}} = 20.26664 - \frac{8423.00}{9865.30}(1.04)^{-28}\times 12.32239 = 16.75817$ so the premium is $\frac{0.220514\times}{16.75817} = \$10,526.88$.

   Using this premium, at age 60, we calculate $\ddot{a}_{60:\overline{20}} = 18.46910 - \frac{8423.00}{9723.05}(1.04)^{-20}\times 12.32239 = 13.59727$, so the policy value is $800000\times -10526.88\times 13.59727 = \$88,583.17$.

Standard Questions

4. *A life insurance company sells 5-year term insurance policies to lives aged 34 for whom the Ultimate part of the lifetable in Table 1 is appropriate. The death benefit is \$600,000. The interest rate is $i = 0.04$. This gives $A_{34} = 0.115052$ and $A_{39} = 0.138327$, and also ${}^2A_{34} = 0.0211068$ and ${}^2A_{39} = 0.0292477$. Using the portfolio premium principle with a 95% probability of profit, they calculate a premium of \$360.62. How many policies are they including in the portfolio?*

   $A_{34:\overline{5}|} = 0.001557329$ $\ddot{a}_{34:\overline{5}|} = 4.627004$

   If premium is P, then EPV of profit is $4.627004P - 934.3974$, and if death occurs in year T for $T \leqslant 5$, then profit is $\frac{1.04}{0.04}(1-1.04^{-T})P - 600000(1.04)^{-T} = 26P - (26P+600000)(1.04)^{-T}$, so the variance of this profit is $(26P+600000)^2\,\mathrm{Var}\left((1.04)^{-T}\right)$.

   The expected value of this profit is $26P - (26P+600000)\mathbb{E}((1.04)^{-T})$

   Conditional on $T \leqslant 5$, T has the following distribution

n	$P(T=n)$
1	$\frac{3.01}{17.56} = 0.1714123$
2	$\frac{3.24}{17.56} = 0.1845103$
3	$\frac{3.49}{17.56} = 0.1987472$
4	$\frac{3.76}{17.56} = 0.2141230$
5	$\frac{4.06}{17.56} = 0.2312073$

so $\mathbb{E}((1.04)^{-T})^2) = 0.7859261$ and $\mathbb{E}((1.04)^{-T})) = 0.8851639$, so the conditional variance is $0.7859261 - 0.8851639^2 = 0.00241097$.

The expected profit if $T \leqslant 5$ is $2.985739P - 531098.34$. If $T > 5$, the profit is $\frac{1.04}{0.04}(1 - 1.04^{-5})P = 4.629895P$

The total variance of profit on a policy is therefore

$$\frac{17.56}{9980.38} \times 0.00241097 \times (26P + 600000)^2 + (1.644156P + 531098.34)^2 \left(\frac{17.56}{9980.38}\right)\left(\frac{9962.82}{9980.38}\right)$$

Substituting $P = 360.62$, we get that the expected profit per policy is $4.627004P - 934.3974 = 734.1927825$ and the variance is 498089419.3. For n policies, The probability that the profit is more than zero is therefore $\Phi\left(\sqrt{n}\frac{734.1927825}{\sqrt{498089419.3}}\right)$. Setting this equal to 0.95 gives

$$\begin{aligned}
\Phi\left(\sqrt{n}\frac{734.1927825}{\sqrt{498088784.6}}\right) &= 0.95 \\
\frac{734.1927825}{\sqrt{498089419.3}} &= \frac{1.644854}{\sqrt{n}} \\
n &= \left(\frac{1.644854}{0.03289701196}\right)^2 = 2500
\end{aligned}$$

so they are using 2500 policies.

5. *A select life aged 39 takes out a whole life insurance with benefit \$600,000. The initial cost of this insurance is \$2000 plus 20% of the first annual premium. The renewal cost is 3% of each subsequent premium. The interest rate is $i = 0.05$. Using the lifetable in Table 1, we can calculate $A_{42} = 0.103456$.*

   (a) Calculate the gross premium for this policy.

   Using the standard recurrence, we calculate

$$\begin{aligned}
A_{[39]+2} &= 0.098913 \\
A_{[39]+1} &= 0.09451229 \\
A_{[39]} &= 0.09026194
\end{aligned}$$

This gives $\ddot{a}_{[39]} = \frac{1.05(1-0.09026194)}{0.05} = 19.1045$. The EPV of benefits is therefore $600000 \times 0.09026194 = \$54,157.16$, while the cost of premiums less expenses is $(19.1045 \times 0.97 - 0.17)P - 2000 = 18.36136P - 2000$. We therefore need to solve

$$
\begin{aligned}
18.36136P - 2000 &= 54157.16 \\
P &= \frac{56157.16}{18.36136} = \$3,058.44
\end{aligned}
$$

(b) Calculate the gross policy value after 2 years.

