ACSC/STAT 4703, Actuarial Models II

FALL 2023
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Practice Final Examination

This Sample examination has more questions than the actual final, in order to cover a wider range of questions. Estimated times are provided after each question to help your preparation.

1. An insurance company has the following data on its policies:

Policy limit	Losses Limited to			
	20,000	50,000	100,000	500,000
20,000	5,400,000			
50,000	4,590,000	6,070,000		
100,000	12,900,000	16,000,000	18,400,000	
500,000	9,200,000	11,100,000	13,800,000	16,200,000

   Use this data to calculate the ILF from $20,000 to $500,000 using

   (a) The direct ILF estimate. [5 mins]

   (b) The incremental method. [5 mins]

2. An insurance company charges a risk charge equal to the square of the average loss amount, divided by 20,000. It has the following data on a set of 1,700 claims from policies with limit $1,000,000.

Losses Limited to	50,000	100,000	500,000	1,000,000
Total claimed	9,500,000	14,060,000	17,220,000	21,390,000

   Calculate the ILF from $500,000 to $1,000,000. [10 mins]

3. An insurer models a loss as following an inverse Weibull distribution with $\tau = 3$ and $\theta = 100$. What are the parameters c_n and d_n that make the distribution of $\frac{M_n - d_n}{c_n}$ converge, where M_n are block maxima of a block of n samples, and what is the limiting distribution? [15 mins]

4. An insurer models aggregate daily losses with a distribution in the MDA of a Gumbel distribution. Of the past 200 years, 29 years included daily losses exceeding $100,000, and 17 years included daily losses exceeding $500,000. What is the probability of a daily loss exceeding $1,000,000 during the next year? [10 mins]

5. A reinsurer offers an excess-of-loss reinsurance contract on a portfolio with attachment point $5,000,000 and a policy limit of $5,000,000. The aggregate loss distribution is estimated to lie in the MDA of a Fréchet distribution with $\xi = 0.2$. The reinsurer estimates that the probability of paying a claim is 0.06 and the probability that the policy limit is reached is 0.0002. What is the expected payment on the contract. [10 mins]

6. An insurer models claims as following a distribution in the MDA of a GEV distribution with $\xi = -2.5$. They find that the probability of a claim exceding $1,000,000 is 0.04 and the probability of a claim exceeding $2,000,000 is 0.008. What is the maximum possible claim under this model?

7. An actuary is reviewing a sample of 75,060 observations that he believes comes from the MDA of a Fréchet distribution. He uses the Hill estimator to estimate ξ. He calculates $\hat{\alpha}_j$ for a range of different thresholds j:

j	$\hat{\alpha}_j$
73,000	2.842
74,000	3.692

Given that $x_{(73000)} = 12493$, which of the following is a possible value for $x_{(74000)}$? Justify your answer.

(i) 12986

(ii) 16986

(iii) 24986

(iv) 29986

[15 mins]

8. Loss amounts follow an exponential distribution with $\theta = 3,000$. The distribution of the number of losses is given in the following table:

Number of Losses	Probability
0	0.64
1	0.28
2	0.08

Assume all losses are independent and independent of the number of losses. The insurance company buys excess-of-loss reinsurance on the part of the loss above $7,500. Calculate the expected payment for this excess-of-loss reinsurance. [15 mins]

9. Claim frequency follows a negative binomial distribution with $r = 0.6$ and $\beta = 0.6$. Claim severity (in thousands) has the following distribution:

Severity	Probability
0	0.352
1	0.384
2	0.217
3 or more	0.047

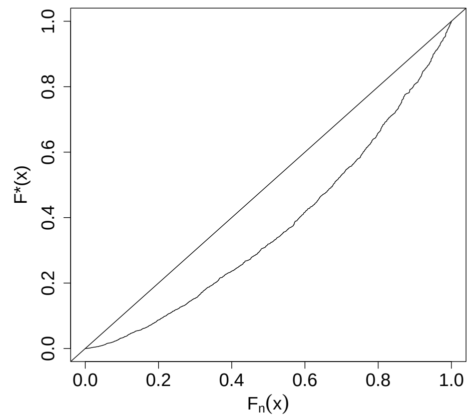
Use the recursive method to calculate the exact probability that aggregate claims are at least 3. [15 mins]

10. Using an arithmetic distribution ($h = 1$) to approximate an inverse Pareto distribution distribution with $\tau = 3$ and $\theta = 6$, calculate the probability that the value is between 3.5 and 6.5, for the approximation using:

(a) The method of rounding. [10 mins]

(b) The method of local moment matching, matching 1 moment on each interval. [15 mins]

11. An actuary is reviewing a sample of 2015 past claims, which she believes come from a Weibull distribution with $\tau = 0.6$ and a value of θ estimated from a previous dataset. She constructs the following p-p plot to compare the sample to this distribution:



(a) The sample included 685 points less than 1,200. What was the value of θ used in the plot? [5 mins.]

(b) Which of the following statements best describes the fit of the Weibull distribution to the data: [5 mins.]

