ACSC/STAT 3720, Life Contingencies I
Winter 2018
Toby Kenney
Homework Sheet 3
Model Solutions

Basic Questions

1. *Calculate the expected benefit of a whole life insurance sold to an individual aged 105, if the death benefit is \$460,000 at the end of the year of death, the lifetable is Table 1, and the interest rate is* $i = 0.04$.

$$
\begin{aligned}
A_{125} &= 1 \\
A_{124} &= \frac{1}{1.04} = 0.961538 \\
A_{123} &= \frac{\left(0.961538 \times \frac{0.01}{0.09} + \frac{0.08}{0.09}\right)}{1.04} = 0.957429 \\
A_{122} &= \frac{\left(0.957429 \times \frac{0.09}{0.37} + \frac{0.28}{0.37}\right)}{1.04} = 0.951582 \\
A_{121} &= 0.947302 \\
A_{120} &= 0.942959 \\
A_{119} &= 0.938364 \\
A_{118} &= 0.93352 \\
A_{117} &= 0.928352 \\
A_{116} &= 0.922879 \\
A_{115} &= 0.917079 \\
A_{114} &= 0.910935 \\
A_{113} &= 0.904445 \\
A_{112} &= 0.897605 \\
A_{111} &= 0.890405 \\
A_{110} &= 0.882844 \\
A_{109} &= 0.874915 \\
A_{108} &= 0.866619 \\
A_{107} &= 0.857957 \\
A_{106} &= 0.848929 \\
A_{105} &= 0.83954
\end{aligned}
$$

So the expected present value of the benefit is $0.839540 \times 460000 = \$386,188.40$.

2. *Calculate the expected benefit, and the variance of the benefit of a 5-year endowment policy with benefit \$500,000 at the end of year of death of the policyholder or after 5 years, whichever*

comes first. The lifetable for this policy is Table 1, and the interest rate is $i = 0.06$. The policy is sold to a select individual aged 36.

We calculate

$$
\begin{aligned}
A_{41:\overline{0}|} &= 1\\
A_{40:\overline{1}|} &= \frac{1}{1.06} = 0.943396226415\\
A_{39:\overline{2}|} &= \frac{\left(0.943396226415 \times \frac{9958.44}{9962.82} + \frac{4.38}{9962.82}\right)}{1.06} = 0.890019916406\\
A_{[36]+2:\overline{3}|} &= \frac{\left(0.890019916406 \times \frac{9962.82}{9966.36} + \frac{3.54}{9966.36}\right)}{1.06} = 0.839678283743\\
A_{[36]+1:\overline{4}|} &= \frac{\left(0.839678283743 \times \frac{9966.36}{9969.20} + \frac{2.84}{9969.20}\right)}{1.06} = 0.792192411113\\
A_{[36]:\overline{5}|} &= \frac{\left(0.792192411113 \times \frac{9969.20}{9971.50} + \frac{2.30}{9971.50}\right)}{1.06} = 0.74739655044
\end{aligned}
$$

The expected square of the benefit is calculated in the same way with double the force of interest, which is $i = (1.06)^2 - 1 = 0.1236$. This gives:

$$
\begin{aligned}
{}^2A_{41:\overline{0}|} &= 1\\
{}^2A_{40:\overline{1}|} &= \frac{1}{1.1236} = 0.889996440014\\
{}^2A_{39:\overline{2}|} &= \frac{\left(0.889996440014 \times \frac{9958.44}{9962.82} + \frac{4.38}{9962.82}\right)}{1.1236} = 0.792136704682\\
{}^2A_{[36]+2:\overline{3}|} &= \frac{\left(0.792136704682 \times \frac{9962.82}{9966.36} + \frac{3.54}{9966.36}\right)}{1.1236} = 0.705064557368\\
{}^2A_{[36]+1:\overline{4}|} &= \frac{\left(0.705064557368 \times \frac{9966.36}{9969.20} + \frac{2.84}{9969.20}\right)}{1.1236} = 0.627579723938\\
{}^2A_{[36]:\overline{5}|} &= \frac{\left(0.627579723938 \times \frac{9969.20}{9971.50} + \frac{2.30}{9971.50}\right)}{1.1236} = 0.558620172144
\end{aligned}
$$

The variance of a \$1 benefit is therefore

$$
{}^2A_{[36]:\overline{5}|} - (A_{[36]:\overline{5}|})^2 = 0.558620172144 - 0.74739655044^2 = 0.000018568534
$$

The expected benefit is thus $500000 \times 0.74739655044 = \$373,698.28$, and the variance of the benefit is $500000^2 \times 0.000018568534 = 4,642,133.5$.

