ACSC/STAT 4703, Actuarial Models II
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Toby Kenney

Homework Sheet 5
Model Solutions

Basic Questions

1. *An insurance company sets the book pure premium for its tennants insurance at \$332. The expected process variance is 8,209 and the variance of hypothetical means is 21,455. If an individual has no claims over the last 6 years, calculate the credibility premium for this individual's next year's insurance using the Bühlmann model.*

   The credibility is

   $$Z = \frac{n}{n + \frac{\text{EPV}}{\text{VHM}}} = \frac{6}{6 + \frac{8209}{21455}} = 0.94005360051$$

   The premium is therefore $0 \times 0.94 + 332 \times 0.05994639949 = \$19.90$.

2. *An insurance company has the following data on a marine insurance policy for a shipping company.*

Year	*1*	*2*	*3*	*4*	*5*
Exposure	*455*	*490*	*476*	*532*	*565*
Aggregate claims	*\$1,202,000*	*\$2,760,000*	*\$5,056,000*	*\$2,410,000*	*\$3,280,000*

   The book premium is \$9,800 per unit of exposure. The variance of hypothetical means per unit of exposure is 1,435,000. The expected process variance per unit of exposure is 42,348,300. Using a Bühlmann-Straub model, calculate the credibility premium for Year 6 if the company has 592 units of exposure.

   The credibility of the company's past history is $\frac{2518}{2518+\frac{42348300}{1435000}} = 0.98841574763$. The company has aggregate claims of \$14,708,000 from 2518 units of exposure, so the average loss per unit of exposure is $\frac{14708000}{2518} = 5841.14376$. The premium in year 6 is therefore $(5841.14376 \times 0.98841574763 + 9800 \times 0.01158425237) \times 592 = \$3,485,106.46$.

3. *An insurance company has the following previous data on aggregate claims:*

Policyholder	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Year 5*	*Mean*	*Variance*
1	*0.00*	*6466.54*	*0.00*	*0.00*	*1430.52*	*1579.41*	*7847453.93*
2	*568.29*	*743.32*	*600.67*	*537.46*	*619.98*	*613.94*	*6221.22*
3	*0.00*	*590.62*	*0.00*	*0.00*	*0.00*	*118.12*	*69766.40*
4	*260.98*	*0.00*	*0.00*	*530.55*	*612.01*	*280.71*	*82541.30*

   Calculate the Bühlmann credibility premium for each policyholder in Year 6.

The mean yearly claims are $\frac{1579.41+613.94+118.12+280.71}{4} = 648.045$. We estimate the expected process variance as $\frac{7847453.93+6221.22+69766.40+82541.30}{4} = 2001495.7125$ and the variance of observed means as $\frac{(1579.41-648.045)^2+(613.94-648.045)^2+(118.12-648.045)^2+(28}{3}$ 428119.80737. The variance of hypothetical means is then $428119.80737 - \frac{2001495.7125}{5} = 27820.66487$.

The credibility of 5 years of experience is therefore $\frac{5}{5+\frac{2001495.7125}{27820.66487}} = 0.0649833630471$

The premiums are therefore:

Policyholder 1	$0.0649833630471 \times 1579.41 + 0.9350166369529 \times 648.045 = \$708.57$
Policyholder 2	$0.0649833630471 \times 613.94 + 0.9350166369529 \times 648.045 = \$645.83$
Policyholder 3	$0.0649833630471 \times 118.12 + 0.9350166369529 \times 648.045 = \$613.61$
Policyholder 4	$0.0649833630471 \times 280.71 + 0.9350166369529 \times 648.045 = \$624.17$

4. *Over a three-year period, an insurance company observes the following numbers of claims:*

No. of claims	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*12*
Frequency	*3401*	*3146*	*1787*	*956*	*444*	*174*	*54*	*29*	*4*	*2*	*2*	*1*

Assuming the number of claims made by an individual in a year follows a Poisson distribution, calculate the credibility estimate for the expected claim frequency in the following year, of an individual who has made a total of 1 claim in the past 3 years.

The average number of claims made by an individual in a three-year period is $\frac{3146\times1+1787\times2+956\times3+444\times4+174\times5+54\times6+29\times7+4\times8+2\times9+2\times10+1\times12}{3401+3146+1787+956+444+174+54+29+4+2+2+1} = 1.2843$

If the number of claims follows a Poisson distribution, then this is also the variance of the number of claims in a three-year period (the expected process variance). The variance of observed means is then

$\frac{3401\times(0-1.2843)^2+3146\times(1-1.2843)^2+1787\times(2-1.2843)^2+956\times(3-1.2843)^2+444\times(4-1.2843)^2+174\times(5-1.2843)^2+54\times(6-1.28}{3401+3146+1787+956+444+174+54+29+4+2}$

1.79865337536 This means that the variance of hypothetical means is $1.79865337536 - 1.2843 = 0.51435337536$, so the credibility of three years of experience is $\frac{1}{1+\frac{1.2843}{0.51435337536}} = 0.285965813317$. The expected annual claim frequency for an individual who has made 1 claim in the past 3 years is

$$\frac{1 \times 0.285965813317 + 1.2843 \times 0.714034186683}{3} = 0.40099997309$$

Standard Questions

5. *Aggregate claims for a given individual policy are modelled as following an exponential distribution. The first 5 years of experience on this policy are:*

Policyholder	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Year 5*	*Mean*	*Variance*
1	*446*	*208*	*533*	*40*	*25*	*250.4*	*53748.3*
2	*1090*	*1896*	*1309*	*62*	*361*	*943.6*	*544664.3*
3	*856*	*74*	*455*	*192*	*521*	*419.6*	*93305.3*
4	*76*	*203*	*560*	*1170*	*124*	*426.6*	*208730.8*

(a) Estimate the EPV and VHM.

