ACSC/STAT 4720, Life Contingencies II
Fall 2016
Toby Kenney
Homework Sheet 5
Model Solutions

Basic Questions

1. *An insurance company sells a 5-year annual life insurance policy to a life aged 29, for whom the lifetable below is appropriate.*

x	l_x	d_x
29	*10000.00*	*0.88*
30	*9999.12*	*0.95*
31	*9998.17*	*1.03*
32	*9997.15*	*1.11*
33	*9996.04*	*1.21*
34	*9994.83*	*1.31*

The annual gross premium is \$152. Initial expenses are \$90 plus 25% of the first premium. The death benefits are \$1,200,000. Renewal costs are 3% of each subsequent premium. The interest rate is $i = 0.03$

(a) Calculate the expected net cash-flows associated with this policy (assuming no reserve). [This is the profit vector for the policy.]

t	Premium (at $t-1$)	Expenses	Interest	Expected Death Benefits	Net Cash Flow
0		128.00			-128.00
1	152	0.00	4.5600	105.6000	50.96
2	152	4.56	4.4232	114.0100	37.85
3	152	4.56	4.4232	123.6226	28.24
4	152	4.56	4.4232	133.2381	18.63
5	152	4.56	4.4232	145.2577	6.61

(b) Which of the following is the internal rate of return of the policy:

The profit signiture is

t	P(in force)	Pr_t	Π_t
0	1.000000	-128.00	-128.00
1	1.000000	50.960000	50.96000
2	0.999912	37.853167	37.84984
3	0.999817	28.240577	28.23541
4	0.999714	18.625094	18.61977
5	0.999603	6.605533	6.60291

(i) $i = 0.041241$

The NPV is

$$50.96(1.041241)^{-1} + 37.85(1.041241)^{-2} + 28.24(1.041241)^{-3} + 18.62(1.041241)^{-4} + 6.60(1.041241)^{-5} - 128$$
$=2.099425$

(ii) $i = 0.049045$

The NPV is

$$50.96(1.049045)^{-1} + 37.85(1.049045)^{-2} + 28.24(1.049045)^{-3} + 18.62(1.049045)^{-4} + 6.60(1.049045)^{-5} - 128$$
$=0.00002071$

(iii) $i = 0.055031$

The NPV is

$$50.96(1.055031)^{-1} + 37.85(1.055031)^{-2} + 28.24(1.055031)^{-3} + 18.62(1.055031)^{-4} + 6.60(1.055031)^{-5} - 128$$
$= -1.57049$

(iv) $i = 0.061620$

The NPV is

$$50.96(1.061620)^{-1} + 37.85(1.061620)^{-2} + 28.24(1.061620)^{-3} + 18.62(1.061620)^{-4} + 6.60(1.061620)^{-5} - 128$$
$= -3.260441$

so (ii) $i = 0.049045$ is the internal rate of return.

2. *An insurance company sells a 5-year annual life insurance policy to a life aged 44, for whom the lifetable below is appropriate.*

x	l_x	d_x
44	*10000.00*	*7.25*
45	*9992.75*	*8.01*
46	*9984.74*	*8.85*
47	*9975.89*	*9.78*
48	*9966.11*	*10.81*
49	*9955.30*	*11.95*

The annual gross premium is \$720. Initial expenses are \$130 plus 20% of the first premium. The death benefits are \$720,000. Renewal costs are 4% of each subsequent premium. The interest rate is $i = 0.03$. *Gross reserves are calculated on the basis* $i = 0.02$, *with mortality following the table.*

(a) Calculate the reserves.

The expected present value of future benefits and future premiums in each year are given below:

Year	EPV future benefits	EPV premiums	Reserve
1	3022.02	3317.86	0.00
2	2562.32	2681.14	0.00
3	2038.06	2031.37	6.69
4	1441.93	1368.18	73.74
5	765.65	691.20	74.45

(b) Calculate the profit signature.

We first calculate the profit vector

t	Reserves	Premium (at $t-1$)	Expenses	Interest	Expected Death Benefits	Change in Reserves	Net Cash Flow
0			274.00				-274.00
1	0.00	720	0.00	21.60	522.00	0.00	219.60
2	0.00	720	28.80	20.74	577.14	6.69	128.10
3	6.69	720	28.80	20.94	638.17	67.05	6.91
4	73.74	720	28.80	22.95	705.86	0.71	7.58
5	74.45	720	28.80	22.97	780.97	-74.45	7.66

The profit signature is then calculated as

t	P(in force)	Pr_t	Π_t
0	1.000000	-274.00	-274.00
1	1.000000	219.60	219.60
2	0.999275	128.10	128.01
3	0.998474	6.91	6.90
4	0.997589	7.58	7.56
5	0.996611	7.66	7.63

(c) Calculate the profit margin at a risk discount rate of $i = 0.06$.