(i) The Weibull distribution assigns too much probability to high values and too little probability to low values.

(ii) The Weibull distribution assigns too much probability to low values and too little probability to high values.

(iii) The Weibull distribution assigns too much probability to tail values and too little probability to central values.

(iv) The Weibull distribution assigns too much probability to central values and too little probability to tail values.

12. A worker's compensation insurance company classifies workplaces as "safe" or "hazardous". Claims from hazardous workplaces follow a Gamma distribution with $\alpha = 0.1021749$, $\theta = 1066798$ (mean \$109,000 and standard deviation \$341,000). Claims from safe workplaces follow a Gamma distribution with $\alpha = 0.01209244$, $\theta = 2646281$ (mean \$32,000 and standard deviation \$261,000). 94% of workplaces are classified as safe.

    [You may need the following values:

$$\Gamma(0.01209244) = 82.13091$$
$$\Gamma(0.1021749) = 9.302457$$

    ]

    (a) Calculate the expectation and variance of claim size for a claim from a randomly chosen workplace. [5 mins.]

    (b) The last 2 claims from a particular workplace are \$488,200 and \$17,400. Calculate the expectation and variance for the next claim size from this workplace. [10 mins.]

13. item An insurance company sets the book pure premium for its home insurance at \$791. The expected process variance is 6,362,000 and the variance of hypothetical means is 341,200. If an individual has no claims over the last 8 years, calculate the credibility premium for this individual's next year's insurance using the Bühlmann model. [5 mins.]

14. An insurance company is reviewing the premium for an individual with the following past claim history:

Year	1	2	3	4	5
Exposure	0.2	1	1	0.4	0.8
Aggregate claims	0	$2,592	0	$147	$1,320

The usual premium per unit of exposure is $2,700. The expected process variance is 123045 and the variance of hypothetical means is 36403 (both per unit of exposure). Calculate the credibility premium for this individual if she has 0.6 units of exposure in year 6. [10 mins.]

15. An insurance company has 3 years of past history on a homeowner, denoted X_1, X_2, X_3. Because the individual moved house at the end of the second year, the third year has a different mean and variance, and is not as correlated with the other two years. It has the following

$$\begin{aligned} \mathbb{E}(X_1) &= 1,322 & \mathrm{Var}(X_1) &= 226,000 \\ \mathbb{E}(X_2) &= 1,322 & \mathrm{Var}(X_2) &= 226,000 \\ \mathbb{E}(X_3) &= 4,081 & \mathrm{Var}(X_3) &= 1,108,000 \\ \mathbb{E}(X_4) &= 4,081 & \mathrm{Var}(X_4) &= 1,108,000 \\ \mathrm{Cov}(X_1, X_2) &= 214 & \mathrm{Cov}(X_1, X_3) &= 181 \\ \mathrm{Cov}(X_2, X_3) &= 181 & \mathrm{Cov}(X_1, X_4) &= 181 \\ \mathrm{Cov}(X_2, X_4) &= 181 & \mathrm{Cov}(X_3, X_4) &= 861 \end{aligned}$$

It uses a formula $\hat{X}_4 = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3$ to calculate the credibility premium in the fourth year. Calculate the values of α_0, α_1, α_2 and α_3. [15 mins.]

16. An insurance company has the following previous data on aggregate claims:

Policyholder	Year 1	Year 2	Year 3	Year 4	Mean	Variance
1	1,210	246	459	1,461	944.00	340158.00
2	0	0	0	0	0.00	0.00
3	0	2,185	0	0	548.25	1202312.25
4	809	0	0	1,725	633.50	674939.00
5	0	0	0	0	0.00	0.00

Calculate the Bühlmann credibility premium for policyholder 3 in Year 5. [15 mins.]

17. An insurance company collects the following claim frequency data for 7,000 customers insured for the past 3 years:

No. of claims	Frequency
0	1,494
1	2,460
2	1,810
3	827
4	302
5	72
6	31
7	3
8	1
> 8	0

It assumes that the number of claims an individual makes in a year follows a Poisson distribution with parameter Λ, which may vary between individuals.

Find the credibility estimate for the expected number of claims per year for an individual who has made 4 claims in the past 3 years. [15 mins.]

18. The following table shows the cumulative losses (in thousands) on claims from one line of business of an insurance company over the past 4 years.

Accident year	Development year 0	1	2	3
2019	890	3372	4563	4823
2020	1307	2653	3453	
2021	2742	6632		
2022	1224			

The earned premiums in each year are given in the following table:

Year	Earned Premiums (000's)
2019	5398
2020	6503
2021	8152
2022	7350

Assume that payments for Accident Year 2019 have been finalised.

Calculate the total outstanding reserves using per-premium losses using:

(a) The chain-ladder method.

(b) The Bornhuetter-Fergusson method. The expected loss ratio is 0.79 and the

[15 mins]

(c) The Bühlmann-Straub Credibility method for per-premium losses.