For an exponential distribution with mean θ, the variance is θ^2. The expected process variance is therefore $\frac{250.4^2+943.6^2+419.6^2+426.6^2}{4} = 327783.21$. The variance of observed means is $\frac{(250.4-510.05)^2+(943.6-510.05)^2+(419.6-510.05)^2+(426.6-510.05)^2}{3} = 90176.2766667$. The variance of hypothetical means is therefore $90176.2766667 - \frac{327783.21}{5} = 24619.6346667$.

(b) Calculate the credibility premium for policyholder 2 in the next year.

The credibility of 5 years of experience is $\frac{5}{5+\frac{327783.21}{24619.6346667}} = 0.273016757585$. The credibility premium for policyholder 2 is therefore $0.273016757585 \times 943.6 + 0.726983242415 \times 510.05 = \$628.42$.

6. *Claim frequency in a year for an individual follows a Poisson with parameter Λt where Λ is the individual's risk factor and t is the individual's exposure in that year. An insurance company collects the following data:*

	Year 1		*Year 2*		*Year 3*	
Policyholder	*Exp*	*claims*	*Exp*	*claims*	*Exp*	*claims*
1	*432*	*2*	*403*	*2*	*448*	*3*
2	*214*	*4*	*270*	*3*	*302*	*6*
3	*303*	*0*	*323*	*1*	*317*	*1*
4	*515*	*3*	*487*	*2*	*502*	*4*

In Year 5, policyholder 2 has 264 units of exposure. Calculate the credibility estimate for claim frequency for policyholder 2.

We summarise each policyholder's total exposure and claims:

Policyholder	Exp	claims	$\hat{\Lambda}$
1	1283	7	0.005455962588
2	786	13	0.016539440204
3	943	2	0.002120890774
4	1504	9	0.005984042553

Taking an equally weighted average of these gives

$$\hat{\lambda} = \frac{0.005455962588 + 0.016539440204 + 0.002120890774 + 0.005984042553}{4} = 0.00752508402975$$

For the Poisson distribution, this is also the EPV, and the variance of the means is

$$\mathrm{Var}(\hat{\Lambda}) = \frac{(0.005456 - 0.007525)^2 + (0.016539 - 0.007525)^2 + (0.002121 - 0.007525)^2 + (0.005984 - 0.007525)^2}{3} = 0.0000390399981$$

The variance due to process variance is

$$\frac{0.00752508402975}{4}\left(\frac{1}{1283}+\frac{1}{786}+\frac{1}{943}+\frac{1}{1504}\right)=0.00000710561112969$$

The variance of hypothetical means is therefore $0.0000390399981183 - 0.00000710561112969 = 0.0000319343869886$. This means the credibility of n units of exposure is $\frac{n}{n+\frac{0.00752508402975}{0.0000319343869886}}$. For policyholder 2, the credibility is therefore $Z = \frac{786}{786+\frac{0.00752508402975}{0.0000319343869886}} = 0.769349704521$.

The expected claim frequency is therefore $264\times(0.769349704521\times0.016539440204+0.230650295479\times0.00752508402975) = 3.81751294027$.

Weighting each policyholder by experience

There were a total of 31 claims from 4516 units of exposure. The average of $\hat{\Lambda}$ is therefore $\frac{31}{4516} = 0.006864481842$

This is the EPV per unit of exposure. We calculate $1283\times(0.005455962588-0.006864481842)^2 + 786\times(0.016539440204-0.006864481842)^2 + 943\times(0.002120890774-0.006864481842)^2 + 1504\times(0.005984042553-0.006864481842)^2 = 0.0985036881817$. The variance due to EPV is $3\times 0.006864481842 = 0.020593445526$. Now we get $0.0985036881817-0.020593445526 = 0.0779102426557$ is an unbiassed estimate for $\sum_{i=1}^{4} m_i(\Lambda_i-\overline{\Lambda})^2$ where Λ_i is the hypothetical mean for the ith policyholder. The variance of hypothetical means is therefore $\frac{0.0779102426557}{4516-\frac{1283^2+786^2+943^2+1504^2}{4516}} = 0.0000234888955211$. The credibility of n units of exposure is therefore $\frac{n}{n+\frac{0.006864481842}{0.0000234888955211}}$. For policyholder 2, the credibility is therefore $Z = \frac{786}{786+\frac{0.006864481842}{0.0000234888955211}} = 0.728963217418$.

The expected claim frequency is therefore $264\times(0.728963217418\times0.016539440204+0.271036782582\times0.006864481842) = 3.67413304315$.