ACSC/STAT 4703, Actuarial Models II

Fall 2020

Toby Kenney

Homework Sheet 3

Model Solutions

Basic Questions

1. *A homeowner's house is valued at \$430,000, but is insured at \$220,000. The insurer requires 70% coverage for full insurance. The home sustains \$9,300 from fire. The policy has a deductible of \$5,000, which decreases linearly to zero when the total cost of the loss is \$15,000. How much does the insurance company reimburse?*

   The proportion of coverage is $\frac{220000}{430000\times 0.7} = 0.730897009967$. The deductible is $5000\times\frac{15000-9300}{10000} = \$2,850$. The company therefore reimburses $(9300-2850)\times 0.730897009967 = \$4,714.29$.

2. *An insurance company has three types of coverages for businesses with different expected loss ratios, and has the following data on recent claims:*

Policy Type	*Policy Year*	*Earned Premiums*	*Expected Loss Ratio*	*Losses paid to date*
Workers' compensation insurance	*2017*	*\$3,000,000*	*0.74*	*\$2,300,000*
	2018	*\$3,600,000*	*0.75*	*\$1,100,000*
	2019	*\$4,100,000*	*0.73*	*\$200,000*
Fire insurance	*2017*	*\$1,100,000*	*0.75*	*\$680,000*
	2018	*\$920,000*	*0.74*	*\$645,000*
	2019	*\$1,080,000*	*0.77*	*\$680,000*
Liability insurance	*2017*	*\$2,400,000*	*0.72*	*\$480,000*
	2018	*\$2,700,000*	*0.73*	*\$740,000*
	2019	*\$2,900,000*	*0.71*	*\$190,000*

Calculate the loss reserves at the end of 2019.

Policy Type	Expected Total Year	Losses paid Losses	Reserves to date	
Workers' compensation insurance	2017	\$2,220,000	\$2,300,000	0
	2018	\$2,700,000	\$1,100,000	\$2,600,000
	2019	\$2,993,000	\$200,000	2,793,000
Fire insurance	2017	\$825,000	\$680,000	\$145,000
	2018	\$680,800	\$645,000	35,800
	2019	\$831,600	\$680,000	\$151,600
Liability insurance	2017	\$1,708,000	\$480,000	\$1,228,000
	2018	\$1,971,000	\$740,000	\$1,231,000
	2019	\$2,059,000	\$190,000	\$1,869,000
Total				\$10,105,400

The total reserves needed are therefore $10,105,400.

3. *The following table shows the paid losses on claims from one line of business of an insurance company over the past 6 years.*

		Development year					
Accident year	*Earned premiums*	*0*	*1*	*2*	*3*	*4*	*5*
2014	*4979*	*549*	*1182*	*730*	*508*	*312*	*339*
2015	*5333*	*605*	*1210*	*737*	*693*	*176*	
2016	*5431*	*731*	*1027*	*778*	*551*		
2017	*5555*	*579*	*1314*	*681*			
2018	*5461*	*807*	*1060*				
2019	*5719*	*727*					

Assume that all payments on claims arising from accidents in 2014 have now been settled. Estimate the future payments arising each year from open claims arising from accidents in each calendar year using

(a) The loss development triangle method

We first compute the cumulative loss development:

	Development year					
Accident year	0	1	2	3	4	5
2014	549	1731	2461	2969	3281	3620
2015	605	1815	2552	3245	3421	
2016	731	1758	2536	3087		
2017	579	1893	2574			
2018	807	1867				
2019	727					

Taking the cumulative sums over columns gives

	Development year					
Accident year	0	1	2	3	4	5
2014	549	1731	2461	2969	3281	3620
2015	1154	3546	5013	6214	6702	
2016	1885	5304	7549	9301		
2017	2464	7197	10123			
2018	3271	9064				
2019	3998					

The loss-development factors are therefore $\frac{9064}{3271} = 2.7710180373$, $\frac{10123}{7197} = 1.40655828818$, $\frac{9301}{7549} = 1.2320837197$, $\frac{6702}{6214} = 1.07853234631$ and $\frac{3620}{3281} = 1.10332215788$.

