

ACSC/STAT 4703, Actuarial Models II

Fall 2020

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Homework Sheet 5

Due: Friday 27th March: 11:59 PM

Basic Questions

1. An insurance company sets the book pure premium for its fire insurance at \$488. The expected process variance is 92,063 and the variance of hypothetical means is 56,243. If a company has aggregate claims of \$23,400 on policies covering a total of 36 properties, calculate the credibility premium for this company's next year's insurance using the Bühlmann model.
2. An insurance company has the following data on a Workers' compensation insurance policy for a company.

| Year | 1 | 2 | 3 | 4 | 5 |
|------------------|-----------|-----------|-----------|-----------|-----------|
| Exposure | 356 | 402 | 550 | 526 | 572 |
| Aggregate claims | \$250,201 | \$293,114 | \$477,136 | \$482,150 | \$499,300 |

The book premium is \$960 per unit of exposure. The variance of hypothetical means per unit of exposure is 589,000. The expected process variance per unit of exposure is 18,323,900. Using a Bühlmann-Straub model, calculate the credibility premium for Year 6 if the company has 611 units of exposure.

3. An insurance company has the following previous data on aggregate claims:

| Policyholder | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 | Mean | Variance |
|--------------|---------|--------|---------|---------|---------|----------|---------------|
| 1 | 0.00 | 0.00 | 2984.19 | 0.00 | 0.00 | 596.838 | 1781077.99122 |
| 2 | 1401.86 | 0.00 | 0.00 | 5422.18 | 3521.14 | 2069.036 | 5589781.11628 |
| 3 | 0.00 | 0.00 | 0.00 | 512.54 | 861.47 | 274.802 | 156811.77912 |
| 4 | 0.00 | 597.94 | 0.00 | 288.63 | 488.99 | 275.112 | 75379.41947 |

Calculate the Bühlmann credibility premium for each policyholder in Year 6.

4. An insurance company observes the following numbers of claims from individuals over a seven-year period — that is, the following table gives the number of claims in the past seven years:

| No. of claims | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---------------|-------|-------|-----|-----|-----|----|----|----|----|---|----|
| Frequency | 1,933 | 1,788 | 891 | 660 | 491 | 58 | 43 | 46 | 23 | 0 | 1 |

Assuming the number of claims made by an individual in a year follows a Poisson distribution, calculate the credibility estimate for the expected claim frequency in the following year, of an individual who has made a total of 1 claim in the past 6 years. [Note that this is a different length of history from the individuals in the dataset.]

Standard Questions

5. Aggregate claims for a given individual policy are modelled as following a Pareto distribution with $\alpha = 6$. The first 5 years of experience on this policy are:

| Policyholder | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 | Mean | Variance |
|--------------|--------|--------|--------|--------|--------|--------|------------|
| 1 | 0.2 | 17.5 | 0.4 | 14.6 | 1.0 | 6.74 | 8.56551 |
| 2 | 1480.6 | 14.5 | 970.7 | 30.9 | 1873.1 | 873.96 | 840.39769 |
| 3 | 700.9 | 79.9 | 1417.4 | 2702.4 | 1.6 | 980.44 | 1118.40544 |
| 4 | 24.7 | 165.0 | 0.0 | 28.0 | 210.9 | 85.72 | 95.33927 |

- (a) Estimate the EPV and VHM.
- (b) Calculate the credibility premium for policyholder 4 in the next year.
6. Claim frequency in a year for an individual follows a Poisson with parameter Λt where Λ is the individual's risk factor and t is the individual's exposure in that year. An insurance company collects the following data:

| Policyholder | Year 1 | | Year 2 | | Year 3 | |
|--------------|--------|--------|--------|--------|--------|--------|
| | Exp | claims | Exp | claims | Exp | claims |
| 1 | 454 | 5 | 531 | 7 | 450 | 3 |
| 2 | 617 | 1 | 616 | 2 | 539 | 0 |
| 3 | 728 | 5 | 651 | 2 | 804 | 3 |
| 4 | 767 | 2 | 761 | 4 | 832 | 3 |

In Year 4, policyholder 3 has 793 units of exposure. Calculate the credibility estimate for claim frequency for policyholder 3.