

ACSC/STAT 4720, Life Contingencies II

Fall 2016

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Homework Sheet 3

Due: Monday 18th October: 12:30 PM

Basic Questions

1. An individual aged 39 has a current salary of \$104,000. The salary scale is $s_y = 1.05^y$. Estimate the individual's final average salary (average of last 3 years working) assuming the individual retires at exact age 65.
2. An employer sets up a DC pension plan for its employees. The target replacement ratio is 60% of final average salary for an employee who enters the plan at exact age 32, with the following assumptions:
 - At age 65, the employee will purchase a continuous life annuity, plus a continuous reversionary annuity for the employee's spouse, valued at 70% of the life annuity.
 - At age 65, the employee is married to someone aged 60.
 - The salary scale is $s_y = 1.04^y$.
 - Mortalities are independent and given by $\mu_x = 0.0000016(1.091)^x$.
 - A fixed percentage of salary is payable monthly in arrear.
 - Contributions earn an annual rate of 6%.
 - The value of the life annuity is based on $\delta = 0.05$.

Calculate the percentage of salary payable monthly to achieve the target replacement rate under these assumptions. [You may use numerical integration to compute the value of the annuities.]

3. The salary scale is given in the following table:

| y | s_y | y | s_y | y | s_y | y | s_y |
|-----|----------|-----|----------|-----|----------|-----|----------|
| 30 | 1.000000 | 39 | 1.350398 | 48 | 1.845766 | 57 | 2.553877 |
| 31 | 1.033333 | 40 | 1.397268 | 49 | 1.912422 | 58 | 2.649694 |
| 32 | 1.067933 | 41 | 1.445983 | 50 | 1.981785 | 59 | 2.749515 |
| 33 | 1.103853 | 42 | 1.496620 | 51 | 2.053975 | 60 | 2.853522 |
| 34 | 1.141149 | 43 | 1.549263 | 52 | 2.129115 | 61 | 2.961903 |
| 35 | 1.179879 | 44 | 1.604000 | 53 | 2.207337 | 62 | 3.074855 |
| 36 | 1.220103 | 45 | 1.660921 | 54 | 2.288777 | 63 | 3.192585 |
| 37 | 1.261887 | 46 | 1.720122 | 55 | 2.373580 | 64 | 3.315310 |
| 38 | 1.305295 | 47 | 1.781702 | 56 | 2.461894 | 65 | 3.443256 |

An employee aged 34 and 7 months has 11 years of service, and a current salary of \$62,000 (for the coming year). She has a defined benefit pension plan with $\alpha = 0.02$ and S_{Fin} is the average of her last 3 years' salary. The employee's mortality is given by $\mu_x = 0.00000224(1.101)^x$. The pension benefit is payable monthly in advance. The interest rate is $i = 0.04$. [This gives $\ddot{a}_{65}^{(12)} = 19.62939$.] Calculate the EPV of the accrued benefit under the assumption that the employee retires at age 65.

Standard Questions

4. An employee aged 58 has been working with a company for 28 years. The employee's salary last year was \$58,000. The salary scale is the same as for Question 3. The service table is given below:

| t | ${}_t p^{(00)}$ | 1 | 2 | 3 |
|----------------|-----------------|-------|---------|------|
| 0 | 10000.00 | 48.48 | 0 | 6.48 |
| 1 | 9945.03 | 45.29 | 0 | 7.29 |
| 2 ⁻ | 9892.45 | | 1137.53 | |
| 2 | 8754.92 | 25.72 | 142.14 | 6.86 |
| 3 | 8580.20 | 21.69 | 128.54 | 7.56 |
| 4 ⁻ | 8422.40 | | 1426.64 | |
| 4 | 6995.76 | 12.81 | 455.53 | 6.95 |
| 5 | 6520.47 | 10.83 | 757.69 | 7.29 |
| 6 | 5744.66 | 9.16 | 416.44 | 7.23 |
| 7 ⁻ | 5311.82 | | 5311.82 | |

Mortality follows a Gompertz model with $B = 0.0000012$ and $C = 1.1$. The member's current salary is \$74,000. If the member withdraws before age 60, he receives a deferred pension starting from age 65, with 2% COLA. The death benefit of the plan is three times the employee's final average salary if the employee is still working at the time of death. If the employee has withdrawn, the death benefit is three times final average salary with COLA of 3%. The accrual rate for the pension is 0.02. Pension payments are made annually in advance. The interest rate is $i = 0.06$.

Calculate the EPV of the accrued benefit. [You may assume that events happen in the middle of each year.]

You are given the following values:

| x | a_x |
|------|----------|
| 60 | 16.52493 |
| 60.5 | 16.49421 |
| 61.5 | 16.43047 |
| 62 | 16.39741 |
| 62.5 | 16.36354 |
| 63.5 | 16.29327 |
| 64.5 | 16.21955 |
| 65 | 16.18134 |

5. An individual aged 39 has 13 years of service, and last year's salary was \$204,000. The salary scale is $s_y = 1.04^y$. The accrual rate is 0.015. The interest rate is $i = 0.03$. There is no death benefit, and no exits other than death or retirement at age 65. Mortality follows a Gompertz law with $B = 0.000002$ and $C = 1.08$. You are given that $\ddot{a}_{65} = 21.85281$. Calculate this year's employer contribution to the plan using
- The projected unit method.
 - The traditional unit method.