This gives the cumulative loss table:

Accident year	Development year					
	0	1	2	3	4	5
2014						3620
2015					3421	3774
2016				3087	3329	3673
2017			2574	3171	3420	3774
2018		1867	2626	3236	3490	3850
2019	727	2015	2834	3491	3765	4154

Using the average loss development factors, we get

$$\frac{1}{5}\left(\frac{1731}{549}+\frac{1815}{605}+\frac{1758}{731}+\frac{1893}{579}+\frac{1867}{807}\right)=2.82817341845$$

$$\frac{1}{4}\left(\frac{2461}{1731}+\frac{2552}{1815}+\frac{2536}{1758}+\frac{2574}{1893}\right)=1.40751923473$$

$$\frac{1}{3}\left(\frac{2969}{2461}+\frac{3245}{2552}+\frac{3087}{2536}\right)=1.23174772397$$

$$\frac{1}{2}\left(\frac{3281}{2969}+\frac{3421}{3245}\right)=1.07966158782$$

$$\frac{3620}{3281}=1.10332215788$$

This gives the cumulative losses

Accident year	Development year					
	0	1	2	3	4	5
2014						3620
2015					3421	3774
2016				3087	3333	3677
2017			2574	3171	3423	3777
2018		1867	2628	3237	3495	3856
2019	727	2056	2894	3565	3849	4246

(b) The Bornhuetter-Ferguson method with expected loss ratio 0.73.

Using the loss-development factors from (a), the proportion of total losses in each development year is

Development year	Cumulative Proportion of total losses	Proportion of total losses
0	$\frac{1}{2.771\times1.4066\times1.2321\times1.0785\times1.1033}=0.174995620242$	0.174995620242
1	$\frac{1}{1.4066\times1.2321\times1.0785\times1.1033}=0.484916020138$	0.309920399896
2	$\frac{1}{1.2321\times1.0785\times1.1033}=0.682062647196$	0.197146627058
3	$\frac{1}{1.0785\times1.1033}=0.840358283424$	0.158295636228
4	$\frac{1}{1.1033}=0.906353591159$	0.065995307735
5	1	0.093646408841

We get the following table:

Accident year	Earned premiums	Expected total payments	Development year				
			1	2	3	4	5
2015	5333	3893.09					365
2016	5431	3964.63				262	371
2017	5555	4055.15			642	268	380
2018	5461	3986.53		786	631	263	373
2019	5719	4174.87	1294	823	661	276	391

Using the average development factors gives

Development year	Cumulative Proportion of total losses	Proportion of total losses
0	$\frac{1}{2.8282\times1.4075\times1.2317\times1.0797\times1.1033} = 0.171209502967$	0.171209502967
1	$\frac{1}{1.4075\times1.2317\times1.0797\times1.1033} = 0.484210165277$	0.31300066231
2	$\frac{1}{1.2317\times1.0797\times1.1033} = 0.681535121278$	0.197324956001
3	$\frac{1}{1.0797\times1.1033} = 0.839479334439$	0.157944213161
4	$\frac{1}{1.1033} = 0.906353591159$	0.06687425672
5	1	0.093646408841

We get the following table:

Accident year	Earned premiums	Expected total payments	Development year				
			1	2	3	4	5
2015	5333	3893.09					365
2016	5431	3964.63				265	371
2017	5555	4055.15			640	271	380
2018	5461	3986.53		787	630	267	373
2019	5719	4174.87	1307	824	659	279	391

4. *An actuary is reviewing the following claims data:*

No. of closed claims

Acc. Year	*Development Year 0*	*1*	*2*	*3*	*4*	*Ult.*
2015	*662*	*1,150*	*1,435*	*1,544*	*1,697*	*2035*
2016	*691*	*1,207*	*1,444*	*1,736*		*2070*
2017	*819*	*1,314*	*1,455*			*2105*
2018	*777*	*1,263*				*2140*
2019	*761*					*2175*

Total paid losses on closed claims (000's)

Acc. Year	*Development Year 0*	*1*	*2*	*3*	*4*
2015	*1,446*	*2,950*	*5,287*	*6,530*	*7,241*
2016	*1,536*	*3,616*	*5,361*	*6,902*	
2017	*2,075*	*3,833*	*5,328*		
2018	*1,636*	*4,067*			
2019	*2,069*				

(a) Calculate tables of percentage of claims closed and cumulative average losses.

Acc. Year	Development Year 0	1	2	3	4
2015	32.5	56.5	70.5	75.9	83.4
2016	33.4	58.3	69.8	83.9	
2017	38.9	62.4	69.1		
2018	36.3	59.0			
2019	35.0				

Acc. Year	Development Year 0	1	2	3	4
2015	2,184	2,565	3,684	4,229	4,267
2016	2,223	2,996	3,713	3,976	
2017	2,534	2,917	3,662		
2018	2,106	3,220			
2019	2,719				

(b) Adjust the total loss table to use the current disposal rate.

We need to multiply by the following factors:

Acc. Year	Development Year 0	1	2	3	4
2015	1.07555300899	1.04437423811	0.980219652891	1.10534154339	1
2016	1.04813613454	1.0121681159	0.990863989577	1	
2017	0.899277223415	0.945466862971	1.		
2018	0.963647391232	1			
2019	1				

This gives the following adjusted total loss:

Acc. Year	Development Year 0	1	2	3	4
2015	1,555	3,081	5,182	7,218	7,241
2016	1,610	3,660	5,312	6,902	
2017	1,866	3,624	5,328		
2018	1,577	4,067			
2019	2,069				

(c) Use the chain ladder method to estimate claim development based on the adjusted numbers. Compare this to the chain ladder method on aggregate payments on closed claims.

