ACSC/STAT 4703, Actuarial Models II

FALL 2022

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Midterm Examination

Tuesday 18th October
16:05– 17:25

Here are some values of the Gamma distribution function with $\theta = 1$ that may be needed for this examination:

x	α	$F(x)$	x	α	$F(x)$	x	α	$F(x)$
245	255	0.2697208	2.5	4	0.2424239	4.375	4	0.6361773
$\left(\frac{7.5}{12}\right)^3$	$\frac{4}{3}$	0.1117140	3.841	2.4	0.8409823	4.875	4	0.7169870
$\left(\frac{9.5}{12}\right)^3$	$\frac{4}{3}$	0.2507382	4.375	3	0.8118663	5.375	4	0.7837292
1.356	2.4	0.2801616	4.875	3	0.8644174	2.156	5	0.06782354
1.941	2.4	0.4612472	5.375	3	0.9035828	3.203	5	0.219922
2.367	2.4	0.5775816	3.875	4	0.5417358	8.542	5	0.9274742

Here are the critical values for a chi-squared distribution:

Degrees of Freedom	Significance level 90%	95%	99%
1	2.705543	3.841459	6.634897
2	4.605170	5.991465	9.210340
3	6.251389	7.814728	11.344867
4	7.779440	9.487729	13.276704
5	9.236357	11.070498	15.086272

1. Using an arithmetic distribution ($h = 1$) to approximate an inverse Weibull distribution with $\tau = 2$ and $\theta = 6$, calculate the probability that the value is more than 8.5, for the approximation using the method of local moment matching, matching 1 moment on each interval.

   [Hint:

$$\int_8^9 x^{-2} e^{-\left(\frac{6}{x}\right)^2}\,dx = 0.00840944$$

   ]

2. Claim frequency follows a negative binomial distribution with $r = 3.3$ and $\beta = 1.5$. Claim severity (in thousands) has the following distribution:

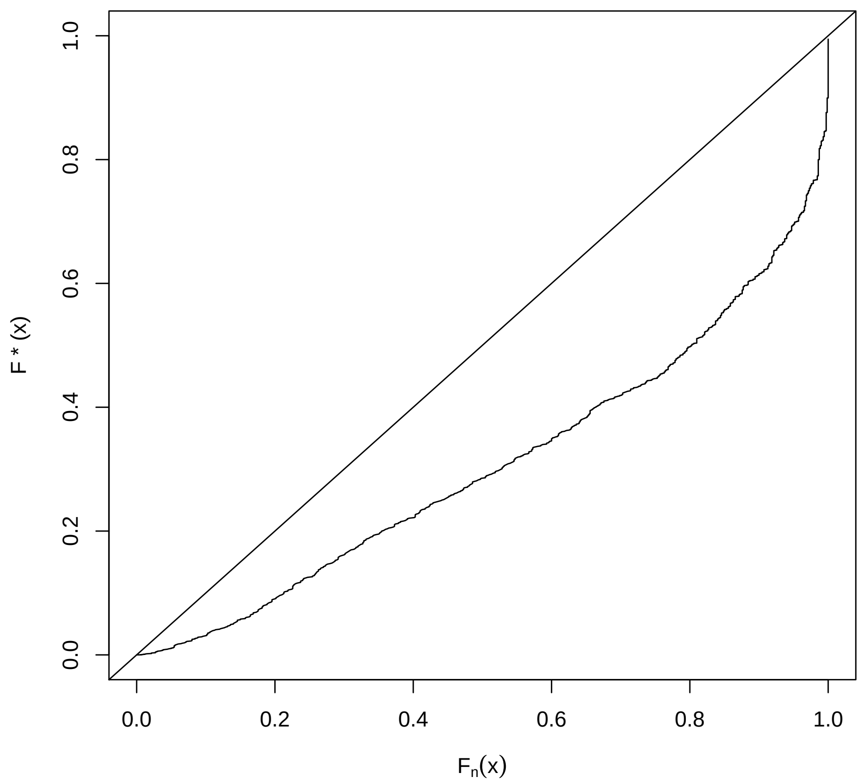
Severity	Probability
0	0.46
1	0.33
2	0.16
$\geqslant 3$	0.05

The expected claim severity per loss is 0.81. The company buys excess-of loss reinsurance for aggregate losses exceeding 2.

