ACSC/STAT 4703, Actuarial Models II

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Homework Sheet 5

Model Solutions

Basic Questions

1. *An insurance company sets the book pure premium for its group health insurance at \$730. The expected process variance is 9,224,000 and the variance of hypothetical means is 433,000. If a company has aggregate claims of \$54,200 over the past 13 years, calculate the credibility premium for this company's next year's insurance using the Bühlmann model.*

   The credibility of 13 years of experience is $Z = \frac{13}{13+\frac{9224000}{433000}} = 0.378980677304$. The credibility premium for this individual is therefore $0.378980677304 \times \frac{54200}{13} + 0.621019322696 \times 730 = \$2,033.40$.

2. *An insurance company has the following data on a commercial auto insurance policy for a company.*

Year	*1*	*2*	*3*	*4*	*5*
Exposure	*4,043*	*4,626*	*4,595*	*4,906*	*5,304*
Aggregate claims	*\$894,100*	*\$1,305,500*	*\$991,400*	*\$1,126,700*	*\$1,295,000*

   The book premium is \$602 per unit of exposure. The variance of hypothetical means per unit of exposure is 92,682. The expected process variance per unit of exposure is 8,046,942,802. Using a Bühlmann-Straub model, calculate the credibility premium for Year 6 if the company has 6,014 units of exposure.

   The company has aggregate claims of \$5,612,700 from 23,474 units of experience. The credibility of the company's experience is $Z = \frac{23474}{23474+\frac{8046942802}{92682}} = 0.212825090105$. Therefore the company's new premium per unit of exposure is $0.212825090105 \times \frac{5612700}{23474} + 0.787174909895 \times 602 = 524.766378625$. The total premium for this company for 6,014 units of exposure is therefore $6014 \times 524.766378625 = \$3,155,945.00$.

3. *An insurance company has the following previous data on aggregate claims:*

Policyholder	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Year 5*	*Mean*	*Variance*
1	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*	*0.000*	*0.00000*
2	*443.61*	*0.00*	*0.00*	*0.00*	*0.00*	*88.722*	*39357.96642*
3	*657.96*	*0.00*	*0.00*	*1299.19*	*0.00*	*391.430*	*338679.69830*
4	*0.00*	*0.00*	*0.00*	*0.00*	*1990.59*	*398.118*	*792489.70962*
5	*0.00*	*2573.86*	*48153.45*	*0.00*	*0.00*	*10145.462*	*452681876.55872*

Calculate the Bühlmann credibility premium for each policyholder in Year 6.

The book premium is the average of the average claims for each individual, i.e. $\frac{0.000+88.722+391.430+398.118+10145.462}{5} = \$2,204.75$. The estimated EPV is the average of the variances for the individuals, that is

$$\frac{0.00000 + 39357.96642 + 338679.69830 + 792489.70962 + 452681876.55872}{5} = 90770480.7866$$

The variance of the observed means is

$$\frac{(0.000 - 2204.75)^2 + (88.722 - 2204.75)^2 + (391.430 - 2204.75)^2 + (398.118 - 2204.75)^2 + (10145.462 - 2204.75)^2}{4} = 19736363.183$$

The part of this due to process variance is $\frac{90770480.7866}{5} = 18154096.1573$.
Therefore, the estimated VHM is $19736363.183 - 18154096.1573 = 1582267.0257$.

The credibility of 5 years of experience is therefore

$$Z = \frac{5}{5 + \frac{90770480.7866}{1582267.0257}} = 0.0801701413288$$

The premiums are therefore:

Policyholder	Premium
1	$0.0801701413288 \times 0.000 + 0.919829858671 \times 2204.75 = \$2,027.99$
2	$0.0801701413288 \times 88.722 + 0.919829858671 \times 2204.75 = \$2,035.11$
3	$0.0801701413288 \times 391.430 + 0.919829858671 \times 2204.75 = \$2,059.38$
4	$0.0801701413288 \times 398.118 + 0.919829858671 \times 2204.75 = \$2,059.91$
5	$0.0801701413288 \times 10145.462 + 0.919829858671 \times 2204.75 = \$2,841.36$

Standard Questions

4. *A insurance company models the number of claims made by a company in a year as a Poisson distribution with mean proportional to their exposure multiplied by a constant that varies between companies. It has the following data from 2020:*

Policyholder	*Exposure*	*No. of Claims*	*Policyholder*	*Exposure*	*No. of Claims*	*Policyholder*	*Exposure*	*No. of Claims*
1	*2005*	*2*	*9*	*586*	*50*	*17*	*319*	*16*
2	*1273*	*0*	*10*	*535*	*13*	*18*	*223*	*2*
3	*1135*	*15*	*11*	*509*	*28*	*19*	*220*	*22*
4	*973*	*0*	*12*	*504*	*1*	*20*	*220*	*0*
5	*861*	*2*	*13*	*496*	*18*	*21*	*189*	*2*
6	*826*	*0*	*14*	*458*	*44*	*22*	*188*	*3*
7	*662*	*33*	*15*	*401*	*0*	*23*	*181*	*0*
8	*619*	*0*	*16*	*394*	*12*	*24*	*147*	*15*

Using this data, calculate the credibility estimate for the expected claim frequency in the following year, for policyholder 3, who made 15 claims from 1135 units of exposure, if that policyholder has 1,310 units of exposure the following year.