3. *A select individual aged 52 purchases a 5-year term insurance policy with a benefit of \$700,000 payable immediately upon the death of the individual. Force of interest is $\delta = 0.038$. The expected benefit from a 5-year term policy for this inidividual with payment at the end of year of death would be \$4,857.62. Using a uniform distribution of deaths assumption, calculate the expected benefit from the policy with payment immediately upon death.*

Under UDD, we have $\overline{A}^{1}_{52:\overline{5}|} = \frac{i}{\delta} A^{1}_{52:\overline{5}|} = \frac{e^{0.038}-1}{0.038} \times \frac{4857.62}{700000}$, so the EPV of the benefit of a continuous policy is $\frac{e^{0.038}-1}{0.038} \times 4857.62 = \$4,951.10$.

4. *An individual aged 36 wants to purchase whole life insurance that pays a benefit at the end of the year of death. The interest rate is $i = 0.04$. The individual has a number of dangerous hobbies and uses the special lifetable:*

x	l_x	d_x
36	10000.00	15.81
37	9984.19	16.23
38	9967.96	16.71
39	9951.25	17.25
40	9934.00	17.86

After age 40, the individual will be too old to participate in these hobbies and will use a standard lifetable, which will give the value $A_{40} = 0.143482$. Calculate the EPV of the benefit for this individual from a whole-life policy which has a death benefit of \$400,000.

Starting from $A_{40} = 0.143482$, we use the standard recurrence:

$$\begin{aligned}
A_{39} &= \frac{\left(0.143482 \times \frac{9934.00}{9951.25} + \frac{17.25}{9951.25}\right)}{1.04} = 0.139391088093 \\
A_{38} &= \frac{\left(0.139391088093 \times \frac{9951.25}{9967.96} + \frac{16.71}{9967.96}\right)}{1.04} = 0.135417103841 \\
A_{37} &= \frac{\left(0.135417103841 \times \frac{9967.96}{9984.19} + \frac{16.23}{9984.19}\right)}{1.04} = 0.131560138348 \\
A_{36} &= \frac{\left(0.131560138348 \times \frac{9984.19}{10000.00} + \frac{15.81}{10000.00}\right)}{1.04} = 0.127820328624
\end{aligned}$$

The EPV of the benefit is therefore $400000 \times 0.127820328624 = \$51,128.13$.

Standard Questions

5. *An individual aged 39 who follows the ultimate part of the lifetable in Table 1 has a 15-year endowment insurance with a benefit of \$600,000 payable at the end of the year of death or after 15 years, whichever is sooner. The individual wants to convert this to a whole life insurance policy. If the current interest rate is $i = 0.04$, what benefit for the whole life policy would have the same EPV as the endowment insurance policy? [The company has already calculated that $A_{39} = 0.138327$ and $A_{54} = 0.236372$.]*

The EPV of the endowment policy is $600000A_{39:\overline{15}|}$. We have that $A_{39:\overline{15}|} = A_{39} + {}_{15}p_{39}(1.04)^{-14}(1 - A_{54}) = 0.138327 + \frac{9838.38}{9962.82}(1.04)^{-15}(1 - 0.236372) = 0.557046381477$, so the EPV is $0.557046381477 \times 600000 = 334227.828886$. The benefit of a whole life policy with this EPV is

$$\frac{334227.828886}{A_{39}} = \frac{334227.828886}{0.138327} = \$2,416,215.41$$

