ACSC/STAT 3720, Life Contingencies I
Winter 2018
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Homework Sheet 8
Model Solutions

Basic Questions

1. *A man aged 43, for whom the ultimate part of the lifetable in Table 1 is appropriate, buys a 20-year term insurance policy with a death benefit of $700,000. (The policy uses a net annual premium.) Seven years later, he wants to surrender the policy. The interest rate is $i = 0.05$. This gives $A_{43} = 0.108129$ $A_{50} = 0.146630$ and $A_{63} = 0.250390$. If the insurance company pays a cash surrender value of 75% of the net policy value, how much does he receive?*

   We have

   $$A^1_{43:\overline{20}|} = A_{43} - {}_{20}p_{43}(1.05)^{-20}A_{63} = 0.108129 - \frac{9638.51}{9942.98}(1.05)^{-20}0.250390 = 0.0166493835187$$

   and

   $$A^1_{50:\overline{13}|} = A_{50} - {}_{13}p_{50}(1.05)^{-13}A_{63} = 0.146630 - \frac{9638.51}{9887.98}(1.05)^{-13}0.250390 = 0.017193006971$$

   We also have

   $$A_{43:\overline{20}|} = A_{43} + {}_{20}p_{43}(1.05)^{-20}(1 - A_{63}) = 0.108129 + \frac{9638.51}{9942.98}(1.05)^{-20}(1 - 0.250390) = 0.381997905749$$

   and

   $$A_{50:\overline{13}|} = A_{50} + {}_{13}p_{50}(1.05)^{-13}(1 - A_{63}) = 0.146630 + \frac{9638.51}{9887.98}(1.05)^{-13}(1 - 0.250390) = 0.53413455028$$

   This gives

   $$\ddot{a}_{43:\overline{20}|} = \frac{1.05}{0.05}(1 - A_{43:\overline{20}|}) = \frac{1.05}{0.05}(1 - 0.381997905749) = 12.9780439793$$

   and

   $$\ddot{a}_{50:\overline{13}|} = \frac{1.05}{0.05}(1 - A_{50:\overline{13}|}) = \frac{1.05}{0.05}(1 - 0.53413455028) = 9.78317444412$$

   The premium is therefore $\frac{700000\times 0.0166493835187}{12.9780439793} = \$898.02$, so the policy value after 7 years is

   $${}_7V = 700000A^1_{50:\overline{13}|} - 898.02\ddot{a}_{50:\overline{13}|} = 700000\times 0.017193006971 - 898.02\times 9.78317444412 = \$3249.62$$

2. *An insurance company sells a whole life insurance policy to a life age 38 for whom the ultimate part of the lifetable in Table 1 is appropriate. The death benefit in the first year is $150,000, and the death benefit increases by 4% each year. The initial premium is $1594.86, and premiums increase by 5% each year for the first 27 years, remain constant for the next 15 years, then stop after that. The interest rate is $i = 0.06$ for the first 10 years, then $i = 0.07$ for the remainder of the policy. Calculate the retrospective policy value after 2 years.*

We use the recurrence

$$ {}_tV = (1+i)^{-1}({}_{t+1}Vp_{x+t} + Bq_{x+t}) - P $$

rewritten as

$$ {}_{t+1}V = \frac{(1+i)({}_tV + P) - Bq_{x+t}}{p_{x+t}} $$

starting from ${}_0V = 0$.

$$ \begin{aligned} {}_1V &= \frac{9966.88}{9962.82}\left(1.06(0+1594.86) - 150000 \times \frac{4.06}{9966.88}\right) \\ &= 1630.11325418 \\ {}_2V &= \frac{9962.82}{9958.44}\left(1.06(1630.11325418 + 1594.86 \times 1.05) - 150000 \times (1.05) \times \frac{4.38}{9962.82}\right) \\ &= \$3,435.27 \end{aligned} $$