The loss development factors are

Development year	Loss development
0/1	$\frac{3080.90400242+3659.99990709+3623.97448577+4067}{1555.249651+1609.93710265+1866.00023859+1576.52713206} = 2.18409545628$
1/2	$\frac{5182.42130483+5312.02184812+5328}{3080.90400242+3659.99990709+3623.97448577} = 1.52654402199$
2/3	$\frac{7217.88027834+6902}{5182.42130483+5312.02184812} = 1.34546255313$
3/4	$\frac{7241}{7217.88027834} = 1.00320311792$

This results in the following estimated cumulative payments

Acc. Year		Development Year 0	1	2	3	4
2014					6902	6924
2015				5328	7169	7192
2016		4067	6208	8353	8380	
2017	2069	4519	6898	9281	9311	

Using the aggregate losses, the loss development factors are

Development year	Loss development
0/1	$\frac{2950+3616+3833+4067}{1446+1536+2075+1636} = 2.16136261766$
1/2	$\frac{5287+5361+5328}{2950+3616+3833} = 1.53630156746$
2/3	$\frac{6530+6902}{5287+5361} = 1.26145755071$
3/4	$\frac{7241}{6530} = 1.1088820827$

This results in the following estimated cumulative payments

Acc.	Development Year					
Year		0	1	2	3	4
2014					6902	7654
2015				5328	6721	7453
2016		4067	6248.13847486	7882	8740	
2017	2069	4472	6870	8666	9610	

Standard Questions

5. *The number of claims on an insurance policy follows a Poisson distribution with mean 0.04. For each claim, there is the following distribution of years to settlement and final claim amount:*

Years to	*Probability*	*Final Claim amount*	
settlement		*Mean*	*Standard Deviation*
0	*0.15*	*700*	*300*
1	*0.25*	*800*	*350*
2	*0.35*	*1,200*	*600*
3	*0.1*	*1,700*	*1,200*
4	*0.1*	*2,600*	*4,200*
5	*0.05*	*3,400*	*6,500*

(a) Calculate the expected loss development ratio.

The expected total payments in each year per policy are given by

Years	Expected payment	Cumulative Expected Payments
0	$0.04 \times 0.15 \times 700 = 4.2$	4.2
1	$0.04 \times 0.25 \times 800 = 8$	12.2
2	$0.04 \times 0.35 \times 1200 = 16.8$	29.0
3	$0.04 \times 0.1 \times 1700 = 6.8$	35.8
4	$0.04 \times 0.1 \times 2600 = 10.4$	46.2
5	$0.04 \times 0.05 \times 3400 = 6.8$	53.0

Thus, the expected loss development ratios are

Development year	Loss development
0/1	$\frac{12.2}{4.2} = 2.90476190476$
1/2	$\frac{29.0}{12.2} = 2.37704918033$
2/3	$\frac{35.8}{29.0} = 1.23448275862$
3/4	$\frac{46.2}{35.8} = 1.2905027933$
4/5	$\frac{53.0}{46.2} = 1.14718614719$

(b) The number of policies sold in the past 5 years is given by

Year	*Policies Sold*
2015	*3,531*
2016	*4,055*
2017	*4,621*
2018	*4,802*
2019	*5,110*

Using a normal approximation for aggregate losses, estimate the 95th percentile of the total payments made in 2020 for these policies.

The number of claims settled in year n for a single policy follows a Poisson distribution with mean $0.04p_n$. The expected total payment in year n for this policy is therefore $0.04p_n\mu_n$, where μ_n is the expected payment for a claim that settles after n years. The variance of the total payment is $0.04p_n(\mu_n^2+\sigma_n^2)$. Thus, the expectation and variance for claims made from each previous year are:

Year	Policies Sold	Expected payment per policy	Variance per policy	Expected aggregate claim	variance of aggregate claim
2015	3,531	6.8	107620	24010.8	380006220
2016	4,055	10.4	97600	42172.	395768000
2017	4,621	6.8	17320	31422.8	80035720
2018	4,802	16.8	25200	80673.6	121010400
2019	5,110	8.0	7625	40880	38963750

Thus the expected total payment in 2020 is $24010.8 + 42172 + 31422.8 + 80673.6 + 40880 = 219159.2$, and the variance of total payments is $380006220 + 395768000 + 80035720 + 121010400 + 38963750 = 1015784090$. Approximating the aggregate payments by a normal distribution, the 95th percentile of total payments in 2020 is $219159.2 + 1.644854\sqrt{1015784090} = \$271,582.95$.