

ACSC/STAT 4703, Actuarial Models II

Fall 2016

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Homework Sheet 3

Due: Friday 14th October: 10:30 PM

**Basic Questions**

1. An insurance company collects the following claim data (in thousands):

$i$	$d_i$	$x_i$	$u_i$	$i$	$d_i$	$x_i$	$u_i$	$i$	$d_i$	$x_i$	$u_i$
1	0	0.6	-	8	0.5	5.6	-	15	2.0	2.5	-
2	0	1.3	-	9	1.0	2.8	-	16	2.0	3.9	-
3	0	2.7	-	10	1.0	4.6	-	17	2.0	6.6	-
4	0	-	10	11	1.0	7.7	-	18	2.0	10.4	-
5	0	-	10	12	1.0	11.3	-	19	2.0	-	15
6	0.5	0.9	-	13	1.0	-	10	20	5.0	7.3	-
7	0.5	1.4	-	14	1.5	3.9	-	21	5.0	8.4	-

Using a Kaplan-Meier product-limit estimator:

- (a) estimate the probability that a random loss exceeds 10.7.
  - (b) estimate the median of the distribution.
  - (c) Use a Nelson-Åalen estimator to estimate the median of the distribution.
2. An insurance company observes the following claim history:

Number of claims	Frequency
0	2089
1	1810
2	799
3	226
4	60
5	14
6	2

Use a Nelson-Åalen estimate to obtain a 95% confidence interval for the probability that a random individual makes more than 4 claims.

3. For the data in Question 1, use Greenwood's approximation to obtain a 95% confidence interval for the probability that a random loss exceeds 10.7, based on the Kaplan-Meier estimator.
- (a) Using a normal approximation
  - (b) Using a log-transformed confidence interval.

4. An insurance company records the following data in a mortality study:

entry	death	exit	entry	death	exit	entry	death	exit
70.2	-	73.3	70.4	-	71.3	71.5	-	71.9
68.5	-	72.3	68.7	71.4	-	70.6	-	72.5
70.9	71.1	-	68.2	-	73.5	69.4	-	73.5
71.4	-	72.4	68.1	-	72.2	70.2	-	74.3
69.9	71.9	-	68.4	-	72.5	69.4	-	72.2
70.1	-	72.6	71.5	-	72.2	70.0	-	72.1
68.7	-	74.2	70.9	71.1	-	70.2	-	72.4
68.8	-	71.4	71.4	-	74.6	69.6	-	73.7
68.4	-	71.2	69.1	-	71.3	70.6	-	73.4
68.3	-	71.7						

Estimate the probability of an individual currently aged exactly 71 dying within the next year using:

- the exact exposure method.
- the actuarial exposure method.

5. An insurance company observes the following claims (in thousands):

2.5 2.9 2.9 3.6 3.8 4.0 4.1 4.8 5.1 5.2 5.9 6.0 6.7  
7.8 8.4

using a kernel density estimate with a uniform kernel with bandwidth 2, estimate the expected loss per claim if the company introduces a deductible of 2.0 on each policy.

6. Using the following table:

Age	No. at start	enter	die	leave	No. at next age
61		0	5	2	1
62		2	6	0	4
63		4	7	1	0
64		10	2	0	8
65		4	6	2	6
66		2	7	0	9

Estimate the probability that an individual aged 62 withdraws from the policy within the next year, conditional on surviving to the end of the year.

## Standard Questions

7. An insurance company collects the following claim data (in thousands):

$i$	$d_i$	$x_i$	$u_i$	$i$	$d_i$	$x_i$	$u_i$	$i$	$d_i$	$x_i$	$u_i$
1	0.0	0.1	-	9	1.0	1.8	-	17	2.0	3.6	-
2	0.0	0.7	-	10	1.0	2.2	-	18	2.0	6.4	-
3	0.0	1.8	-	11	1.0	2.6	-	19	2.0	9.6	-
4	0.0	-	5	12	1.0	11.3	20	20	2.0	-	15
5	0.5	-	10	13	1.0	-	20	21	5.0	5.3	-
6	0.5	-	20	14	1.5	4.5	-	22	5.0	7.5	-
7	0.5	-	10	15	1.5	-	20	23	5.0	8.5	-
8	1.0	1.6	-	16	2.0	2.4	-	24	5.0	-	10

It is attempting to choose a deductible for a new policy. The company has set the policy limit to 12.0. Customer satisfaction surveys have shown that at most 20% of claims should exceed the policy limit. Using a Kaplan-Meier product limit estimator, find the largest deductible they can apply while still meeting this criterion.