3. *A man aged 55, whose mortality follows the ultimate part of Table 1, buys a whole life insurance with a benefit of \$300,000. The interest rate is* $i = 0.06$*, which gives* $A_{55} = 0.136941$*,* $A_{56} = 0.143702$*,* $A_{59} = 0.165721$ *and* $A_{60} = 0.173662$*. Using a Full preliminary term of 1 year, calculate the policy value after 4 years.*

   We calculate $\ddot{a}_{56} = \frac{1.06}{0.06}(1 - 0.143702) = 15.1279313334$. Using a full preliminary term, the premium is $\frac{300000 \times 0.143702}{15.1279313334} = 2849.73530418$. After 4 years, we have $\ddot{a}_{59} = \frac{1.06}{0.06}(1 - 0.165721) = 14.738929$, so the policy value is $300000 \times 0.165721 - 2849.73530418 \times 14.738929 = \$7,714.25$.

Standard Questions

4. *A man aged 48, who is a select life on Table 1 buys a 7-year annual term insurance policy with a benefit of \$400,000. The interest rate is* $i = 0.04$*, and you are given the following values:*

x	A_x^1	$\ddot{a}_x$
$[48] : \overline{7}\vert$	*0.006523115898*	*6.2263058803*
$[48] + 1 : \overline{6}\vert$	*0.006179751315*	*5.438664541*
$[48] + 2 : \overline{5}\vert$	*0.00569192132*	*4.6197316859*
$51 : \overline{4}\vert$	*0.004945851624*	*3.7681188687*
$[51] : \overline{4}\vert$	*0.004097052617*	*3.7700064296*

   The insurance company pays a cash surrender value of 85% of the policy value.

   (a) If he is still a select life at age 51, how much money would he save by surrendering his current policy and buying a new 4-year policy for the same coverage?

   The premium is $\frac{400000 \times 0.006523115898}{6.2263058803} = \$419.07$. After 3 years, the policy value is ${}_3V = 400000 \times 0.004945851624 - 419.07 \times 3.7681188687 = 399.23507529$, so at 85% cash surrender value, the surrender value is $0.85 \times 399.23507529 = \$339.35$. If the life uses this as a lump-sum premium, the premiums for a new policy are $\frac{400000 \times 0.004097052617 - 339.35}{3.7700064296} = 344.69$, so he will save \$74.38 on each premium.

(b) If the insurance company introduces a full preliminary term to its policy value, how many years FPT would it need to introduce to the policy so that surrendering and repurchasing would no longer save the policyholder money.

If the cash surrender value is C, the new premium is

$$\frac{400000 \times 0.004097052617 - C}{3.7700064296} = 434.699801553 - \frac{C}{3.7700064296}$$

For the policyholder not to save money, we need this to be at least the current premium of \$419.07, so we need

$$\begin{aligned}434.699801553 - \frac{C}{3.7700064296} &\geqslant 419.07 \\ \frac{C}{3.7700064296} &\leqslant 434.699801553 - 419.07 = 15.629801553 \\ C &\leqslant 58.9244523482\end{aligned}$$

This requires ${}_3V \leqslant \frac{58.9244523482}{0.85} = 69.3228851155$. Under the full preliminary term, the policy value is calculated using a different premium, P, so we need to ensure that

$$\begin{aligned}400000 \times 0.004945851624 - 3.7681188687P &\leqslant 69.3228851155 \\ 3.7681188687P &\geqslant 1909.01776448 \\ P &\geqslant 506.623551698\end{aligned}$$

Under a Full preliminary term, P is the premium that would be charged for starting the policy at the end of the full preliminary term. If the preliminary term is t years, then the premium used is

$$\frac{400000 \times A^{1}_{[48]+t:\overline{7-t}|}}{\ddot{a}_{[48]+t:\overline{7-t}|}}$$

We evaluate this for $t = 1$, $t = 2$ and $t = 3$:

t	P
1	454.505054939
2	492.835662934
3	525.020764614

So a full preliminary term of 3 years is necessary to avoid this.

