ACSC/STAT 4703, Actuarial Models II
FALL 2016
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Sample Midterm Examination

This Sample examination has more questions than the actual midterm, in order to cover a wider range of questions. Estimated times are provided after each question to help your preparation.

Here are some values of the Gamma distribution function with $\theta = 1$ that will be needed for this examination:

x	α	$F(x)$
245	255	0.2697208
$\left(\frac{7.5}{12}\right)^3$	$\frac{4}{3}$	0.1117140
$\left(\frac{9.5}{12}\right)^3$	$\frac{4}{3}$	0.2507382
2.5	1	0.917915
2.5	2	0.7127025
2.5	3	0.4561869
2.5	4	0.2424239

1. Loss amounts follow an exponential distribution with $\theta = 60,000$. The distribution of the number of losses is given in the following table:

Number of Losses	Probability
0	0.04
1	0.54
2	0.27
3	0.15

   Assume all losses are independent and independent of the number of losses. The insurance company buys excess-of-loss reinsurance on the part of the loss above $150,000. Calculate the expected payment for this excess-of-loss reinsurance.

2. Aggregate payments have a compund distribution. The frequency distribution is negative binomial with $r = 4$ and $\beta = 12$. The severity distribution is a Gamma distribution with $\alpha = 8$ and $\theta = 3000$. Use a normal approximation to aggregate payments to estimate the probability that aggregate payments are more than $2,000,000.

3. Claim frequency follows a negative binomial distribution with $r = 5$ and $\beta = 2.9$. Claim severity (in thousands) has the following distribution:

Severity	Probability
0	0
1	0.600
2	0.220
3	0.166

Use the recursive method to calculate the exact probability that aggregate claims are at least 4.

4. Using an arithmetic distribution ($h = 1$) to approximate a Weibull distribution with $\tau = 3$ and $\theta = 12$, calculate the probability that the value is between 3.5 and 8.5, for the approximation using:

   (a) The method of rounding.

   (b) The method of local moment matching, matching 1 moment on each interval. [$\Gamma\left(\frac{4}{3}\right) = 0.8929795$.]

5. An insurance company has the following portfolio of home insurance policies:

Type of driver	Number	Probability claim	mean of claim	standard deviation
Good driver	600	0.02	\$2,500	\$2,000
Average driver	1400	0.06	\$3,800	\$3,200
Bad driver	500	0.13	\$7,000	\$3,600

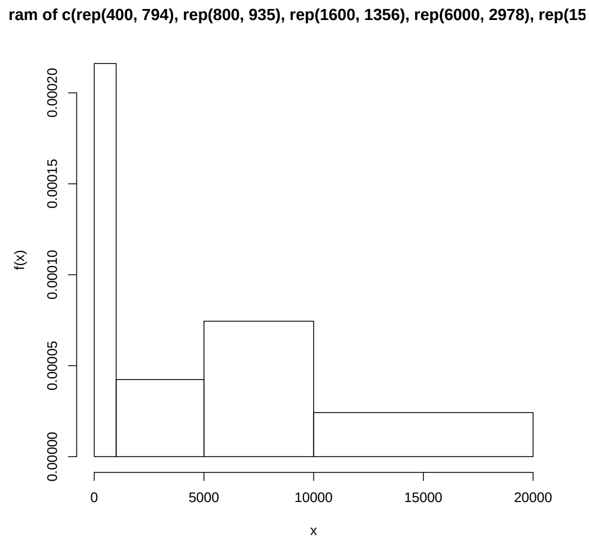
   Calculate the cost of reinsuring losses above \$5,000,000, if the loading on the reinsurance premium is one standard deviation above the expected claim payment on the reinsurance policy, using a Pareto approximation for the aggregate losses on this portfolio.

6. For the following dataset:

   ```
   0.2 0.2 0.4 0.7 1.8 2.1 2.3 3.0 3.5 3.9 4.1 4.2 4.6
   5.1 5.7 6.6 8.2 11.4
   ```

   Calculate a Nelson-Åalen estimate for the probability that a random sample is more than 2.7.

7. The histogram below is obtained from a sample of 8,000 claims.



Which interval included most claims?

8. An insurance company collects the following data on insurance claims:

Claim Amount	Number of Policies
Less than \$5,000	232
\$5,000–\$20,000	147
\$20,000–\$100,000	98
More than \$100,000	23

The policy currently has no deductible and a policy limit of \$100,000. The company wants to determine how much would be saved by introducing a deductible of \$2,000 and a policy limit of \$50,000. Using the ogive to estimate the empirical distribution, how much would the expected claim amount be reduced by the new deductible and policy limit?

