# ACSC/STAT 4703, Actuarial Models II

FALL 2022

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Homework Sheet 1

Due: Tuesday 27th September: 17:30

## Basic Questions

1. Aggregate payments have a compound distribution. The frequency distribution is negative binomial with $r = 3$ and $\beta = 0.5$. The severity distribution is gamma with shape $\alpha = 2.3$ and scale $\theta = 400$. Use a gamma approximation to aggregate payments to estimate the probability that aggregate payments are more than 4,000.

2. Loss amounts follow a gamma distribution with shape $\alpha = 1.3$ and scale $\theta = 1500$. The distribution of the number of losses is given in the following table:

| Number of Losses | Probability |
|---|---|
| 0 | 0.930 |
| 1 | 0.024 |
| 2 | 0.015 |
| 3 | 0.031 |

   Assume all losses are independent and independent of the number of losses. The insurance company buys excess-of-loss reinsurance on the part of the loss above \$5,000. Calculate the expected payment for this excess-of-loss reinsurance.

3. Claim frequency follows a negative binomial distribution with $r = 4.8$ and $\beta = 1.2$. Claim severity (in thousands) has the following distribution:

| Severity | Probability |
|---|---|
| 1 | 0.24 |
| 2 | 0.30 |
| 3 | 0.26 |
| 4 or more | 0.20 |

   Use the recursive method to calculate the exact probability that aggregate claims are at least \$4,000.

4. Use an arithmetic distribution ($h = 1$) to approximate a Pareto distribution distribution with shape $\alpha = 3.5$ and scale $\theta = 6.6$.

(a) Using the method of rounding, calculate the mean of the arithmetic approximation. [You can evaluate this numerically: use 5,000 terms in the sum.]

(b) Using the method of local moment matching, matching 1 moment on each interval, estimate the probability that the value is larger than 3.5.

## Standard Questions

5. An insurance company models loss frequency as negative binomial with $r = 3$ and unknown $\beta$, and loss severity as gamma with shape $\alpha = 0.6$ and scale $\theta = 2400$. There is a per-loss deductable of \$500 for the policy.

   A reinsurance company models aggregate losses using a Pareto distribution with parameters fitted using the method of moments. Using this model, they calculate the cost of stop-loss reinsurance with attachment point \$10,000 and loading of 20% as \$4,000. What is the value of $\beta$?

   [You should get an equation for $\beta$, which can easily be solved by a grid-search (calculating a large number of values to find the correct one).]

   (b)

6. The number of claims an insurance company receives follows a negative binomial distribution with $r = 68$ and $\beta = 1.6$. Claim severity follows a negative binomial distribution with $r = 7.2$ and $\beta = 12$. Calculate the probability that aggregate losses exceed \$12,000.

   (a) Starting the recurrence 6 standard deviations below the mean [You need to calculate 15,000 terms of the recurrence for $f_s$.]

   (b) Using a suitable convolution.