6. *A woman aged 35 buys a house with a mortgage of \$400,000. She amortises this amount with annual payments over a period of 25 years at $i = 0.05$. She takes out mortgage insurance, which pays off the outstanding balance (principle plus interest) of the mortgage at the end of the year in which she dies. [Assume that the mortgage company does not charge a penalty for early repayment in this case.] If the insurance company uses an interest rate $i = 0.03$ and the ultimate part of the life table from Table 1, calculate the expected present value of the benefit on this policy. You are given the following values, some of which may be useful:*

i	$A_{35:\overline{26}\rvert}$
−0.03	2.1951
−0.02830189	2.09808
−0.02	1.68411
−0.01941748	1.65847
−0.01904762	1.64239
0.01904762	0.615276
0.01941748	0.609564
0.02	0.600679
0.02830189	0.487784
0.03	0.467563
0.05	0.285768
0.06	0.224397

The mortgage payments R satisfy

$$\begin{aligned}
Ra_{\overline{25}\rvert 0.05} &= 400000\\
R\frac{1-1.05^{-25}}{0.05} &= 400000\\
R &= \frac{20000}{1-1.05^{-25}}\\
&= 28380.9829197
\end{aligned}$$

After n years, the accumulated value of the original debt is $400000(1.05)^n$, while the accumulated value of payments made (excluding the payment after n years, because that payment won't be made if the individual dies) is $R\frac{1.05^n-1.05}{0.05}$, so the outstanding balance is

$$\begin{aligned}
&400000(1.05)^n - \frac{R}{0.05}(1.05^n - 1.05)\\
=&(400000 - \frac{28380.9829197}{0.05})1.05^n + 1.05\frac{28380.9829197}{0.05}\\
=&596000.641314 - 167619.6583941.05^n
\end{aligned}$$

The EPV of the benefit of the mortgage insurance is therefore

$$596000.641314A^{1}_{35:\overline{25}\rvert} - 167619.658394A^{*1}_{35:\overline{25}\rvert}$$

where $A^{*1}_{35:\overline{25}\rvert}$ uses the "real" rate of interest $\frac{0.03-0.05}{1.05} = -0.0190476190476$. To use the numbers in the table, we have to note that after 26 years, the mortgage is paid off, so $596000.641314 - 167619.6583941.05^{26} = 0$, so

$$596000.641314A^{1}_{35:\overline{25}\rvert}-167619.658394A^{*1}_{35:\overline{25}\rvert} = 596000.641314A^{1}_{35:\overline{26}\rvert}-167619.658394A^{*1}_{35:\overline{26}\rvert} = 596000.641314A_{35:}$$

This gives that the EPV is

$$596000.641314 \times 0.467563 - 167619.6583941.64239 = \$3371.00$$

[Note: for mathematical convenience, we have assumed exact mortgage payments. In practice, mortgage payments would be rounded up to the nearest cent, with a slightly reduced final payment. The same method can still be used — the constants in the outstanding balance formula are changed slightly, and the formula for converting to an endowment insurance needs to be adjusted for the reduction in final payment.]