5. *A woman aged 26, whose mortality follows the ultimate part of the lifetable in Table 1, buys a whole life insurance policy with a death benefit of \$300,000. The interest rate is $i = 0.05$. This results in a net annual premium of \$756.51. The insurance company offers a cash surrender value of 85% of the policy value. The woman plans to convert the policy to a 15-year term insurance policy with a death benefit of \$2,600,000 when her first child is born. For how many years will this conversion be possible without increasing the premium?*

   You are given the following values of A_x:

x	A_x	${}_{15}p_x$	$A^{1}_{x:\overline{15}\vert}$	$\ddot{a}_{x:\overline{15}\vert}$
26	*0.0502926*	0.994423907148	0.0029515555728	10.8929843891
27	*0.0526386*	0.993980293318	0.0031740604145	10.89279289
28	*0.0550919*	0.993494349355	0.0034183734372	10.892571011
29	*0.0576569*	0.992962006049	0.0036859504938	10.8923292837
30	*0.0603386*	0.992379181784	0.0039801123633	10.8920392016
31	*0.0631417*	0.991739793475	0.004303595058	10.8917047559
32	*0.0660706*	0.991040725161	0.0046577985553	10.8913280225
33	*0.0691308*	0.990274841014	0.0050474677784	10.8908814396
34	*0.0723272*	0.989434981105	0.0054749700878	10.8903876176
35	*0.0756648*	0.988515964876	0.0059432495561	10.8898370617

If the woman changes the policy at age x, the policy value will be

$$300000A_x - 756.51\ddot{a}_x = 300000A_x - 756.51\frac{1.05}{0.05}(1 - A_x) = 315886.71A_x - 15886.71$$

The surrender value will therefore be

$$0.85(315886.71A_x - 15886.71) = 268503.7035A_x - 13503.7035$$

The premium for the 15-year term insurance policy, the premium does not increase provided

$$\frac{2600000A^{1}_{x:\overline{15}\vert} - (268503.7035A_x - 13503.7035)}{\ddot{a}_{x:\overline{15}}} \leqslant 756.51$$

If we set

$$f(x) = \frac{2600000A^{1}_{x:\overline{15}\vert} - (268503.7035A_x - 13503.7035)}{\ddot{a}_{x:\overline{15}}}$$

then we can calculate

x	$f(x)$
26	704.490005362
27	699.783940407
28	697.64018487
29	698.297251531
30	702.426601575
31	710.565566084
32	722.940208488
33	740.550151301
34	763.839172359
35	793.388668017

We see that she can convert the policy without increasing the premium provided her age is at most 33.

