ACSC/STAT 4703, Actuarial Models II

FALL 2023

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Midterm Examination

Thursday 19th October

13:05– 14:25

Here are some values of the Gamma distribution function with $\theta = 1$ that may be needed for this examination:

x	α	$F(x)$	x	α	$F(x)$	x	α	$F(x)$
245	255	0.2697208	2.5	4	0.2424239	4.375	4	0.6361773
$\left(\frac{7.5}{12}\right)^3$	$\frac{4}{3}$	0.1117140	3.841	2.4	0.8409823	4.875	4	0.7169870
$\left(\frac{9.5}{12}\right)^3$	$\frac{4}{3}$	0.2507382	4.375	3	0.8118663	5.375	4	0.7837292
1.356	2.4	0.2801616	4.875	3	0.8644174	2.156	5	0.06782354
1.941	2.4	0.4612472	5.375	3	0.9035828	3.203	5	0.219922
2.367	2.4	0.5775816	3.875	4	0.5417358	8.542	5	0.9274742

Here are the critical values for a chi-squared distribution:

Degrees of Freedom	Significance level 90%	95%	99%
1	2.705543	3.841459	6.634897
2	4.605170	5.991465	9.210340
3	6.251389	7.814728	11.344867
4	7.779440	9.487729	13.276704
5	9.236357	11.070498	15.086272

1. Using an arithmetic distribution ($h = 1$) to approximate a Generalised Pareto distribution with $\xi = -4$ and $\beta = 50$, calculate the probability that the value is more than 4.5, for the approximation using the method of local moment matching, matching 1 moment on each interval.

2. Claim frequency follows a Poisson distribution with $\lambda = 3.5$. Claim severity (in thousands) has the following distribution:

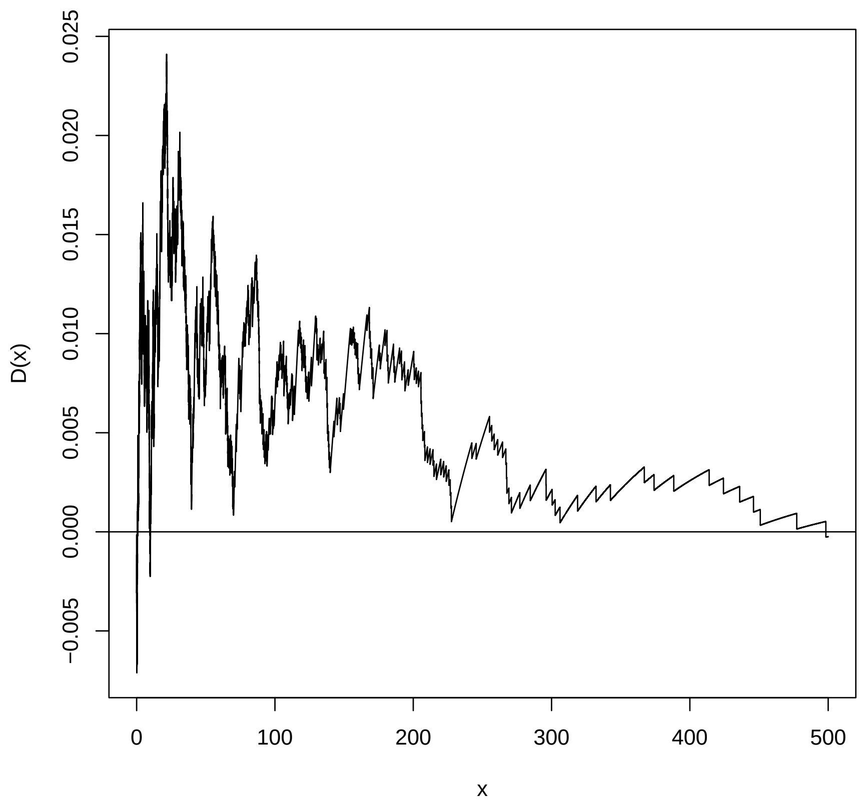
Severity	Probability
0	0.62
1	0.24
2	0.07
$\geqslant 3$	0.07

The expected claim severity per loss is 0.58. The company buys excess-of loss reinsurance for aggregate losses exceeding 2.

(a) Use the recursive method to calculate the probability that the reininsurance makes a payment.

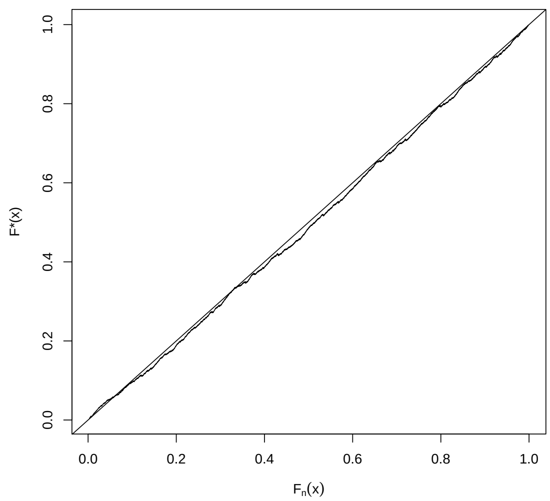
(b) What is the expected payment on the reinsurance? [Hint: first calculate the insurer's expected payment with this reinsurance policy. Then consider the expected total payments between the insurer and the reinsurer.]

3. An insurance company collects a sample of 1265 claims. Based on previous experience, it believes these claims might follow a Weibull distribution with $\theta = 36$ and $\tau = 0.7$. To test this, it computes the following plot of $D(x) = F^*(x) - F_n(x)$.

