ACSC/STAT 4703, Actuarial Models II

FALL 2021

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Homework Sheet 2

Due: Thursday 7th October: 11:30 AM

Basic Questions

1. An insurance company has the following portfolio of medical malpractice insurance policies:

Type of worker	Number	Probability of claim	mean claim (millions)	standard deviation (millions)
Family Doctor	1,500	0.00015	\$1.3	\$0.9
Surgeon	600	0.00074	\$4.4	\$5.6
Nurse	2,400	0.00136	\$0.5	\$0.4

They model aggregate losses using a gamma distribution. Calculate the cost of reinsuring losses above \$10,000,000, if the loading on the reinsurance premium is one standard deviation above the expected claim payment on the reinsurance policy.

2. An insurance company is modelling claim data as following a log-normal distribution with $\sigma = 1$. It collects the following sample of claims:

```
2.7 3.9 5.0 5.6 6.0 6.5 6.8 8.3 9.7 10.6 10.7 10.9
11.1 11.1 11.5 11.8 12.6 12.7 13.5 14.0 15.7 16.2 16.3
20.8 21.5 23.8 28.7 29.8 31.0 31.7 33.9 35.2 39.8 40.8
48.8 49.6 70.6 74.1 84.2 86.6
```

```
X<- c(2.7,3.9,5.0,5.6,6.0,6.5,6.8,8.3,9.7,10.6,10.7,
10.9,11.1,11.1,11.5,11.8,12.6,12.7,13.5,14.0,15.7,16.2,
16.3,20.8,21.5,23.8,28.7,29.8,31.0,31.7,33.9,35.2,39.8,
40.8,48.8,49.6,70.6,74.1,84.2,86.6)
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The MLE for μ is 2.845. Graphically compare this empirical distribution with the best log-normal distribution with $\sigma = 1$. Include the following plots:

(a) Comparisons of $F(x)$ and $F^*(x)$

(b) Comparisons of $f(x)$ and $f^*(x)$

(c) A plot of $D(x)$ against x.

(d) A *p*-*p* plot of $F(x)$ against $F^*(x)$.

3. For the data in Question 2, calculate the following test statistics for the goodness of fit of the log-normal distribution with $\sigma = 1$ and $\mu = 2.845$:

   (a) The Kolmogorov-Smirnov test.

   (b) The Anderson-Darling test.

   (c) The chi-square test, dividing into the intervals 0–10, 10–20, 20–40 and more than 40.

4. For the data in Question 2, perform a likelihood ratio test to determine whether a log-normal distribution with fixed $\sigma = 1$, or a log-normal distribution with σ freely estimated is a better fit for the data. [For the general log-normal distribution, the MLE is $\sigma^2 = 0.7305957$ and $\mu = 2.845$.]

5. For the data in Question 2, use AIC and BIC to choose between a log-normal distribution with $\sigma = 1$ for the data and a transformed gamma distribution. [The MLE for the transformed gamma distribution is $\alpha = 0.02372$, $\theta = 87.77037$ and $\tau = 26.26$.]

Standard Questions

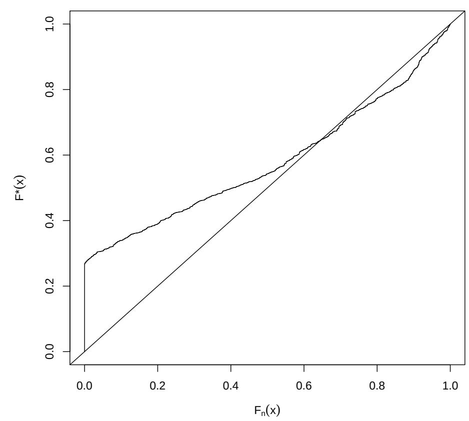
6. A health insurer divides insureds into three categories: non-smoker; occasional smoker; and heavy smoker. The number of claims made by an individual follows a negative binomial distribution with parameters r and β. It has the following portfolio of policies.

Category	Number insured	r	β of claim	mean claim	standard deviation
non-smoker	3,422	0.3	2.2	\$860	\$ 83,620
occasional smoker	1,053	0.9	2.4	\$1,220	\$113,190
heavy smoker	410	1.2	4.8	\$1,740	\$179,420

   The insurance company models the aggregate losses as following a Pareto distribution with the correct mean and variance. It wants to buy stop-loss reinsurance for its policies. The reinsurance company uses the same Pareto distribution to model aggregate losses and sets its premium at 125% of expected payments on the policy. The insurer sets the premium for the part of the losses that it covers as one standard deviation above expected aggregate payments it makes on the portfolio (and directly adds the reinsurer's premium to this). What attachment point for the reinsurance results in the smallest total premium for the policy? [You may need to numerically solve the derivative equal to zero. You may find the substitution $t = \frac{\theta}{\theta+a}$ helpful.]

7. An insurance company collects a sample of 700 past claims, and attempts to fit a distribution to the claims. Based on experience with other claims, the company believes that an inverse gamma distribution with $\alpha = 3.4$

and $\theta = 1,000$ may be appropriate to model these claims. It constructs the following *p*-*p* plot to compare the sample to this distribution:



(a) What was the smallest data point of the sample?

(b) Which of the following statements best describes the fit of the inverse gamma distribution to the data:

(i) The inverse gamma distribution assigns too much probability to high values and too little probability to low values.

(ii) The inverse gamma distribution assigns too much probability to low values and too little probability to high values.

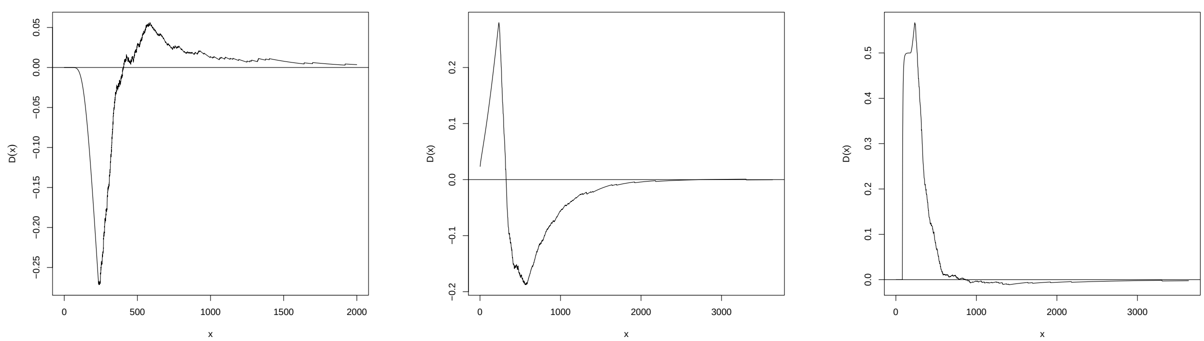
(iii) The inverse gamma distribution assigns too much probability to tail values and too little probability to central values.

(iv) The inverse gamma distribution assigns too much probability to central values and too little probability to tail values.

Justify your answer.

(c) Which of the following plots is the $D(x)$ plot of this model on this data? Justify your answer.

(i) (ii) (iii)


D(x)
x
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
0
500
1000
1500
2000
0.2
0.1
0.0
−0.1
−0.2
0
1000
2000
3000
0.5
0.4
0.3
0.2
0.1
0.0