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66
26	9997.00	9995.83	9994.40	9992.66
27	9995.14	9993.90	9992.38	9990.52
28	9993.16	9991.84	9990.22	9988.24
29	9991.05	9989.65	9987.92	9985.80
30	9988.81	9987.30	9985.46	9983.18
31	9986.40	9984.80	9982.82	9980.38
32	9983.83	9982.11	9979.99	9977.37
33	9981.07	9979.23	9976.95	9974.13
34	9978.11	9976.13	9973.68	9970.64
35	9974.93	9972.79	9970.16	9966.88
36	9971.50	9969.20	9966.36	9962.82
37	9967.80	9965.33	9962.25	9958.44
38	9963.81	9961.14	9957.82	9953.69
39	9959.50	9956.61	9953.02	9948.55
40	9954.84	9951.71	9947.82	9942.98
41	9949.79	9946.41	9942.19	9936.94
42	9944.32	9940.66	9936.08	9930.38
43	9938.39	9934.41	9929.45	9923.26
44	9931.96	9927.64	9922.25	9915.52
45	9924.97	9920.28	9914.42	9907.10
46	9917.37	9912.28	9905.91	9897.94
47	9909.11	9903.58	9896.65	9887.98
48	9900.13	9894.11	9886.57	9877.13
49	9890.36	9883.80	9875.59	9865.30
50	9879.71	9872.57	9863.63	9852.42
51	9868.12	9860.34	9850.59	9838.38
52	9855.48	9847.01	9836.39	9823.08
53	9841.72	9832.48	9820.90	9806.39
54	9826.71	9816.64	9804.02	9788.18
55	9810.34	9799.37	9785.60	9768.33
56	9792.49	9780.52	9765.51	9746.67
57	9773.03	9759.97	9743.60	9723.05
58	9751.79	9737.56	9719.69	9697.28
59	9728.63	9713.10	9693.62	9669.17
60	9703.36	9686.43	9665.17	9638.51
61	9675.80	9657.33	9634.15	9605.07
62	9645.73	9625.59	9600.31	9568.61
63	9612.94	9590.98	9563.42	9528.85
64	9577.18	9553.24	9523.19	9485.52
65	9538.19	9512.09	9479.35	9438.30
66	9495.69	9467.25	9431.58	9386.86
67	9449.37	9418.39	9379.54	9330.85
68	9398.90	9365.17	9322.87	9269.88
69	9343.95	9307.23	9261.20	9203.55
70	9284.12	9244.18	9194.11	9131.43
71	9219.03	9175.59	9121.17	9053.07
72	9148.24	9101.03	9041.91	8967.97
73	9071.30	9020.03	8955.85	8875.63

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
74	8987.73	8932.10	8862.49	8775.52
75	8897.04	8836.71	8761.27	8667.10
76	8798.69	8733.34	8651.66	8549.78
77	8692.13	8621.41	8533.09	8423.00
78	8576.81	8500.36	8404.95	8286.16
79	8452.13	8369.60	8266.68	8138.66
80	8317.52	8228.53	8117.67	7979.93
81	8172.36	8076.57	7957.35	7809.41
82	8016.08	7913.13	7785.15	7626.56
83	7848.11	7737.67	7600.54	7430.89
84	7667.89	7549.66	7403.05	7221.99
85	7474.92	7348.64	7192.27	6999.51
86	7268.77	7134.21	6967.86	6763.22
87	7049.07	6906.07	6729.62	6513.04
88	6815.55	6664.05	6477.46	6249.02
89	6568.09	6408.10	6211.48	5971.42
90	6306.70	6138.35	5931.96	5680.73
91	6031.59	5855.15	5639.41	5377.67
92	5743.19	5559.08	5334.61	5063.27
93	5442.15	5250.97	5018.61	4738.86
94	5129.44	4931.97	4692.79	4406.12
95	4806.33	4603.54	4358.89	4067.08
96	4474.39	4267.51	4018.96	3724.10
97	4135.60	3926.04	3675.44	3379.91
98	3792.25	3581.66	3331.11	3037.57
99	3447.02	3237.23	2989.05	2700.39
100	3102.90	2895.94	2652.63	2371.88
101	2763.19	2561.21	2325.37	2055.64
102	2431.39	2236.61	2010.90	1755.27
103	2111.15	1925.80	1712.81	1474.18
104	1806.12	1632.34	1434.48	1215.44
105	1519.82	1359.55	1178.94	981.65
106	1255.46	1110.36	948.70	774.71
107	1015.81	887.14	745.58	595.71
108	802.96	691.49	570.56	444.87
109	618.23	524.17	423.71	321.41
110	462.04	385.00	304.13	223.65
111	333.80	272.80	210.00	149.10
112	231.99	185.53	138.71	94.62
113	154.19	120.34	87.07	56.74
114	97.30	73.90	51.50	31.84
115	57.78	42.55	28.41	16.52
116	31.92	22.69	14.43	7.81
117	16.15	11.04	6.63	3.30
118	7.34	4.79	2.69	1.21
119	2.90	1.79	0.93	0.37
120	0.95	0.55	0.26	0.09
121	0.23	0.13	0.05	0.01
122	0.03	0.02	0.01	0.00