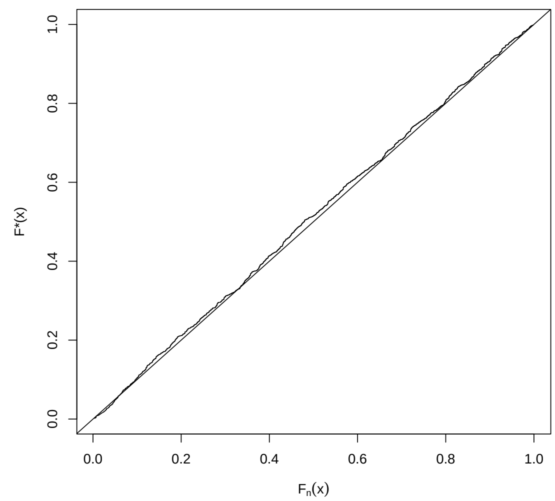


(a) How many of the claims in their sample were more than 200?

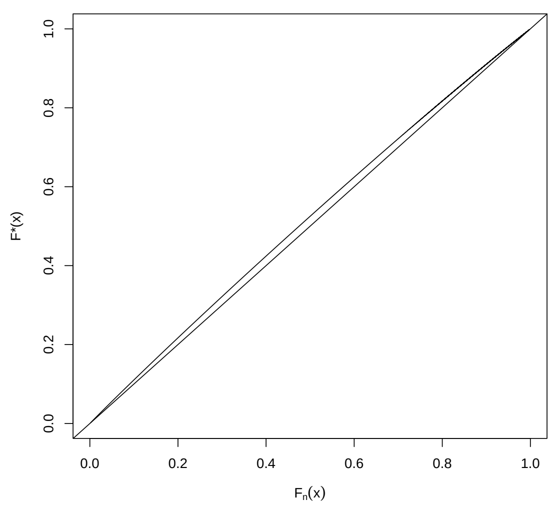
(b) Which of the following is a p-p plot of this data?



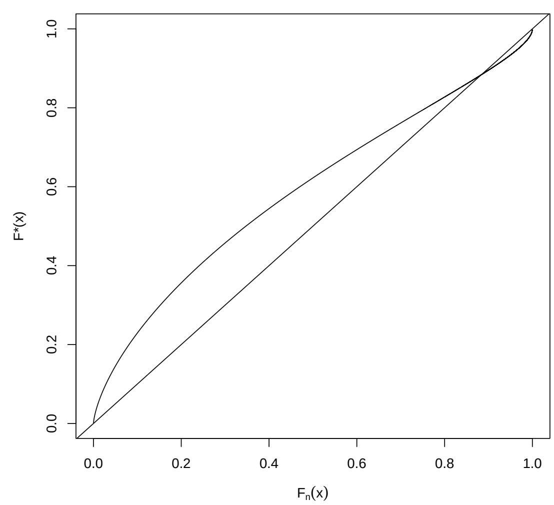
(i)



(ii)



(iii)



(iv)

Justify your answer.

4. An insurance company collects the following sample:

   `0.13 0.23 0.23 0.27 0.59 1.26 1.28 1.76 2.33 5.04 6.89 8.16 9.09 13.86 15.92 16.89`

   They model this as following a distribution with the following distribution function:

i	x_i	$F(x)$	$i^2(\log(F(x_{i+1})) - \log(F(x_i)))$	$(16-i)^2(\log(1-F(x_i)) - \log(1-F(x_{i+1})))$
0		0		37.43461
1	0.13	0.1360401	0.106478454	4.56953483
2	0.23	0.1513248	0.002172436	0.02179601
3	0.23	0.1514070	4.050981349	20.96220820
4	0.27	0.2374793	6.400518565	28.10090523
5	0.59	0.3542889	0.669214036	2.15965902
6	1.26	0.3639008	2.301255097	4.65792993
7	1.28	0.3879223	0.980911115	1.28980418
8	1.76	0.3957662	2.119481049	1.80639628
9	2.33	0.4090922	12.271987807	7.69244240
10	5.04	0.4760137	7.227572221	3.45541285
11	6.89	0.5116917	16.664763579	6.05162201
12	8.16	0.5872482	15.074621810	4.27021623
13	9.09	0.6520573	16.286311186	3.36337902
14	13.86	0.7180227	4.404637982	0.53652559
15	15.92	0.7343413	60.369489479	7.60702336
16	16.89	0.9603355	10.360980876	
Total			159.291377	111.020333

Calculate the Anderson-Darling statistic for this model and this data.

5. An insurance company collects a sample of 1952 claims. They want to decide whether this data is better modelled as following a Gumbel distribution or a Fréchet distribution. After calculating MLE estimates for the parameters (2 parameters for the Gumbel and 3 for the Fréchet), log-likelihoods for the two distributions are:

Distribution	log-likelihood
Gumbel	-5049.35
Fréchet	-5048.89

Use AIC to decide whether the Gumbel distribution or the Fréchet distribution is a better fit for the data.

6. An insurer's premium for a policy with limit $1,000,000 is the expected loss plus a risk charge equal to the square of the expected loss divided by $4,000. The pure premium ILF from $1,000,000 to $2,000,000 is 1.17. By buying excess-of-loss reinsurance with attachment point $1,000,000 and limit $1,000,000, and loading 25%, the insurer is able to charge a premium of $880 for policies with limit $2,000,000. What is the premium for a policy with limit $1,000,000?