

ACSC/STAT 4703, Actuarial Models II

Fall 2016

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Homework Sheet 4

Due: Wednesday 26th October: 10:30 PM

Basic Questions

1. An insurance company models number of claims an individual makes in a year as following a negative binomial distribution with $\beta = 2.1$, and R an unknown parameter with prior distribution a gamma distribution with $\alpha = 4$ and $\theta = 0.01$.

(a) What is the probability that a random individual makes exactly 4 claims?

(b) The company now observes the following claim frequencies:

Number of claims	Frequency
0	36
1	27
2	15
3	5
4	2
5	1

What is the probability that $R > 0.4$? [You may use numerical integration to calculate this.]

(c) Calculate the predictive probability that this individual makes 5 claims next year. [You may use numerical integration to calculate this.]

2. An insurance company models loss sizes as following a Gamma distribution with $\alpha = 3$, and finds that the posterior distribution for Θ is an exponential distribution with $\theta = 1400$. Calculate the Bayes estimate for Θ based on a loss function:

(a) $l(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2$

(b) $l(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^4$

3. An insurance company models annual claim frequencies as following a Poisson distribution with parameter Λ , where the prior distribution for Λ is a Gamma distribution with $\alpha = 3$ and $\theta = 0.06$. They observe a total of 5 claims in 15 years. Calculate a 95% credibility interval for Λ .

(a) Using an HPD interval.

(b) With equal probability above and below the interval.

4. Calculate a conjugate prior distribution for the parameter α of a Pareto distribution.

Standard Questions

5. An insurance company models number of claims made by an individual in a year as following a Poisson distribution where the parameter Λ follows a Gamma distribution with $\alpha = 3$ and $\theta = 0.06$. The company monitors the individual's claim history. If the individual's expected number of claims per year has decreased by 10% or more, the individual receives a discount on their premium. Suppose the individual's actual rate of claims is $\lambda = 0.25$, what is the probability that this individual ever receives a discount on their premium?