ACSC/STAT 4703, Actuarial Models II
Fall 2016
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Homework Sheet 6
Model Solutions

Basic Questions

1. *An insurance company sells home insurance. It estimates that the standard deviation of the aggregate annual claim is \$6,321 and the mean is \$1,025.*

   (a) How many years history are needed for an individual or group to be assigned full credibility? (Use $r = 0.05$*,* $p = 0.95$*.)*

   For n years of history, the standard deviation of the mean annual claim is $\frac{6321}{\sqrt{n}}$, and the mean of the mean annual claim is \$1,025. An individual has full credibility if a 95% confidence interval (which has width 1.96 times the standard deviation) is contained in the interval $[0.95\mu, 1.05\mu] = [973.75, 1076.25]$. That is, if

$$\begin{aligned}1.96 \times \frac{6321}{\sqrt{n}} &\leqslant 51.25\\ \sqrt{n} &\geqslant \frac{1.96 \times 6321}{51.25}\\ n &\geqslant \left(\frac{1.96 \times 6321}{51.25}\right)^2\\ &= 58438.09\end{aligned}$$

   So 58439 years are needed. (Though for partial credibility, it makes sense to use 58438.09.)

   The standard premium for this policy is \$1,025. An individual has claimed a total of \$62,300 in the last 10 years.

   (b) What is the Credibility premium for this individual, using limited fluctuation credibility?

   The standard for full credibility is 58438.09 years, so we get $Z = \sqrt{\frac{10}{58438.09}} = 0.01308133$, and the credibility premium is $0.01308133 \times 6230 + 0.98691867 \times 1025 = \$1,093.09$.

2. *A car insurance company classifies drivers as good or bad. Annual claims from good drivers follow a Pareto distribution with* $\alpha = 6$ *and* $\theta = 4000$*. Annual claims from bad drivers follow a Pareto distribution with shape* $\alpha = 4$ *and* $\theta = 5000$*. 80% of individuals are good drivers.*

(a) Calculate the expectation and variance of the aggregate annual claims from a randomly chosen driver.

The expected annual claims are

$$0.8 \times \frac{4000}{5} + 0.2 \times \frac{5000}{3} = \$973.33$$

The variance of annual claims is

$$0.2 \times 0.8 \left(\frac{5000}{3} - 800\right)^2 + 0.8 \times \frac{4000^2}{5^2 \times 4} + 0.2 \times \frac{5000^2}{3^2 \times 2} = 525955.56$$

(b) Given that a driver's annual claims over the past 3 years are \$8,000, \$3,500 and \$500, what are the expectation and variance of the driver's claims next year?

If the driver is good, then the likelihood of these claim values is

$$6^3 \frac{4000^{18}}{12000^7 \times 7500^7 \times 4500^7} = 8.305168 \times 10^{-06}$$

If the driver is bad, the likelihood of these claims is

$$4^3 \frac{5000^{12}}{13000^5 \times 8500^5 \times 5500^5} = 0.0001884496$$

The likelihood that a driver is good and makes these claims is $8.305168 \times 10^{-06} \times 0.8 = 6.6441350 \times 10^{-06}$. The likelihood that the driver is bad and makes these claims is $0.0001884496 \times 0.2 = 0.00003768992$, so the overall likelihood that a driver makes these claims is 0.00004433405. The posterior probability that the driver is good is therefore

$$\frac{6.6441350 \times 10^{-06}}{0.00004433405} = 0.1498653$$

The new expectation is therefore

$$0.1498653 \times 800 + (1 - 0.1498653) \times \frac{5000}{3} = \$1,536.78$$

and the new variance is

$$0.1498653(1-0.1498653) \left(\frac{5000}{3} - 800\right)^2 + 0.1498653 \times \frac{4000^2}{5^2 \times 4} + (1-0.1498653) \frac{5000^2}{3^2 \times 2} = 1300417$$

3. *The number of claims made by an individual in a year follows a Poisson distribution with mean Λ, where the value of Λ follows a Gamma distribution with $\alpha = 4.2$ and $\theta = 0.05$. Given that an individual has made no claims in the past 10 years, what is the expected number of claims made in the next year?*

   The posterior distribution for Λ is a Gamma distribution with $\alpha = 4.2$ and $\theta = \frac{0.05}{1+0.05\times 10} = \frac{1}{30}$. The expected number of claims is therefore $\frac{4.2}{30} = 0.14$.

Standard Questions

4. *For a certain insurance policy, the book premium is based on average claim frequency of 0.6 claims per year, and average claim severity of \$2,030. A particular group has made 350 claims from 987 policies in the last year. The average claim severity is \$3,414. Estimate the credibility premium for this group using limited fluctuation credibility if the standard for full credibility is:*

   (a) 603 claims for claim frequency, 940 claims for severity.