(a) Use the recursive method to calculate the probability that the reininsurance makes a payment.

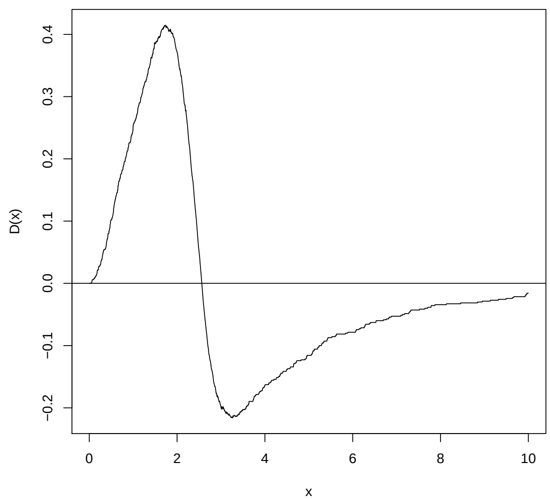
(b) What is the expected payment on the reinsurance? [Hint: first calculate the insurer's expected payment with this reinsurance policy. Then consider the expected total payments between the insurer and the reinsurer.]

3. An insurance company collects a sample of 700 claims. Based on previous experience, it believes these claims might follow an inverse Pareto distribution with $\theta = 0.7$ and $\tau = 3.9$. To test this, it computes the following p-p plot.

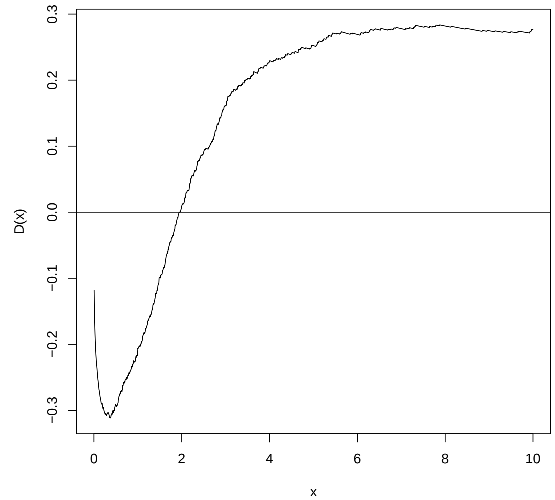


(a) How many of the claims in their sample were between 3 and 11?

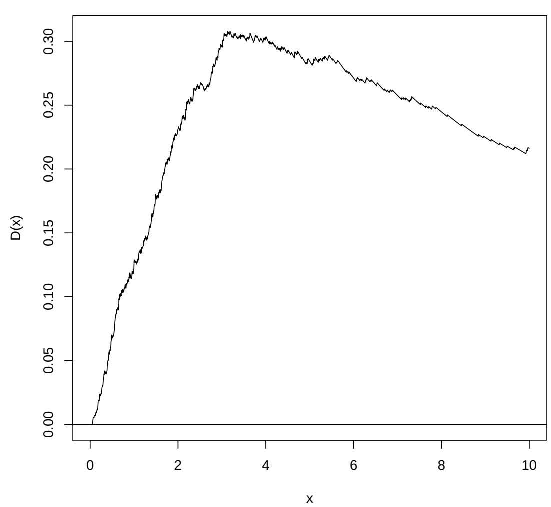
(b) Which of the following is a plot of $D(x) = F_n(x) - F^*(x)$ for this data?



