ACSC/STAT 4703, Actuarial Models II

FALL 2022
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Homework Sheet 2

Due: Thursday 6th October: 17:30

Basic Questions

1. An insurance company has the following portfolio of liability insurance policies:

Type of product	Number	Probability of claim	mean claim (millions)	standard deviation (millions)
Electrical	2,800	0.00125	\$1.3	\$33.1
Health	4,300	0.00374	\$2.2	\$21.6
Other	3,700	0.00072	\$0.8	\$24.5

They model aggregate losses using a Pareto distribution. Calculate the cost of reinsuring losses above \$10,000,000, if there is a 30% loading on the reinsurance premium.

2. An insurance company is modelling claim data as following a Weibull distribution with $\tau = 0.4$. It collects the following sample of claims:

```
0.1 0.2 0.2 0.4 0.4 0.8 0.8 1.1 1.5 1.5 1.8 2.5 3.2
3.3 3.4 4.1 4.8 5.1 10.7 20.1 25.3 26.1 36.2 45.8 48.7
50.5 56.3 75.6 81.5 81.5 81.6 108.2 170.8 186.9 345.9
356.0 412.7 523.1 1439.0 1998.8
```

```
 X<- c( 0.1, 0.2, 0.2, 0.4, 0.4, 0.8, 0.8, 1.1, 1.5, 1.5
1.8, 2.5, 3.2, 3.3, 3.4, 4.1, 4.8, 5.1, 10.7, 20.1
25.3, 26.1, 36.2, 45.8, 48.7, 50.5, 56.3, 75.6, 81.5, 81.5
81.6,108.2,170.8,186.9,345.9,356.0,412.7,523.1,1439.0,1998.8)
```

The MLE for θ is 4.657714. Graphically compare this empirical distribution with the best Weibull distribution with $\tau = 0.4$. Include the following plots:

(a) Comparisons of $F(x)$ and $F^*(x)$

(b) Comparisons of $f(x)$ and $f^*(x)$

(c) A plot of $D(x)$ against x.

(d) A *p*-*p* plot of $F(x)$ against $F^*(x)$.

3. For the data in Question 2, calculate the following test statistics for the goodness of fit of the Weibull distribution with $\tau = 0.4$ and $\theta = 4.657714$:

   (a) The Kolmogorov-Smirnov test.

   (b) The Anderson-Darling test.

   (c) The chi-square test, dividing into the intervals 0–1,1–5,5–50 and more than 50.

4. For the data in Question 2, perform a likelihood ratio test to determine whether a Weibull distribution with fixed $\tau = 0.4$, or a transformed gamma distribution with α and τ freely estimated is a better fit for the data. [For the transformed gamma distribution, the MLE is $\alpha = 0.0273$, $\tau = 7.415$ and $\theta = 2045.683$.]

5. For the data in Question 2, use AIC and BIC to choose between a Weibull distribution with $\tau = 0.4$ for the data and a Pareto distribution. [The MLE for the Pareto distribution is $\alpha = 0.4108$, $\theta = 2.112$.]

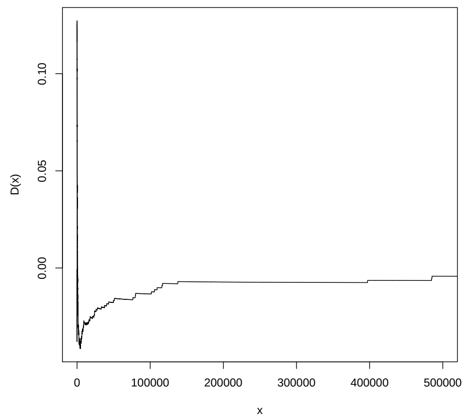
Standard Questions

6. An auto insurer divides insureds into three categories: new drivers; average drivers; and good drivers. The number of claims made by an individual follows a negative binomial distribution with parameters r and β. The insurance company has the following portfolio of policies.

Category	Number insured	r	β of claim	mean claim	standard deviation
new driver	1,300	0.3	22.6	\$639	\$250,340
average driver	6,422	3.2	0.8	\$1,430	\$180,460
safe driver	1,105	0.9	1.8	\$1,100	\$105,660

   The insurance company models the aggregate losses as following an Pareto distribution with the correct mean and variance. It charges a 10% loading on its premiums. It wants to buy stop-loss reinsurance for its policies. The reinsurance company uses the same Pareto distribution to model aggregate losses and sets its premium at 125% of expected payments on the policy. The insurer is willing to pay 30% of its total premiums towards reinsurance. What attachment point can it set to achieve this?

7. An insurance company collects a sample of 900 past claims, and attempts to fit a distribution to the claims. Based on experience with other claims, the company believes that an inverse Pareto distribution with $\tau = 3.1$ and $\theta = 35$ may be appropriate to model these claims. It constructs the following plot of $D(x) = F_n(x) - F^*(x)$ to compare the sample to this distribution:



(a) How many data points in the sample were more than 100,000?

(b) Which of the following statements best describes the fit of the inverse gamma distribution to the data:

(i) The inverse Pareto distribution assigns too much probability to high values and too little probability to low values.

(ii) The inverse Pareto distribution assigns too much probability to low values and too little probability to high values.

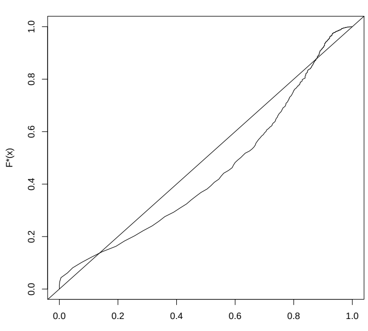
(iii) The inverse Pareto distribution assigns too much probability to tail values and too little probability to central values.

(iv) The inverse Pareto distribution assigns too much probability to central values and too little probability to tail values.

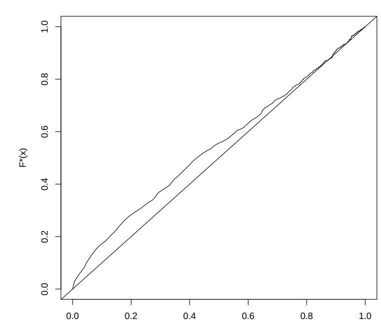
Justify your answer.

(c) Which of the following plots is the *p*-*p* plot of this model on this data? Justify your answer.

(i)



(ii)



(iii)