ACSC/STAT 4720, Life Contingencies II
Fall 2015
Toby Kenney
Homework Sheet 2
Due: Friday 7th October: 12:30 PM

Basic Questions

1. The following is a multiple decrement table giving probabilities of death and surrender for a life insurance policy:

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
53	10000.00	39.60	1.62
54	9958.78	39.43	1.74
55	9917.61	39.26	1.86
56	9876.49	39.09	2.00
57	9835.40	38.91	2.15
58	9794.34	38.74	2.31
59	9753.28	38.57	2.49
60	9712.22	38.39	2.69
61	9671.15	38.21	2.90
62	9630.03	38.02	3.14

A life insurance policy has a death benefit of \$300,000 payable at the end of the year of death. Premiums are payable at the beginning of each year. Calculate the premium for a 10-year policy sold to a life aged 53 if there is no-payment to policyholders who surrender their policy, and the interest rate is $i = 0.04$.

2. Update the multiple decrement table from the previous question with the following mortality probabilities

x	l_x	d_x
53	10000.00	1.25
54	9998.75	1.33
55	9997.41	1.42
56	9995.99	1.51
57	9994.48	1.61
58	9992.87	1.72
59	9991.15	1.84
60	9989.31	1.97
61	9987.34	2.10
62	9985.24	2.25

using:

(a) UDD in the multiple decrement table.

(b) UDD in the independent decrements.

3. The mortalities for a husband and wife (whose lives are assumed to be independent) aged 49 and 54 respectively, are given in the following tables:

x	l_x	d_x
49	10000.00	1.25
50	9998.75	1.31
51	9997.44	1.38
52	9996.06	1.45
53	9994.61	1.53
54	9993.08	1.62
55	9991.46	1.71
56	9989.75	1.81
57	9987.94	1.92
58	9986.01	2.04

x	l_x	d_x
54	10000.00	1.24
55	9998.76	1.33
56	9997.43	1.42
57	9996.01	1.52
58	9994.49	1.63
59	9992.87	1.74
60	9991.12	1.87
61	9989.25	2.01
62	9987.25	2.15
63	9985.09	2.31

The interest rate is $i = 0.04$.

(a) They want to purchase a 10-year last survivor insurance policy with a death benefit of $4,000,000. Annual premiums are payable while both are alive. Calculate the net premium for this policy using the equivalence principle.

(b) They want to purchase a 10-year reversionary annuity, which will provide an annuity to the wife of $40,000 at the start of each year if the husband is dead and the wife is alive. Calculate the net premium for this policy using the equivalence principle. For the wife, $a_{64} = 19.86$.

4. A husband is 58; the wife is 72. Their lifetables while both are alive, and the lifetable for the husband if the wife is dead, are given below:

x	l_x	d_x
72	10000.00	7.92
73	9992.08	8.61
74	9983.47	9.36
75	9974.11	10.17
76	9963.93	11.06
77	9952.87	12.03
78	9940.84	13.08
79	9927.77	14.22
80	9913.54	15.47
81	9898.07	16.82

x	l_x	d_x
58	10000.00	3.49
59	9996.51	3.80
60	9992.70	4.14
61	9988.56	4.51
62	9984.05	4.92
63	9979.13	5.36
64	9973.77	5.85
65	9967.92	6.39
66	9961.53	6.97
67	9954.56	7.61

x	l_x	d_x
58	10000.00	3.94
59	9996.06	4.30
60	9991.76	4.68
61	9987.08	5.10
62	9981.97	5.57
63	9976.41	6.08
64	9970.33	6.63
65	9963.70	7.24
66	9956.46	7.91
67	9948.55	8.64

Calculate the annual premium for a 10-year term insurance policy sold to the husband with death benefit $200,000. The interest rate is $i = 0.05$. Use the UDD assumption for handling changes to the husband's mortality in the event of the wife's death.

Standard Questions

5. A couple want to receive the following:

- While both are alive, they would like to receive a pension of $80,000 per year.
- If the husband is alive and the wife is not, they would like to receive a pension of $45,000 per year.
- If the wife is alive and the husband is not, they would like to receive a pension of $65,000 per year.

- When the husband dies: if the wife is still alive, they would like a death benefit of $130,000; otherwise, they would like a death benefit of $85,000.

Construct a combination of insurance and annuity policies that achieve this combination of benefits.

6. A husband aged 49 and wife aged 46 have the following transition intensities:

$$\mu_{xy}^{01} = 0.000002y + 0.000000001x$$
$$\mu_{xy}^{02} = 0.0000015x + 0.000000003y$$
$$\mu_{xy}^{03} = 0.000042 + 0.000011x + 0.000013y$$
$$\mu_{x}^{13} = 0.000004x$$
$$\mu_{x}^{23} = 0.000003y$$

They want to purchase a reversionary annuity, which will pay a continuous annuity at a rate of $70,000 per year when the husband is alive and the wife is dead. Premiums are payable continuously while both are alive. Force of interest is $\delta = 0.04$.

(a) Calculate the annual rate of continuous premium.

(b) Calculate the policy value after 5 years if both are still alive.