ACSC/STAT 4703, Actuarial Models II
Fall 2015
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Homework Sheet 3
Due: Friday 16th October: 10:30 PM

Basic Questions

1. An insurance company collects the following claim data (in thousands):

i	d_i	x_i	u_i	i	d_i	x_i	u_i	i	d_i	x_i	u_i
1	0	0.4	-	8	1.0	-	15	15	2.0	-	10
2	0	1.6	-	9	1.0	4.6	-	16	2.0	-	10
3	0	-	20	10	1.0	-	15	17	2.0	2.6	-
4	0	1.8	-	11	1.0	1.3	-	18	2.0	-	20
5	0	-	10	12	1.5	-	10	19	2.0	14.6	-
6	0.5	1.9	-	13	1.5	6.8	-	20	5.0	-	15
7	0.5	1.6	-	14	1.5	1.9	-	21	5.0	8.4	-

   Using a Kaplan-Meier product-limit estimator:

   (a) estimate the probability that a random loss exceeds 17.3.

   (b) estimate the median of the distribution.

   (c) Use a Nelson-Åalen estimator to estimate the median of the distribution.

2. An insurance company observes the following claim history:

Number of claims	Frequency
0	2,846
1	701
2	360
3	202
4	114
5	56
6	12
7	0
8	2

   Use a Nelson-Åalen estimate to obtain a 95% confidence interval for the probability that a random individual makes more than 5 claims.

3. For the data in Question 1, use Greenwood's approximation to obtain a 95% confidence interval for the probability that a random loss exceeds 17.3, based on the Kaplan-Meier estimator.

   (a) Using a normal approximation

   (b) Using a log-transformed confidence interval.

4. An insurance company records the following data in a mortality study:

entry	death	exit	entry	death	exit	entry	death	exit
60.4	-	64.4	61.6	-	64.2	62.1	-	63.9
62.7	-	63.7	60.8	-	63.8	62.9	-	64.5
63.4	-	64.4	64.3	-	66.3	61.8	63.7	-
61.2	-	63.2	63.3	-	66.3	60.2	60.6	-
62.2	-	65.2	62.8	-	64.8	63.8	65.2	-
60.9	-	62.9	61.3	-	63.3	62.1	63.4	-
63.0	-	65.6	62.1	-	65.1			

Estimate the probability of an individual currently aged exactly 63 dying within the next year using:

(a) the exact exposure method.

(b) the actuarial exposure method.

Standard Questions

5. An insurance company collects the following claim data (in thousands):

i	d_i	x_i	u_i	i	d_i	x_i	u_i	i	d_i	x_i	u_i
1	0	0.7	-	8	1.0	-	20	15	2.0	4.1	-
2	0	1.3	-	9	1.0	4.2	-	16	2.0	-	15
3	0	-	10	10	1.0	-	10	17	2.0	2.9	-
4	0	11.8	-	11	1.0	1.5	-	18	2.0	8.6	-
5	0.5	-	15	12	1.0	-	10	19	5.0	-	10
6	0.5	-	15	13	1.5	4.8	-	20	5.0	-	15
7	1.0	3.6	-	14	1.5	2.9	-	21	5.0	18.4	-

It is attempting to price a new policy with a deductible of 1.0. Using a Kaplan-Meier estimator, calculate the probability that a random claim on a policy with a deductible of 1.0 exceeds 5.0.

6. An insurance company has historical data from 2,861 claims. It finds that 1,830 are less than \$5,000, 793 are between \$5,000 and \$20,000, 168 are between \$20,000 and \$100,000, and the remaining 40 are more than \$100,000. Calculate a 95% confidence interval for the probability that a random claim is more than \$30,000.

7. An insurance company observes the following claims (in thousands):

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0.8 2.3 5.7 4.2 11.6 8.7 3.0 7.4 1.5 15.2 9.3 2.5
3.8
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using a kernel density estimate with a triangular kernel with bandwidth 1, estimate the expected loss per claim if the company introduces a deductible of 2.0 on each policy.

8. Using the following table:

Age	No. at start	enter	die	leave	No. at next age
58	0	2	1	0	1
59	1	6	0	1	6
60	6	12	1	2	15
61	15	9	0	0	24
62	22	10	2	3	27
63	27	4	3	2	26
64	26	0	2	1	23

Estimate the probability that an individual aged 61 withdraws from the policy within the next two years, conditional on surviving to the end of those two years.