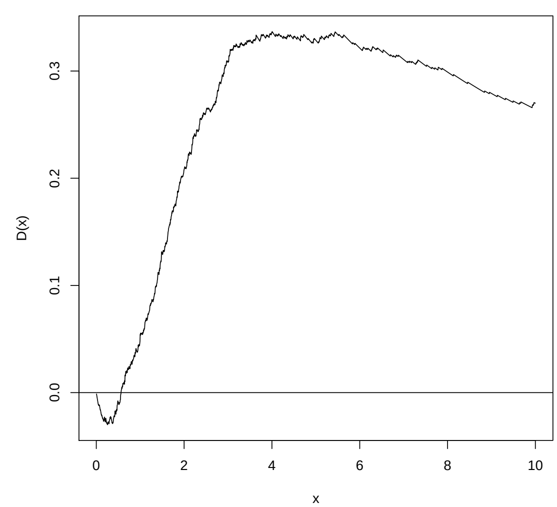
(i)



(ii)



(iii)



(iv)

Justify your answer.

4. An insurance company collects the following sample:

   `0.06 0.32 0.61 0.67 1.16 2.53 4.02 5.09 10.27 15.83 17.64 17.84 20.00 20.92 24.44 42.52 63.80 71.84`

   They model this as following a distribution with the following distribution function:

x	$F(x)$	$i^2(\log(F(x_{i+1})) - \log(F(x_i)))$	$(18-i)^2(\log(1-F(x_i)) - \log(1-F(x_{i+1})))$
0.06	0.003940665	3.6581388	52.466840
0.32	0.152854776	1.3462645	21.656703
0.61	0.214016777	0.8161002	6.703294
0.67	0.234330430	5.0055286	26.831221
1.16	0.320402414	1.5140605	5.855879
2.53	0.340406390	4.2591188	11.325024
4.02	0.383158609	13.8026392	32.511770
5.09	0.507824310	1.0051731	1.992625
10.27	0.515863074	2.8523340	3.893844
15.83	0.534352306	9.9014547	10.302913
17.64	0.589968955	12.5596075	10.958980
17.84	0.654497867	17.0641094	13.339362
20.00	0.736838758	13.0086484	9.131373
20.92	0.795796371	5.5330811	2.957822
24.44	0.818581810	22.8721986	10.549221
42.52	0.906170573	4.0967235	1.524215
63.80	0.920788516	6.6446792	1.260842
71.84	0.942204512	19.2886673	
		145.228528	223.261928

Calculate the Anderson-Darling statistic for this model and this data.

5. An insurance company collects a sample of 1500 claims. They want to decide whether this data is better modeled as following an inverse exponential distribution, or a generalised Pareto distribution. After calculating MLE estimates for the parameters (1 parameter for the inverse exponential and 3 for the generalised Pareto), log-likelihoods for the two distributions are:

Distribution	log-likelihood
Inverse Exponential	-4244.75
Generalised Pareto	-4236.89

Use a BIC to decide whether the generalised Pareto distribution or the inverse exponential distribution is a better fit for the data.

6. A homeowner's house has a value of \$860,000. The insurer requires 75% coverage for full insurance. The deductible is \$4,000, decreasing linearly to zero for losses of \$12,000. The home sustains \$7,000 of damage from fire. The insurer reimburses \$3,300. For what value was the home insured?

7. The following table shows the cumulative losses (in thousands) on claims from one line of business of an insurance company over the past 4 years.

	Development year			
Accident year	0	1	2	3
2018	5539	6003	6829	7108
2019	6243	6792	7314	
2020	6217	7209		
2021	6372			

Using the mean for calculating loss development factors, esimate the total reserve needed for payments to be made in **2023** (two years in the future) using the Bornhuetter-Fergusson method. The expected loss ratio is 0.74 and the earned premiums in each year are given in the following table:

Year	Earned Premiums (000's)
2018	8537
2019	8764
2020	9023
2021	9285

[Assume no more payments are made after development year 3.]