ACSC/STAT 4703, Actuarial Models II
Fall 2015
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Homework Sheet 2
Due: Friday 9th October: 10:30 PM

Basic Questions

1. An insurance company has the following portfolio of home insurance policies:

Type of driver	Number	Probability claim	mean of claim	standard deviation
Small home	800	0.12	\$30,000	\$15,000
Medium home	2100	0.07	\$55,000	\$42,000
Large home	500	0.06	\$100,000	\$83,000

Calculate the cost of reinsuring losses above \$20,000,000, if the loading on the reinsurance premium is one standard deviation above the expected claim payment on the reinsurance policy:

(a) using a gamma approximation for the aggregate losses on this portfolio.

(b) using a compound Poisson approximation, with Poisson mean set to match the probability of no loss, and modelling the loss distribution for each type as a distribution taking only two values with probability 0.5 — that is, for example, for small homes, the claim amount is either \$15,000 or \$45,000 with probability 0.5 for each, so that the mean and standard deviation are as in the table. [You may approximate the secondary distribution by a normal distribution. You may use the fact that for a standard normal, the excess-of-loss random variable $(Z-a)_+$ has mean $\frac{e^{-\frac{a^2}{2}}}{\sqrt{2\pi}}-a(1-\Phi(a))$, and variance $(1-\Phi(a))\left(a^2\Phi(a)+2a\frac{e^{-\frac{a^2}{2}}}{\sqrt{2\pi}}+1\right)+a\frac{e^{-\frac{a^2}{2}}}{\sqrt{2\pi}}-\frac{e^{-a^2}}{2\pi}$.]

2. Calculate the Emprirical distribution function and the cumulative hazard rate function for the following dataset

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2.2 3.4 1.6 2.8 1.3 2.2 3.3 2.8 3.1 2.0 1.3 3.3 2.5
1.9 2.0 1.8 2.4 2.9 3.1
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3. For the sample from Question 2, calculate a Nelson-Åalen estimate for the probability that a random sample is more than 2.7.

4. Draw a histogram of the following distribution:

Claim Amount	Number of Claims
Less than \$5,000	70
\$5,000–\$10,000	152
\$10,000–\$20,000	115
\$20,000–\$50,000	34
\$50,000–\$100,000	29

Standard Questions

5. In a group life insurance policy, the insurer divides lives into three categories: "preferred", "standard" and "impaired". The probabilities of dying for individuals in the three categories are 0.002, 0.005 and 0.015 respectively. The group currently includes 18 preferred lives, 497 standard lives and 46 impaired lives. The premium for each life is set so that aggregate premiums are one standard deviation above the mean for aggregate claims. An additional 20 impaired lives are added to the group. By how much should the premium per life increase?

6. An insurance company collects the following data on insurance claims:

Claim Amount	Number of Policies
Less than \$5,000	230
\$5,000–\$10,000	152
\$10,000–\$50,000	112
\$50,000–\$200,000	44
\$200,000–\$1,000,000	64
More than \$1,000,000	16

   The policy currently has no deductible and a policy limit of \$1,000,000. The company wants to determine how much would be saved by introducing a deductible of \$2,000 and a policy limit of \$500,000. Using the ogive to estimate the empirical distribution, how much would the expected claim amount be reduced by the new deductible and policy limit?

7. An insurance company calculates the expected payment per claim as \$11,342 based on a sample of 3,000 claims with a policy limit of \$1,000,000. The company finds that these claims included 19 claims which exceeded the policy limit. The loss amounts for these claims were:

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1,263,000 4,712,000 1,630,000 1,103,000 2,241,000 1,420,000
1,337,000 1,831,000 1,903,000 1,557,000 1,670,000 1,098,000
1,226,000 2,947,000 1,346,000 1,660,000 1,403,000 1,185,000
1,631,000
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   Calculate the expected payment per loss if the policy limit is increased to \$2,000,000.