ACSC/STAT 4720, Life Contingencies II
Fall 2015
Toby Kenney
Homework Sheet 4
Due: Friday 13th November: 12:30 PM

Basic Questions

1. A life aged 46 has mortality given in the table below. The yield rate is in another table below

x	l_x	d_x		Term (years)	Yield rate
46	10000.00	1.01		1	0.018
47	9998.99	1.10		2	0.021
48	9997.89	1.21		3	0.028
49	9996.69	1.32		4	0.031
50	9995.37	1.45		5	0.033

   Calculate the net annual premium for a 5-year endowment insurance with benefit $150,000 sold to this life.

2. An insurance company sells N one-year life insurance policies to lives aged 60. The death benefit is $130,000, payable at the end of the year to lives which die during the year. The company uses $q_{60} = 0.00022$ and $i = 0.04$ to calculate the premium for the policy. This results in a net premium of $130000 \times 0.00022(1.04)^{-1} = \$27.5$.

   However, q_{60} is an estimated probability based on past data, and the true value is normally distributed with mean 0.00022 and standard deviation 0.00002. The interest rate cannot be fixed, and the actual interest rate obtained is normally distributed with mean 0.04 and standard deviation 0.005.

   Calculate the expected aggregate profit of the policies, and the variance of this aggregate profit.

3. An insurance company sells a pension to a life aged 66. The life has mortality following a Gompertz law with $B = 0.00000091$ and $C = 1.11$. The annual pension payment is currently $65,000. Every year this payment increases by inflation, which follows a log-normal distribution with $\mu = 0$ and $\sigma^2 = 0.04$ (and inflation in each year is independent and independent of interest rates). The insurance company uses fixed income investments with a 10-year period to fix the interest rate, and the interest rates are reset every 10 years. The current interest rate is $i = 0.06$, and interest rates at future times are independent and normally distributed with mean 0.04 and standard deviation 0.01. Simulate 10000 PV future loss random variables.

   (a) Use these to estimate the EPV future loss.

   (b) Construct a 95% confidence interval for the EPV future loss.

Standard Questions

4. An insurance company sells a large number of 4-year life insurance policies to lives aged 54, with the following lifetable

x	l_x	d_x
54	10000.00	4.95
55	9995.05	5.34
56	9989.71	5.76
57	9983.95	6.23
58	9977.71	6.75

They can fix the interest rate for the first two years at 4%. For the last two years, they can either fix the interest rate for the last two years at 4%, or they can use the market rates in two years' time. If they do this, the market rates will follow a log-normal distribution with $\mu = -3.8$ and $\sigma^2 = 0.3$. What is the probability that it does worse by using market rates than by fixing the rates at $i = 0.04$?

5. An insurance company has N 10-year term insurance policies sold to lives aged 37 with mortality following the lifetable below.

x	l_x	d_x
37	10000.00	2.39
38	9997.61	2.41
39	9995.19	2.44
40	9992.76	2.46
41	9990.30	2.48
42	9987.82	2.51
43	9985.30	2.54
44	9982.76	2.57
45	9980.19	2.61
46	9977.58	2.65

The current interest rate is $i = 0.04$ and is fixed for 5 years. After 5 years, the insurance company will fix interest rates at the new interest rate for the remainder of the policy. The new interest rate is normally distributed with mean 0.03 and variance 0.005. For what value of N is the variance due to uncertainty in the interest rate larger than the variance due to uncertainty in mortality?