At a risk discount rate of $i = 0.06$, the NPV is

$$219.60(1.06)^{-1} + 128.01(1.06)^{-2} + 6.90(1.06)^{-3} + 7.56(1.06)^{-4} + 7.63(1.06)^{-5} - 274 = \$64.58$$

The NPV of premiums received is

$$720\left(1.000000 + 0.999275(1.06)^{-1} + 0.998474(1.06)^{-2} + 0.997589(1.06)^{-3} + 0.996611(1.06)^{-4}\right) = 3210.02$$

so the profit margin is $\frac{64.58}{3210.02} = 2.012\%$.

3. *For the policy in Question 2:*

(a) Calculate the reserves and profit signature for a general premium. [You may assume that P is such that the reserves are zero in Years 1 and 2.]

For a premium P, the expected present value of future benefits and future premiums in each year are given below:

Year	EPV future benefits	EPV premiums (less expenses)	Reserve
1	3022.02	$4.612330P$	0
2	2562.32	$3.728080P$	0
3	2036.58	$2.823656P$	$2036.58 - 2.823656P$
4	1439.72	$1.901089P$	$1439.72 - 1.901089P$
5	763.81	$0.960000P$	$763.81 - 0.960000P$

We assume that $721.26 < P < 757.31$ so that the first three reserves are zero.

Now we calculate the profit vector

t	Premium (at $t-1$)	Expenses	Interest	Exp. Death Benefits	Change in Reserves	Net Cash Flow
0	$130+0.2P$					$-(130+0.2P)$
1	P	0.00	$0.03P$	522.00	0.00	$1.03P-522.00$
2	P	$0.04P$	$0.0292P$	577.14	$2036.58-2.823656P$	$3.812856P-2613.72$
3	P	$0.04P$	$61.09-0.05590968P$	637.71	$0.922567P-596.86$	$20.24-0.018577P$
4	P	$0.04P$	$43.19-0.028233P$	704.78	$0.941089P-675.91$	$14.32-0.009322P$
5	P	$0.04P$	22.91	779.08	$0.960000P-763.81$	7.64

The profit signature is then calculated as

t	P(in force)	Pr_t	Π_t
0	1.000000	-274.00	-274.00
1	1.000000	$1.03P-522.00$	$1.03000000P-522.00$
2	0.999275	$3.812856P-2613.72$	$3.810092P-2611.83$
3	0.999199	$20.24-0.018577P$	$20.22-0.018562P$
4	0.999115	$14.32-0.009322P$	$14.31-0.00931375P$
5	0.999022	7.63	7.62

(b) Calculate the premium that gives an internal rate of return of $i=0.10$.

At $i=0.10$, the NPV is

$$(1.03P-522.00)(1.1)^{-1}+(3.810092P-2611.83)(1.1)^{-2}+(20.22-0.018562P)(1.1)^{-3}+(14.31-0.00931375P)(1.1)^{-4}+(7.62)(1$$

Setting this to zero gives

$$\begin{aligned} 4.064893P &= 2877.39 \\ P &= \$707.86 \end{aligned}$$

4. *For a 5-year term insurance policy sold to a life aged 44, and actuary performs the following profit test without reserves:*

Year	*Premium*	*Expenses*	*Interest*	*Expected Death Benefits*	Pr_t
0		*1500*			-1500
1	*5900*	*0*	*177.00*	*4216.80*	*1860.20*
2	*5900*	*80*	*174.60*	*4806.66*	*1187.94*
3	*5900*	*80*	*174.60*	*5478.02*	*516.58*
4	*5900*	*80*	*174.60*	*6243.89*	-249.29
5	*5900*	*80*	*174.60*	*7117.12*	-1122.52

Calculate the reserves needed to ensure that all cash flows are non-negative.