After 2 years, we have $A_{[39]+2} = 0.098913$, so $\ddot{a}_{[39]+2} = \frac{1.05(1-0.098913)}{0.05} = 18.92283$ so the policy value is

$$
{}_2V = 600000 \times 0.098913 - 3058.44 \times 0.97 \times 18.92283 = \$3,209.69
$$

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66
26	9997.00	9995.83	9994.40	9992.66
27	9995.14	9993.90	9992.38	9990.52
28	9993.16	9991.84	9990.22	9988.24
29	9991.05	9989.65	9987.92	9985.80
30	9988.81	9987.30	9985.46	9983.18
31	9986.40	9984.80	9982.82	9980.38
32	9983.83	9982.11	9979.99	9977.37
33	9981.07	9979.23	9976.95	9974.13
34	9978.11	9976.13	9973.68	9970.64
35	9974.93	9972.79	9970.16	9966.88
36	9971.50	9969.20	9966.36	9962.82
37	9967.80	9965.33	9962.25	9958.44
38	9963.81	9961.14	9957.82	9953.69
39	9959.50	9956.61	9953.02	9948.55
40	9954.84	9951.71	9947.82	9942.98
41	9949.79	9946.41	9942.19	9936.94
42	9944.32	9940.66	9936.08	9930.38
43	9938.39	9934.41	9929.45	9923.26
44	9931.96	9927.64	9922.25	9915.52
45	9924.97	9920.28	9914.42	9907.10
46	9917.37	9912.28	9905.91	9897.94
47	9909.11	9903.58	9896.65	9887.98
48	9900.13	9894.11	9886.57	9877.13
49	9890.36	9883.80	9875.59	9865.30
50	9879.71	9872.57	9863.63	9852.42
51	9868.12	9860.34	9850.59	9838.38
52	9855.48	9847.01	9836.39	9823.08
53	9841.72	9832.48	9820.90	9806.39
54	9826.71	9816.64	9804.02	9788.18
55	9810.34	9799.37	9785.60	9768.33
56	9792.49	9780.52	9765.51	9746.67
57	9773.03	9759.97	9743.60	9723.05
58	9751.79	9737.56	9719.69	9697.28
59	9728.63	9713.10	9693.62	9669.17
60	9703.36	9686.43	9665.17	9638.51
61	9675.80	9657.33	9634.15	9605.07
62	9645.73	9625.59	9600.31	9568.61
63	9612.94	9590.98	9563.42	9528.85
64	9577.18	9553.24	9523.19	9485.52
65	9538.19	9512.09	9479.35	9438.30
66	9495.69	9467.25	9431.58	9386.86
67	9449.37	9418.39	9379.54	9330.85
68	9398.90	9365.17	9322.87	9269.88
69	9343.95	9307.23	9261.20	9203.55
70	9284.12	9244.18	9194.11	9131.43
71	9219.03	9175.59	9121.17	9053.07
72	9148.24	9101.03	9041.91	8967.97
73	9071.30	9020.03	8955.85	8875.63
74	8987.73	8932.10	8862.49	8775.52
75	8897.04	8836.71	8761.27	8667.10
76	8798.69	8733.34	8651.66	8549.78
77	8692.13	8621.41	8533.09	8423.00
78	8576.81	8500.36	8404.95	8286.16
79	8452.13	8369.60	8266.68	8138.66
80	8317.52	8228.53	8117.67	7979.93
81	8172.36	8076.57	7957.35	7809.41
82	8016.08	7913.13	7785.15	7626.56
83	7848.11	7737.67	7600.54	7430.89
84	7667.89	7549.66	7403.05	7221.99
85	7474.92	7348.64	7192.27	6999.51
86	7268.77	7134.21	6967.86	6763.22
87	7049.07	6906.07	6729.62	6513.04
88	6815.55	6664.05	6477.46	6249.02
89	6568.09	6408.10	6211.48	5971.42
90	6306.70	6138.35	5931.96	5680.73
91	6031.59	5855.15	5639.41	5377.67
92	5743.19	5559.08	5334.61	5063.27
93	5442.15	5250.97	5018.61	4738.86
94	5129.44	4931.97	4692.79	4406.12
95	4806.33	4603.54	4358.89	4067.08
96	4474.39	4267.51	4018.96	3724.10
97	4135.60	3926.04	3675.44	3379.91
98	3792.25	3581.66	3331.11	3037.57
99	3447.02	3237.23	2989.05	2700.39
100	3102.90	2895.94	2652.63	2371.88
101	2763.19	2561.21	2325.37	2055.64
102	2431.39	2236.61	2010.90	1755.27
103	2111.15	1925.80	1712.81	1474.18
104	1806.12	1632.34	1434.48	1215.44
105	1519.82	1359.55	1178.94	981.65
106	1255.46	1110.36	948.70	774.71
107	1015.81	887.14	745.58	595.71
108	802.96	691.49	570.56	444.87
109	618.23	524.17	423.71	321.41
110	462.04	385.00	304.13	223.65
111	333.80	272.80	210.00	149.10
112	231.99	185.53	138.71	94.62
113	154.19	120.34	87.07	56.74
114	97.30	73.90	51.50	31.84
115	57.78	42.55	28.41	16.52
116	31.92	22.69	14.43	7.81
117	16.15	11.04	6.63	3.30
118	7.34	4.79	2.69	1.21
119	2.90	1.79	0.93	0.37
120	0.95	0.55	0.26	0.09
121	0.23	0.13	0.05	0.01
122	0.03	0.02	0.01	0.00