ACSC/STAT 4703, Actuarial Models II
Fall 2020
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Homework Sheet 1
Due: Friday 31st January: 14:30 PM

Basic Questions

1. Aggregate payments have a compund distribution. The frequency distribution is negative binomial with $r = 2$ and $\beta = 1.9$. The severity distribution is a gamma distribution with $\alpha = 0.7$ and $\theta = 18000$. Use a Pareto approximation to aggregate payments to estimate the probability that aggregate payments are more than $350,000.

2. Loss amounts follow a Pareto distribution with $\alpha = 4$ and $\theta = 120,000$. The distribution of the number of losses is given in the following table:

Number of Losses	Probability
0	0.47
1	0.11
2	0.27
3	0.15

   Assume all losses are independent and independent of the number of losses. The insurance company buys excess-of-loss reinsurance on the part of the loss above $200,000. Calculate the expected payment for this excess-of-loss reinsurance.

   [You may use the following formula for independant Pareto distributions X and Y with $\alpha = 4$:

$$\begin{aligned}&\mathbb{E}((X+Y-a)_+)\\&=\frac{4\theta^8}{3(2\theta+a)^7}\left(30\log\left(\frac{\theta+a}{\theta}\right)+15\left(\frac{2\theta+a}{\theta}-\frac{2\theta+a}{\theta+a}\right)+\frac{7}{2}\left(\frac{(2\theta+a)^2}{\theta^2}-\frac{(2\theta+a)^2}{(\theta+a)^2}\right)\right.\\&\qquad\left.+\left(\frac{(2\theta+a)^3}{\theta^3}-\frac{(2\theta+a)^3}{(\theta+a)^3}\right)+\frac{1}{4}\left(\frac{(2\theta+a)^4}{\theta^4}-\frac{(2\theta+a)^4}{(\theta+a)^4}\right)\right)+\frac{\theta^4(\theta+a)+\theta a^4}{3(\theta+a)^4}\end{aligned}$$

   You may use numerical integration to find the expected payment if there are 3 losses. Hint: find the expected payment by conditioning on the first loss.]

3. An insurance company models loss frequency as binomial with $n = 88$, $p = 0.11$, and loss severity as exponential with $\theta = 20,000$. Calculate the expected aggregate payments if there is a policy limit of $80,000 and a deductible of $15,000 applied to each claim.

4. Claim frequency follows a negative binomial distribution with $r = 2$ and $\beta = 4.1$. Claim severity (in thousands) has the following distribution:

Severity	Probability
1	0.4
2	0.39
3	0.14
4	0.05
5 or more	0.02

Use the recursive method to calculate the exact probability that aggregate claims are at least 5.

5. Use an arithmetic distribution ($h = 1$) to approximate a Pareto distribution with $\alpha = 4$ and $\theta = 60$.

(a) Using the method of rounding, calculate the mean of the arithmetic approximation. [You can evaluate this numerically: use 10,000 terms in the sum.]

(b) Using the method of local moment matching, matching 1 moment on each interval, estimate the probability that the value is larger than 18.5.

Standard Questions

6. The number of claims an insurance company receives follows a negative binomial distribution with $r = 64$ and $\beta = 37$. Claim severity follows a negative binomial distribution with $r = 14$ and $\beta = 1.4$. Calculate the probability that aggregate losses exceed $32,000.

(a) Starting the recurrence 6 standard deviations below the mean [You need to calculate the recurrence up to $f_s(100,000)$.]

(b) Using a suitable convolution.