There were a total of 278 claims from 13924 units of exposure, so the mean claim frequency is $\frac{278}{13924} = 0.0199655271474$ per unit of exposure. For the Poisson distribution, this is also the average EPV per unit of exposure.

The estimator for variance of the hypothetical means is

$$\frac{\sum_{i=1}^{24} m_i \left(\frac{X_i}{m_i} - 0.0199655271474\right)^2 - 23 \times 0.0199655271474}{\sum_{i=1}^{21} m_i - \frac{\sum_{i=1}^{24} {m_i}^2}{\sum_{i=1}^{24} m_i}}$$

```
HW5Q4<-cbind(c(2005,1273,1135, 973, 861, 826, 662, 619, 586, 535, 509, 504, 496,
                458, 401, 394, 319, 223, 220, 220, 189, 188, 181, 147),
             c(2, 0,15, 0, 2, 0,33, 0,50,13,28, 1,18,44, 0,12,16, 2,22, 0, 2,
               3, 0,15))

colSums(HW5Q4)

EPV<-sum(HW5Q4[,2])/sum(HW5Q4[,1])
VHM<-(sum(HW5Q4[,1]*(HW5Q4[,2]/HW5Q4[,1]-EPV)^2)-23*EPV)/
    (sum(HW5Q4[,1])-sum(HW5Q4[,1]^2)/sum(HW5Q4[,1])))
```

This gives 0.0009077470062 as the estimated VHM.

We calculate the book premium as the credibility-weighted average of the experience of the companies.

```
#calculate credibility
Credibility<-HW5Q4[,1]/(HW5Q4[,1]+EPV/VHM)
CredWeightAve<-sum(HW5Q4[,2]/HW5Q4[,1]*Credibility)/sum(Credibility)
```

We calculate the credibility-weighted average frequency as 0.02815448622.

The credibility of 1135 units of exposure is

$$Z = \frac{1135}{1135 + \frac{0.0199655271474}{0.0009077470062}} = 0.980989890162$$

Thus the expected claim frequency for Policyholder 3 is

$$1310\left(0.980989890162 \times \frac{15}{1135} + 0.019010109838 \times 0.02815448622\right) = 17.6847956066$$

5. *Aggregate claims for a given individual policy are modelled as following a Pareto distribution with $\alpha = 3$ and θ varying between individuals. The first 5 years of experience on this policy are:*

Policyholder	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Year 5*	*Mean*	*Variance*
1	*663.93*	*1018.96*	*588.18*	*768.51*	*600.04*	*727.924*	*31581.77793*
2	*622.80*	*723.56*	*637.23*	*843.72*	*743.15*	*714.092*	*7994.85627*
3	*11.81*	*14.33*	*12.13*	*15.87*	*13.40*	*13.508*	*2.76212*
4	*1625.20*	*2125.83*	*2325.31*	*1592.02*	*3829.92*	*2299.656*	*832055.28003*
5	*140.58*	*142.46*	*138.79*	*330.92*	*273.83*	*205.316*	*8259.46703*

(a) Estimate the EPV and VHM.

The mean claim amount is $\frac{727.924+714.092+13.508+2299.656+205.316}{5} = 792.0992$. The mean of the Pareto distribution is $\frac{\Theta}{2}$, while the variance is $\frac{3\Theta^2}{4}$. Thus, the EPV is $\frac{3}{4}\mathbb{E}(\Theta^2) = \frac{3}{4}\left(\mathbb{E}(\Theta)^2 + \mathrm{Var}(\Theta)\right)$. Meanwhile, the hypothetical means are $\frac{\Theta}{2}$, so the VHM is $\mathrm{Var}\left(\frac{\Theta}{2}\right) = \frac{1}{4}\mathrm{Var}(\Theta)$. We have that $\frac{1}{2}\mathbb{E}(\Theta) = 792.0992$, so $\mathbb{E}(\Theta) = 1584.1984$. The variance of observed means is

$$\mathrm{VHM} + \frac{\mathrm{EPV}}{5} = \frac{1}{4}\mathrm{Var}(\Theta) + \frac{3}{5 \times 4}\left(1584.1984^2 + \mathrm{Var}(\Theta)\right)$$

From the data, the estimated variance of observed means is

$$\frac{1}{4}\left((727.924 - 792.0992)^2 + (714.092 - 792.0992)^2 + (13.508 - 792.0992)^2 + (2299.656 - 792.0992)^2 + (205.316 - 792.0992)^2\right) = 808362.4663$$

Thus we have

$$\begin{aligned} \frac{1}{4}\mathrm{Var}(\Theta) + \frac{3}{5 \times 4}\left(1584.1984^2 + \mathrm{Var}(\Theta)\right) &= 808362.466323 \\ 8\,\mathrm{Var}(\Theta) + 3 \times 1584.1984^2 &= 20 \times 808362.466323 \\ \mathrm{Var}(\Theta) &= 1079774.45185 \end{aligned}$$

Thus, the EPV is

$$\frac{3}{4}\left(1584.1984^2 + 1079774.45185\right) = 2692094.26681$$

and the VHM is

$$\frac{1079774.45185}{4} = 269943.612963$$

(b) Calculate the credibility premium for Policyholder 5 in the next year.

The credibility of 5 years' of experience is

$$Z = \frac{5}{5 + \frac{2692094.26681}{269943.612963}} = 0.33393882597$$

Thus the premium for Policyholder 5 is

$$0.33393882597 \times 205.316 + 0.66606117403 \times 792.0992 = \$596.15$$