In order for the Year 5 cash flows to be non-negative, the reserve has to be $1122.52(1.03)^{-1}=1089.82$. The probability of paying this reserve to a policy in force at the start of Year 4 is $\frac{9326.11}{9524.35}=0.9791859812$, so the expected reserve payment at the end of Year 4 is $0.9762762824\times 1089.82=1067.14$. Adding this to the current Year 4 cash flows makes the net cash flow at end of Year 4 -1316.43. The reserve needed to cover this is $1316.43(1.03)^{-1} = 1278.09$. The expected reserve payment at the end of Year 3 is $1278.09\frac{9524.35}{9701.49} = 1254.75$. This makes the net-cash flow at end of Year 3 $516.58 - 1254.75 = -738.17$. To cover this, the Year 3 reserve needs to be $738.17\frac{9701.49}{9859.44} = 726.35$. With this reserve payment, the Year 2 cash-flow is still positive, so no reserves are needed in Years 1 or 2. In summary the reserves are:

Year	Reserve
1	0.00
2	0.00
3	726.35
4	1278.09
5	1089.82

Standard Questions

5. *A couple purchase a 5-year last survivor insurance policy. Annual Premiums of $49,830 are payable while both are alive. If one life is dead, there are no premiums or benefits. If both lives die within the 5-year period, a benefit of $1,000,000 is payable. The husband is 74 and the wife is 81. Their lifetables are given below. Assume both lives are independent.*

x	l_x	d_x
74	*10000.00*	*591.85*
75	*9408.15*	*628.62*
76	*8779.53*	*662.27*
77	*8117.26*	*691.27*
78	*7425.99*	*713.96*
79	*6712.03*	*728.54*

x	l_x	d_x
81	*10000.00*	*1113.81*
82	*8886.19*	*1114.43*
83	*7771.76*	*1097.45*
84	*6674.31*	*1061.21*
85	*5613.10*	*1004.92*
86	*4608.18*	*928.94*

Initial expenses are $3,000, and renewal expenses are $80 at the start of each subsequent year while both are alive, and $60 at the start of each year while only one is alive. The interest rate is $i = 0.04$. Use a profit test without reserves to determine the NPV of this policy at a risk discount rate of $i = 0.10$.

We first perform the profit test in the both alive state

t	Premium (at $t-1$)	Expenses	Interest	Expected Death Benefits	Net Cash Flow
0		3000.00			−3000.00
1	49830	0	1993.20	6592.08	45231.12
2	49830	80	1990.00	8379.56	43360.44
3	49830	80	1990.00	10651.95	41088.05
4	49830	80	1990.00	13540.45	38199.55
5	49830	80	1990.00	17212.67	34527.33

Then in the husband alive wife dead state

t	Premium (at $t-1$)	Expenses	Interest	Expected Death Benefits	Net Cash Flow
0		3000.00			-3000.00
1	0	0	0	59185.00	-59185.00
2	0	60	-2.40	66816.54	-66878.94
3	0	60	-2.40	75433.42	-75495.82
4	0	60	-2.40	85160.51	-85222.91
5	0	60	-2.40	96143.41	-96205.81

Then in the wife alive husband dead state

t	Premium (at $t-1$)	Expenses	Interest	Expected Death Benefits	Net Cash Flow
0		3000.00			-3000.00
1	0	0	0	111381.00	-111381.00
2	0	60	-2.40	125411.45	-125473.85
3	0	60	-2.40	141209.97	-141272.37
4	0	60	-2.40	158999.21	-159061.61
5	0	60	-2.40	179031.19	-179093.59

Now we calculate the profit signature:

Year	P(Both)	P(Husband)	P(Wife)	NCF(Both)	NCF(Husband)	NCF(Wife)	Π_t
0	1	0	0	-3000			-3000
1	1.000000	0.000000	0.000000	45231.12	-59185.00	-111381.00	45231.12
2	0.836026	0.104789	0.052593	43360.44	-66878.94	-125473.85	22643.25
3	0.682324	0.195629	0.094852	41088.05	-75495.82	-141272.37	-133.78
4	0.541771	0.269955	0.125660	38199.55	-85222.91	-159061.61	-22298.60
5	0.416828	0.325771	0.144482	34527.33	-96205.81	-179093.59	-42824.83

The NPV at a risk discount rate $i = 0.1$ is therefore

$$45231.12(1.1)^{-1} + 22643.25(1.1)^{-2} - 133.78(1.1)^{-3} - 22298.60(1.1)^{-4} - 42824.83(1.1)^{-5} - 3000 = \$14,911.03$$