ACSC/STAT 4703, Actuarial Models II
Fall 2016
Toby Kenney
Homework Sheet 7
Model Solutions

Basic Questions

1. *An insurance company sets the book pure premium for its home insurance premium at \$1,132. The expected process variance is 261,244 and the variance of hypothetical means is 89,402. If an individual has no claims over the last 8 years, calculate the credibility premium for this individual's next year's insurance using the Bühlmann model.*

   The credibility of 8 years experience is

   $$Z = \frac{8}{8 + \frac{261244}{89402}} = 0.7324581$$

   so the credibility premium is

   $$(1 - 0.7324581) \times 1132 = \$302.86$$

2. *An insurance company has the following data on an insurance policy for a company that rents out apartments.*

Year	*1*	*2*	*3*	*4*
Exposure	*835*	*884*	*952*	*944*
Aggregate claims	*\$122,000*	*\$106,000*	*\$153,000*	*\$149,000*

   The book premium is \$500 per unit of exposure. The variance of hypothetical means per unit of exposure is 880. The expected process variance per unit of exposure is 6,300. Using a Bühlmann-Straub model, calculate the credibility premium for Year 5 if the company has 1,063 units of exposure.

   The company has a history of 3615 units of exposure. The credibility of its experience is therefore

   $$\frac{3615}{3615 + \frac{6300}{880}} = 0.9980235$$

   The credibility premium per unit of exposure is therefore

   $$0.9980235 \times \frac{530000}{3615} + 0.0019765 \times 500 = \$147.31$$

3. *An insurance company has the following previous data on aggregate claims:*

Policyholder	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Year 5*	*Mean*	*Variance*
1	*0.00*	*158.74*	*1674.34*	*0.00*	*0.00*	*366.616*	*539144.09148*
2	*135.41*	*0.00*	*0.00*	*29.10*	*152.90*	*63.482*	*5602.81662*
3	*0.00*	*0.00*	*0.00*	*0.00*	*588.49*	*117.698*	*69264.09602*
4	*0.00*	*0.00*	*88.22*	*233.61*	*1424.39*	*349.244*	*370346.60373*

Calculate the Bühlmann credibility premium for each policyholder in Year 6.

Based on these data, the mean aggregate claims is $\frac{366.616+63.482+117.698+349.244}{4} = \frac{897.040}{4} = 224.260$. The EPV is

$$\frac{539144.09148 + 5602.81662 + 69264.09602 + 370346.60373}{4} = 984357.607854 = 246089.4019625$$

The variance of the observed means is therefore

$$\frac{(366.616 - 224.260)^2 + (63.482 - 224.260)^2 + (117.698 - 224.260)^2 + (349.244 - 224.260)^2}{3} = 24363.75$$

This is the VHM plus the conditional variance of the means. This conditional variance of the means is the EPV divided by 5. Therefore the VHM is $24363.75 - \frac{246089.4019625}{5} = 241216.651555$

The credibility of 5 years of experience is therefore

$$Z = \frac{5}{5 + \frac{241216.651555}{24363.75}} = 0.335556136552$$

The credibility premiums are therefore

Policyholder	Credibility premium
1	$0.3355561 \times 366.616 + 0.6644439 \times 224.26 = \$272.03$
2	$0.3355561 \times 63.482 + 0.6644439 \times 224.26 = \$170.31$
3	$0.3355561 \times 117.698 + 0.6644439 \times 224.26 = \$188.50$
4	$0.3355561 \times 349.244 + 0.6644439 \times 224.26 = \$266.20$

4. Over a three-year period, an insurance company observes the following numbers of claims:

No. of claims	*Frequency*
0	*1,856*
1	*2,901*
2	*2,465*
3	*1,387*
4	*760*
5	*386*
6	*159*
7	*51*
8	*19*
9	*13*
10	*3*

Assuming the number of claims made by an individual in a year follows a Poisson distribution, calculate the credibility estimate for the expected claim frequency in the following year, of an individual who has made a total of 7 claims in the past 3 years.