   The credibility estimate for claim frequency is $\sqrt{\frac{350}{603}}\frac{350}{987}+0.6\left(1-\sqrt{\frac{350}{603}}\right)=$ 0.413047. The credibility estimate for claim severity is

$$\sqrt{\frac{350}{940}} \times 3414 + \left(1-\sqrt{\frac{350}{940}}\right) \times 2030 = \$2,874.51$$

   The credibility premium is therefore $0.413047 \times 2874.51 = \$1,187.31$.

   (b) 1106 policies for claim frequency, 940 claims for severity.

   The credibility estimate for claim frequency is $\sqrt{\frac{987}{1106}}\frac{350}{987}+0.6\left(1-\sqrt{\frac{987}{1106}}\right)=$ 0.3681869. As in part (a), the credibility estimate for claim severity is \$2,874.51

   The credibility premium is therefore $0.3681869 \times 2874.51 = \$1058.39$.

   (c) 1523 policies for aggregate claims.

   The group's average aggregate claims per policy is

$$\frac{350 \times 3414}{987} = \$1,210.638$$

   The credibility estimate is therefore

$$\sqrt{\frac{987}{1523}} \times 1210.638 + \left(1-\sqrt{\frac{987}{1523}}\right) \times 2030 \times 0.6 = \$1,212.07$$

5. *An insurance company has 3 years of past history on a driver, denoted X_1, X_2, X_3. It uses a formula $\hat{X}_4 = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3$ to calculate the credibility premium in the fourth year. It has the following information on the driver:*

   - *In a given year, the expected aggregate claim is \$800 plus 5% of the value of the car.*
   - *In a given year, the variance of the aggregate claim is \$800,000 plus 12 times the value of the car.*
   - *The value of the car is \$19,500 in the first year.*

- *The value of the car decreases by 15% every year.*
- *The correlation (recall $Corr(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}}$) between aggregate claims in years i and j is $e^{-5\sqrt{|i-j|}}$.*

Find a set of equations which can determine the values of α_0, α_1, α_2 and α_3. [You do not need to solve these equations.]

The company needs to choose α_0, α_1, α_2 and α_3 to satisfy:

$$\begin{aligned}
\mathbb{E}(X_4) &= \alpha_0 + \alpha_1\mathbb{E}(X_1) + \alpha_2\mathbb{E}(X_2) + \alpha_3\mathbb{E}(X_3)\\
\mathrm{Cov}(X_4, X_1) &= \alpha_1 \mathrm{Var}(X_1) + \alpha_2 \mathrm{Cov}(X_2, X_1) + \alpha_3 \mathrm{Cov}(X_3, X_1)\\
\mathrm{Cov}(X_4, X_2) &= \alpha_1 \mathrm{Cov}(X_1, X_2) + \alpha_2 \mathrm{Var}(X_2) + \alpha_3 \mathrm{Cov}(X_3, X_2)\\
\mathrm{Cov}(X_4, X_3) &= \alpha_1 \mathrm{Cov}(X_1, X_3) + \alpha_2 \mathrm{Cov}(X_2, X_3) + \alpha_3 \mathrm{Var}(X_3)
\end{aligned}$$

We calculate the following:

Year i	Value of car	$\mathbb{E}(X_i)$	$\mathrm{Var}(X_i)$
1	19500.00	1775.00	1034000.00
2	16575.00	1628.75	998900.00
3	14088.75	1504.44	969065.00
4	11975.44	1398.77	943705.25

We now calculate the covariances:

$$\begin{aligned}
\mathrm{Cov}(X_1, X_2) &= \sqrt{1034000.00 \times 998900.00}e^{-5} = 6847.765\\
\mathrm{Cov}(X_1, X_3) &= \sqrt{1034000.00 \times 969065.00}e^{-5\sqrt{2}} = 850.180\\
\mathrm{Cov}(X_1, X_4) &= \sqrt{1034000.00 \times 943705.25}e^{-5\sqrt{3}} = 171.229\\
\mathrm{Cov}(X_2, X_3) &= \sqrt{998900.00 \times 969065.00}e^{-5} = 6629.26\\
\mathrm{Cov}(X_2, X_4) &= \sqrt{998900.00 \times 943705.25}e^{-5\sqrt{2}} = 824.619\\
\mathrm{Cov}(X_3, X_4) &= \sqrt{969065.00 \times 943705.25}e^{-5} = 6443.506
\end{aligned}$$

This gives the equations:

$$\begin{aligned}
1398.77 &= \alpha_0 + 1775.00\alpha_1 + 1628.75\alpha_2 + 1504.44\alpha_3\\
171.229 &= 1034000\alpha_1 + 6847.765\alpha_2 + 850.180\alpha_3\\
824.619 &= 6847.765\alpha_1 + 998900\alpha_2 + 6629.26\alpha_3\\
6443.506 &= 850.180\alpha_1 + 6629.26\alpha_2 + 969065\alpha_3
\end{aligned}$$