ACSC/STAT 4720, Life Contingencies II
Fall 2016

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Homework Sheet 7

Due: Friday 2nd December: 12:30 PM

Basic Questions

1. An equity-linked insurance policy has the following properties:

   - Annual premiums are \$6,000.
   - Expense charges are 10% of the first premium and 0.5% of subsequent premiums.
   - There is a year-end management fee of 1.5% of fund value.
   - There is a year-end death benefit of 150% of fund value.
   - Surrenders receive full fund value.
   - GMMB is the total of the premiums paid.
   - The annual return is 6%.
   - The insurer's initial expenses are \$700 plus 30% of the first premium.
   - The insurer's renewal expenses are 0.5% of each subsequent premium.
   - Mortality is given by $q_x = 0.0003 + 0.00002x$.
   - The policy is sold to a life aged 47.
   - The policy matures in 5 years.
   - Surrenders happen at a rate of 2% per year.

   (a) Calculate the projected fund value up to maturity of the policy.

   (b) Calculate the profit signature of the policy.

   (c) If the annual return is $i = 0.01$, what is the profit signature?

2. For an equity-linked insurance policy with the following properties:

   - Annual premiums are \$10,000.
   - Expense charges are 6% of the first premium and 1% of subsequent premiums.
   - There is a year-end management fee of 0.6% of fund value.
   - There is a year-end death benefit of 120% of fund value.
   - Surrenders receive full fund value.
   - GMMB is the total of the premiums paid.
   - The insurer's initial expenses are \$600 plus 20% of the first premium.
   - The insurer's renewal expenses are 0.4% of each subsequent premium.

- Mortality is given by $q_x = 0.0002 + 0.00001x$.
- The policy is sold to a life aged 52.
- The policy matures in 5 years.
- Surrenders happen at a rate of 1% per year.

(a) Use the following random numbers from a uniform distribution to simulate 5 years of annual returns following a log-normal distribution with $\mu = 0.04$ and $\sigma = 0.07$.

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0.42398186 0.82146466 0.88083835 0.38797765 0.05112565 0.59871460 0.51560631 0.96433468
0.10803186 0.70858266
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(b) Use the simulated returns to calculate the account values for the next 5 years.

(c) Calculate the profit signature for the policy for these simulated returns.

Standard Questions

3. An equity-linked insurance policy has the following properties:

   - Annual premiums are \$11,000.
   - Expense charges are 10% of the first premium and 1% of subsequent premiums.
   - There is a year-end management fee of 1.3% of fund value.
   - There is a year-end death benefit of 150% of fund value.
   - Surrenders receive full fund value.
   - GMMB is 110% of the total of the premiums paid.
   - The insurer's initial expenses are \$200 plus 20% of the first premium.
   - The insurer's renewal expenses are 0.5% of each subsequent premium.
   - Mortality is given by $q_x = 0.0002 + 0.00003x$.
   - The policy is sold to a life aged 55.
   - The policy matures in 5 years.
   - Surrenders happen at a rate of 2% per year.
   - Annual returns are log-normally distributed with $\mu = 0.04$ and $\sigma = 0.18$.

   Simulate 5000 sets of 5-years' worth of annual returns. [Please include your code with your answer.]

   (a) Calculate the expected NPV of the policy at a risk discount rate of 10%.

   (b) Calculate the value of the Management expense fee needed to ensure that the probability of a net loss (negative NPV) is at most 10%, and the expected NPV is at least \$500.

4. For the policy in the previous question, suppose the fund value at the beginning of year 4 (before premiums are received) is \$39,230. Use a simulation to calculate a 95% quantile reserve at the start of year 4, if the reserve makes an annual return of $i = 0.03$.