9. An insurance company collects the following claim data (in thousands):

i	d_i	x_i	u_i	i	d_i	x_i	u_i	i	d_i	x_i	u_i
1	0	0.8	-	8	0.5	-	5	15	2.0	-	5
2	0	1.3	-	9	1.0	1.2	-	16	2.0	-	10
3	0	-	20	10	1.0	-	15	17	2.0	2.4	-
4	0	4.4	-	11	1.0	1.8	-	18	2.0	-	5
5	0	-	10	12	1.0	-	10	19	2.0	11.6	-
6	0.5	1.4	-	13	1.0	6.3	-	20	5.0	-	15
7	0.5	1.8	-	14	2.0	4.9.	-	21	5.0	5.9	-

Using a Kaplan-Meier product-limit estimator:

(a) estimate the probability that a random loss exceeds 3.

(b) Use Greenwood's approximation to obtain a 95% confidence interval for the probability that a random loss exceeds 3, based on the Kaplan-Meier estimator, using a normal approximation.

(c) Use Greenwood's approximation to find a log-transformed confidence interval for the probability that a random loss exceeds 3.

10. An insurance company records the following data in a mortality study:

entry	death	exit	entry	death	exit	entry	death	exit
51.3	-	58.4	56.5	-	58.2	55.3	-	59.9
54.7	-	59.7	54.7	-	59.8	53.3	59.1	
53.8	-	58.5	57.9	-	61.3	56.7	58.4	-
57.3	-	58.3	58.0	-	59.3	52.4	58.9	-
52.8	-	60.6	58.4	-	59.8	57.7	58.8	-
58.7	-	59.5	53.0	-	58.3	58.3	60.4	-
53.3	-	62.4	53.1	-	60.1	58.1	58.4	-

Estimate the probability of an individual currently aged exactly 58 dying within the next year using:

(a) the exact exposure method.

(b) the actuarial exposure method.

11. An insurance company observes the following claims (in thousands):

```
3.1 4.6 17.2 6.5 3.8 2.0 5.7 9.2 8.3 7.5 9.8 3.2 6.1
5.8 9.2 3.7 4.4
```

using a kernel density estimate with a uniform kernel with bandwidth 0.8, estimate the expected payment per claim if the company introduces a deductible of 1.5 on each policy.

12. Using the following table:

Age	No. at start	enter	die	leave	No. at next age
48	26	43	2	13	54
49	54	39	7	17	69
50	69	46	14	28	73
51	73	22	13	44	38

Estimate the probability that an individual aged 49 withdraws from the policy within the next two years, conditional on surviving to the end of those two years.

13. An insurance company models number of claims an individual makes in a year as following a Poisson distribution with Λ an unknown parameter with prior distribution a gamma distribution with $\alpha = 3$ and $\theta = 0.12$.

(a) What is the probability that a random individual makes exactly 2 claims?

(b) The company observes the following claim frequencies:

Number of claims	Frequency
0	234
1	104
2	44
3	12
4	6

What is the posterior probability that $\Lambda > 0.6$?

(c) Calculate the predictive probability that an individual makes no claims next year.

14. An insurance company models loss sizes as following a Weibull distribution with $\tau = 3$, and finds that the posterior distribution for Θ is a Pareto distribution with $\alpha = 4$ and $\theta = 1100$. Calculate the Bayes estimate for Θ based on a loss function:

(a) $l(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2$

(b) $l(\hat{\theta}, \theta) = |\hat{\theta} - \theta|^3$

(i) 422.35

(ii) 494.30

(iii) 560.87

(iv) 616.47

15.

16. An insurance company models claim frequency per year as following a Poisson distribution with mean Λ, where the prior distribution for Λ is a Gamma distribution with $\alpha = 5$ and $\theta = 6$. Over a 10-year period, they observe a total of 374 claims.

(a) Calculate the posterior distribution of Λ.

You are given the following values of the distribution function of the posterior distribution for Λ. Calculate a 95% credibility interval for Λ.

x	$F(x)$
34.12	0.0231
34.18	0.0250
34.35	0.0310
34.48	0.0364
41.60	0.9706
41.68	0.9731
41.74	0.9749
41.98	0.9810

(b) Using an HPD interval.

(c) With equal probability above and below the interval.

17. Claim severity follows a Pareto distribution with $\theta = 1000$ and α unknown. Which of the following is a conjugate prior distribution for α? Justify your answer.

(i) Log-normal distribution

(ii) Inverse Weibull distribution

(iii) Gamma distribution

(iv) Inverse Pareto distribution