The total number of claims is 18572, from 10000 policies, so the average claim frequency per 3-year period is 1.8572. The variance of the number of claims is therefore

$$\left(\frac{1^2\times 2901+2^2\times 2{,}465+3^2\times 1{,}387+4^2\times 760+5^2\times 386+6^2\times 159+7^2\times 51+8^2\times 19+9^2\times 13+10^2\times 3}{10000} - 1.8572^2\right)\times\frac{10000}{9999} = 2.3356417$$

Since the variance of a Poisson distribution is equal to the mean, we have that the EPV is 1.8572. The variance of observed claim frequencies is the EPV plus the VHM, so the VHM is $2.3356417 - 1.8572 = 0.4784417$. The credibility of the past history is therefore $\frac{1}{1+\frac{0.0.4784417}{1.8572}} = 0.7951562$. Therefore, the expected claim frequency from an individual who has made a total of 7 claims in the past 3 years is

$$0.7951562 \times \frac{7}{3} + 0.2048438 \times 1.8572 = 2.235800$$

Standard Questions

5. *Aggregate claims for a given individual policy are modelled as following a Pareto distribution with $\alpha = 4$. The first 4 years of experience on this policy are:*

Policyholder	*Year 1*	*Year 2*	*Year 3*	*Year 4*
1	*2061*	*1928*	*448*	*1663*
2	*785*	*690*	*294*	*711*
3	*984*	*958*	*3206*	*1260*
4	*3040*	*415*	*7003*	*1714*

(a) Estimate the EPV and VHM based on the method of moment estimates for each θ. [That is for each policyholder estimate a value of θ that makes the mean observations for that policyholder equal to the observed mean.]

The mean of a Pareto distribution is $\frac{\theta}{\alpha-1}$, so the moment estimator for θ in our case is 3 times the mean of the observed data. The variance of a Pareto distribution is $\frac{\theta^2}{(\alpha-1)^2(\alpha-2)} = \frac{\theta^2}{18}$ is the square of the mean divided by 2.

The mean aggregate claims and variances of aggregate claims are:

Policyholder	mean	variance
1	1525	1162812.5
2	620	192200.0
3	1602	1283202.0
4	3043	4629924.5
Mean	1697.5	1817034.75

Therefore the EPV is 1817034.75.

The variance of the estimated means is $\frac{(1525-1697.5)^2+(620-1697.5)^2+(1602-1697.5)^2+(3043-1697.5)^2}{3} = 1003417.66667$. The part of the variance due to the EPV is $\frac{1817034.75}{4} = 454258.6875$ The VHM is therefore $1003417.66667-454258.6875 = 549158.979167$

(b) Calculate the credibility premium for policyholder 2 in the next year.

The credibility of 4 years of experience is therefore

$$Z = \frac{4}{4 + \frac{549158.979167}{1817034.75}} = 0.929750793608$$

so the credibility premium is

$$0.9297508 \times 620 + 0.0702492 \times 1697.5 = \$695.69$$

6. *Claim frequency in a year for an individual follows a Poisson with parameter Λt where Λ is the individual's risk factor and t is the individual's exposure in that year. An insurance company collects the following data:*

	Year 1		*Year 2*		*Year 3*	
Policyholder	*Exp*	*claims*	*Exp*	*claims*	*Exp*	*claims*
1	*130*	*1*	*403*	*2*	*231*	*0*
2	*241*	*7*	*373*	*11*	*379*	*15*
3	*86*	*0*	*371*	*0*	*407*	*1*
4	*64*	*3*	*275*	*7*	*255*	*3*

In Year 4, policyholder 2 has 264 units of exposure. Calculate the credibility estimate for claim frequency for policyholder 2.

We calculate the mean number of claims per unit of exposure for the 4 policyholders:

Policyholder	Mean claim frequency
1	0.00392670157068
2	0.0332326283988
3	0.00115740740741
4	0.0218855218855

The average of these is 0.0150505648156, so this is the estimated mean claim frequency on a per-policyholder basis. (On a per unit of exposure basis, we would take the total 50 claims over the total 3215 units of exposure, for a frequency of 0.01555210). Since the mean and variance of a Poisson distribution are equal, this is also the EPV.

The variance of the policyholders mean claim frequencies is

$$\frac{(0.00392670-0.01505056)^2+(0.03323262-0.01505056)^2+(0.00115742-0.01505056)^2+(0.02188552-0.01505056)^2}{3} = 0.0002313547$$

The variances of these due to the estimation are:

$$\frac{0.00392670157068}{764} + \frac{0.0332326283988}{993} + \frac{0.00115740740741}{864} + \frac{0.0218855218855}{594} = 1.91976158156 \times 10^{-05}$$

Therefore the VHM is $0.000231354743517 - 1.91976158156 \times 10^{-05} = 0.000212157127701$

The credibility of 993 units of exposure is therefore

$$\frac{993}{993 + \frac{0.0150505648156}{0.000212157127701}} = 0.93332273637$$

Therefore the credibility estimate for policyholder 2 is

$$264\left(0.9333227 \times \frac{33}{993} + 0.0666773 \times 0.0150505648156\right) = 8.453359$$