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66
26	9997.00	9995.83	9994.40	9992.66
27	9995.14	9993.90	9992.38	9990.52
28	9993.16	9991.84	9990.22	9988.24
29	9991.05	9989.65	9987.92	9985.80
30	9988.81	9987.30	9985.46	9983.18
31	9986.40	9984.80	9982.82	9980.38
32	9983.83	9982.11	9979.99	9977.37
33	9981.07	9979.23	9976.95	9974.13
34	9978.11	9976.13	9973.68	9970.64
35	9974.93	9972.79	9970.16	9966.88
36	9971.50	9969.20	9966.36	9962.82
37	9967.80	9965.33	9962.25	9958.44
38	9963.81	9961.14	9957.82	9953.69
39	9959.50	9956.61	9953.02	9948.55
40	9954.84	9951.71	9947.82	9942.98
41	9949.79	9946.41	9942.19	9936.94
42	9944.32	9940.66	9936.08	9930.38
43	9938.39	9934.41	9929.45	9923.26
44	9931.96	9927.64	9922.25	9915.52
45	9924.97	9920.28	9914.42	9907.10
46	9917.37	9912.28	9905.91	9897.94
47	9909.11	9903.58	9896.65	9887.98
48	9900.13	9894.11	9886.57	9877.13
49	9890.36	9883.80	9875.59	9865.30
50	9879.71	9872.57	9863.63	9852.42
51	9868.12	9860.34	9850.59	9838.38
52	9855.48	9847.01	9836.39	9823.08
53	9841.72	9832.48	9820.90	9806.39
54	9826.71	9816.64	9804.02	9788.18
55	9810.34	9799.37	9785.60	9768.33
56	9792.49	9780.52	9765.51	9746.67
57	9773.03	9759.97	9743.60	9723.05
58	9751.79	9737.56	9719.69	9697.28
59	9728.63	9713.10	9693.62	9669.17
60	9703.36	9686.43	9665.17	9638.51
61	9675.80	9657.33	9634.15	9605.07
62	9645.73	9625.59	9600.31	9568.61
63	9612.94	9590.98	9563.42	9528.85
64	9577.18	9553.24	9523.19	9485.52
65	9538.19	9512.09	9479.35	9438.30
66	9495.69	9467.25	9431.58	9386.86
67	9449.37	9418.39	9379.54	9330.85
68	9398.90	9365.17	9322.87	9269.88
69	9343.95	9307.23	9261.20	9203.55
70	9284.12	9244.18	9194.11	9131.43
71	9219.03	9175.59	9121.17	9053.07
72	9148.24	9101.03	9041.91	8967.97
73	9071.30	9020.03	8955.85	8875.63
74	8987.73	8932.10	8862.49	8775.52
75	8897.04	8836.71	8761.27	8667.10
76	8798.69	8733.34	8651.66	8549.78
77	8692.13	8621.41	8533.09	8423.00
78	8576.81	8500.36	8404.95	8286.16
79	8452.13	8369.60	8266.68	8138.66
80	8317.52	8228.53	8117.67	7979.93
81	8172.36	8076.57	7957.35	7809.41
82	8016.08	7913.13	7785.15	7626.56
83	7848.11	7737.67	7600.54	7430.89
84	7667.89	7549.66	7403.05	7221.99
85	7474.92	7348.64	7192.27	6999.51
86	7268.77	7134.21	6967.86	6763.22
87	7049.07	6906.07	6729.62	6513.04
88	6815.55	6664.05	6477.46	6249.02
89	6568.09	6408.10	6211.48	5971.42
90	6306.70	6138.35	5931.96	5680.73
91	6031.59	5855.15	5639.41	5377.67
92	5743.19	5559.08	5334.61	5063.27
93	5442.15	5250.97	5018.61	4738.86
94	5129.44	4931.97	4692.79	4406.12
95	4806.33	4603.54	4358.89	4067.08
96	4474.39	4267.51	4018.96	3724.10
97	4135.60	3926.04	3675.44	3379.91
98	3792.25	3581.66	3331.11	3037.57
99	3447.02	3237.23	2989.05	2700.39
100	3102.90	2895.94	2652.63	2371.88
101	2763.19	2561.21	2325.37	2055.64
102	2431.39	2236.61	2010.90	1755.27
103	2111.15	1925.80	1712.81	1474.18
104	1806.12	1632.34	1434.48	1215.44
105	1519.82	1359.55	1178.94	981.65
106	1255.46	1110.36	948.70	774.71
107	1015.81	887.14	745.58	595.71
108	802.96	691.49	570.56	444.87
109	618.23	524.17	423.71	321.41
110	462.04	385.00	304.13	223.65
111	333.80	272.80	210.00	149.10
112	231.99	185.53	138.71	94.62
113	154.19	120.34	87.07	56.74
114	97.30	73.90	51.50	31.84
115	57.78	42.55	28.41	16.52
116	31.92	22.69	14.43	7.81
117	16.15	11.04	6.63	3.30
118	7.34	4.79	2.69	1.21
119	2.90	1.79	0.93	0.37
120	0.95	0.55	0.26	0.09
121	0.23	0.13	0.05	0.01
122	0.03	0.02	0.01	0.00