19. An insurance company collects the following run-off table for incremental losses.

Accident year	Development year 0	1	2	3	4	5	6	7	8
2014	1027	942	403	264	374	143	67	24	11
2015	1096	1022	498	302	472	174	85	43	
2016	1109	1161	545	354	522	133	74		
2017	1153	1392	694	373	634	339			
2018	1336	1511	688	404	586				
2019	1280	1429	694	433					
2020	1449	1602	728						
2021	1702	1899							
2022	1693								

The earned premiums for each year were as follows:

Accident year	Earned Premiums
2014	4324
2015	4720
2016	4939
2017	5873
2018	6343
2019	6869
2020	7205
2021	7795
2022	7538

They have used the chain-ladder method to estimate claims reserves.

Use the Spearman's rank correlation to test whether the Development year 0 and 2 payments are correlated in different accident years.

20. An insurance company has collected the following run-off table for incremental per-premium losses.

	Development year								
Accident year	0	1	2	3	4	5	6	7	8
2014	0.1427	0.2342	0.2033	0.0714	0.0874	0.0243	0.0167	0.0164	0.0114
2015	0.1496	0.2422	0.1842	0.0702	0.0772	0.0224	0.0285	0.0143	
2016	0.1809	0.2261	0.1754	0.0854	0.0822	0.0283	0.0174		
2017	0.1753	0.2392	0.1793	0.0773	0.0734	0.0539			
2018	0.1536	0.2311	0.1808	0.0724	0.0686				
2019	0.1780	0.2429	0.1848	0.0633					
2020	0.1549	0.2602	0.1883						
2021	0.1702	0.2299							
2022	0.1693								

Use the binomial test to determine whether Calendar year 2021 is unusual.

21. An insurance company uses a Poisson model for outstanding claims. They estimate the following parameters:

Accident Year i	μ_i	Dev. Year j	γ_j
0	1523	0	0.124
1	1952	1	0.382
2	2120	2	0.290
3	2084	3	0.147
4	2302	4	0.057

It is currently the start of calendar year 5 (so development year 0 has just finished for accident year 4). Using these estimated values, what is the probability that the outstanding claims exceed 3,700?

22. An insurance company collects the following cumulative run-off triangle:

	Development year			
Accident year	0	1	2	3
0	5539	6003	6829	7108
1	6243	6792	7314	
2	6217	7209		
3	6372			

and estimates the following reserves using the chain-ladder method:

Accident Year i	$\hat{C}_{i,J}$	Dev. Year j	f_j	γ_j	β_j
0	7108	0	1.1113950775	0.782059819772	0.782059819772
1	7612.81476056	1	1.10535365377	0.087117614236	0.869177434008
2	8294.04873842	2	1.04085517645	0.091571018443	0.960748452451
3	8147.71432939	3		0.039251547549	1

Under Mack's model, what is the mean squared error of the outstanding claims, including both process variance and squared estimation error?

23. An actuary is reviewing the following incremental loss development triangles:

No. of claims reported

Acc. Year	Development Year 0	1	2	3
2019	843	159	9	0
2020	862	164	11	
2021	830	166		
2022	844			

No. of claims finalised

Acc. Year	Development Year 0	1	2	3
2019	428	338	186	59
2020	442	352	203	
2021	435	325		
2022	451			

Payments (000's)

Acc. Year	Development Year 0	1	2	3
2019	211	200	144	71
2020	213	231	153	
2021	227	176		
2022	209			

(a) Use the chain-ladder method to predict the numbers of claims settled in each year

(b) Estimate the outstanding claims

24. An insurer classifies policies into three classes — single, couple and family. The experience from policy year 2016 is:

Age Class	Current differential	Earned premiums	Loss payments
Single	0.74	4,740	3,940
Couple	0.93	4,490	3,880
Family	1	5,670	4,930

The base premium was \$420. Claim amounts are subject to 4% annual inflation. If the expense ratio is 25%, calculate the new premiums for each age class for policy year 2018. [15 mins]

25. An insurer has different premiums for personal and commercial vehicles. Its experience for accident year 2016 is given below. There was a rate change on 1st August 2015, which affects some policies in 2016.

Type	Differential before rate change	Current differential	Earned premiums	Loss payments
Personal	1	1	11,300	9,800
Commercial	1.51	1.67	7,600	6,300

Before the rate change, the base premium was \$950. The current base premium is \$1,020. Assuming that policies were sold uniformly over the year, calculate the new premimums for policy year 2018 assuming 6% annual inflation and a permissible loss ratio of 0.75. [15 mins]

26. An insurance company has the following data for accident year 2017:

		Earned Premiums: House	Earned Premiums: Appartment	Loss Payments: House	Loss Payments: Appartment
Differential		1	0.88	1	0.88
Halifax	1	5,200	4,100	4,150	3,600
Dartmouth	0.84	3,700	2,900	2,080	2,430
Bedford	1.25	4,400	2,500	3,820	2,030

The base premium in 2017 was \$840. Calculate new premiums for policy year 2018 using inflation of 3% per year and expense ratio of 0.2.