

ACSC/STAT 4703, Actuarial Models II

FALL 2021

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Homework Sheet 1

Due: Tuesday 28th September: 11:30 AM

Basic Questions

1. Aggregate payments have a compound distribution. The frequency distribution is Binomial with $n = 7$ and $p = 0.4$. The severity distribution is Pareto with shape $\alpha = 2.3$ and scale $\theta = 600$. Use a Pareto approximation to aggregate payments to estimate the probability that aggregate payments are more than 10,000.
2. Loss amounts follow a Gamma distribution with shape $\alpha = 4.6$ and scale $\theta = 500$. The distribution of the number of losses is given in the following table:

Number of Losses	Probability
0	0.880
1	0.064
2	0.035
3	0.021

Assume all losses are independent and independent of the number of losses. The insurance company buys excess-of-loss reinsurance on the part of the loss above \$1,000. Calculate the expected payment for this excess-of-loss reinsurance.

3. Claim frequency follows a negative binomial distribution with $r = 1.8$ and $\beta = 2.2$. Claim severity (in thousands) has the following distribution:

Severity	Probability
1	0.69
2	0.24
3	0.06
4 or more	0.01

Use the recursive method to calculate the exact probability that aggregate claims are at least \$4,000.

4. Use an arithmetic distribution ($h = 1$) to approximate a Weibull distribution with shape $\tau = 0.5$ and scale $\theta = 8.2$.

- (a) Using the method of rounding, calculate the mean of the arithmetic approximation. [You can evaluate this numerically: use 2,000 terms in the sum.]
- (b) Using the method of local moment matching, matching 1 moment on each interval, estimate the probability that the value is larger than 5.5.

Standard Questions

5. An insurance company models loss frequency as Poisson with parameter $\lambda = 6$, and loss severity as Pareto with shape $\alpha = 2.5$ and scale $\theta = 2400$. One reinsurance company uses a gamma distribution to model aggregate losses, fitted by the method of moments, and sells stop-loss reinsurance with attachment point \$10,000 for a loading of 100% based on the estimated payments under this model. Another reinsurance company uses a Pareto distribution to model aggregate losses and charges a 20% loading.
 - (a) Which reinsurance company is cheaper if each policy includes a deductible of \$500.
 - (b) Show that a deductible $d = 519.285$ is a local maximum for the second reinsurer's premium.
6. The number of claims an insurance company receives follows a binomial distribution with $n = 128$ and $p = 0.64$. Claim severity follows a negative binomial distribution with $r = 8.3$ and $\beta = 24$. Calculate the probability that aggregate losses exceed \$20,000.
 - (a) Starting the recurrence 6 standard deviations below the mean [You need to calculate 30000 terms of the recurrence for f_s .]
 - (b) Using a suitable convolution.