ACSC/STAT 4720, Life Contingencies II

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Homework Sheet 6

Model Solutions

Basic Questions

1. *A policyholder aged 58 buys a 5-year type A universal life insurance policy. The death benefit is \$450,000. The policyholder pays a premium of \$6,400 at the start of each year. The lifetable for the policyholder is:*

x	l_x	d_x
58	*10000.00*	*137.45*
59	*9862.55*	*149.49*
60	*9713.06*	*162.35*
61	*9550.71*	*176.05*
62	*9374.66*	*190.57*
63	*9184.09*	*205.89*

The cost of insurance is based on 105% of mortality in the above table and $i = 0.04$*. Expense charges are 2% of the account value (after each premium is paid). Assume the credited interest rate is* $i = 0.05$*.*

(a) Calculate the projected account value for the next 5 years.

t	AV_{t-1}	P_t	EC_t	CoI_t	interest	AV_t
1	0.00	6400	128.00	61.54	310.52	6520.98
2	6520.98	6400	258.42	65.93	303.78	13226.46
3	13226.46	6400	392.53	70.47	296.85	20121.63
4	20121.63	6400	530.43	75.16	289.72	27211.84
5	27211.84	6400	672.24	79.96	282.39	34502.63

(b) Suppose the insurer earns an interest rate, $i = 0.06$*, and mortality follows the above table, initial expenses are \$1,300 and renewal expenses are 1% of account value each year after the first. Suppose there are no surrenders. Calculate the profit margin of this policy at a risk discount rate of* $i = 0.10$*.*

t	AV_{t-1}	P_t	E_t	I_t	EDB_t	EAV_t	$\Pr_t$
0			1300				−1300.00
1	0.00	6400	0.00	384.00	64.95	6520.08	198.97
2	6520.98	6400	65.21	771.35	70.64	13224.48	331.99
3	13226.46	6400	132.26	1169.65	76.72	20118.37	468.76
4	20121.63	6400	201.22	1579.23	83.20	27207.05	609.39
5	27211.84	6400	272.12	2000.38	90.06	34496.06	753.99

The profit signature is then given by

t	P(in force)	$\Pr_t$	Π_t
0	1.0000000	-1300.00	-1300.00
1	1.0000000	198.97	198.97
2	0.9998626	331.99	331.94
3	0.9997131	468.76	468.63
4	0.9995507	609.39	609.12
5	0.9993747	753.99	753.52

At a risk discount rate of $i = 0.1$, the NPV is therefore

$$198.97(1.1)^{-1} + 331.94(1.1)^{-2} + 468.63(1.1)^{-3} + 609.12(1.1)^{-4} + 753.52(1.1)^{-5} - 1300 = \$391.22$$

The EPV of premiums received is

$$6000(1 + 0.9998626(1.1)^{-1} + 0.9997131(1.1)^{-2} + 0.9995507(1.1)^{-3} + 0.9993747(1.1)^{-4}) = \$26,679.93$$

The profit margin is therefore $\frac{391.22}{26679.93} = 1.466\%$.

2. *A life aged 37 buys a 5-year type B universal life insurance policy with additional death benefit \$350,000. The annual premium is \$7,400. Mortality is as shown in the following table:*

x	l_x	d_x
37	*10000.00*	*11.86*
38	*9988.14*	*12.77*
39	*9975.37*	*13.75*
40	*9961.62*	*14.80*
41	*9946.82*	*15.93*
42	*9930.88*	*17.15*
43	*9913.73*	*18.47*

The credited interest rate is $i = 0.04$. *Cost of insurance is based on mortality in the above table and* $i = 0.02$. *Expense charges are 1.5% of account value.*

(a) Project the account value for the next 5 years.

t	AV_{t-1}	P_t	EC_t	CoI_t	interest	AV_t
1	0.00	7400	111.00	406.96	275.28	7157.32
2	7157.32	7400	218.36	438.71	556.01	14456.26
3	14456.26	7400	327.84	472.98	842.22	21897.66
4	21897.66	7400	439.46	509.80	1133.94	29482.33
5	29482.33	7400	553.23	549.54	1431.18	37210.74

(b) Assume that the insurance company earns interest $i = 0.08$*; Mortality is 105% of the mortality in the lifetable. Initial expenses are \$3,700; renewal expenses are 1% of premiums paid. The surrender charges and surrender rates are:*

Year	*Charge*	*rate*
1	*\$4,200*	*1%*
2	*\$3,200*	*2%*
3	*\$1,400*	*3%*
4	*\$400*	*3%*
5	*\$0*	*100%*

Which of the following is the internal rate of return of the policy:

(i) $i = 0.09906$

(ii) $i = 0.10320$

(iii) $i = 0.11382$

(iv) $i = 0.12034$

We calculate the profit vector

t	AV_{t-1}	P_t	E_t	I_t	EDB_t	ESB_t	EAV_t	$\Pr_t$
0			3700					−3700
1	0.00	7400	0	592.00	423.59	29.54	7077.34	419.58
2	7157.32	7400	74	1158.67	465.96	224.84	14149.03	738.24
3	14456.26	7400	74	1742.58	512.62	614.08	21211.45	1144.75
4	21897.66	7400	74	2337.89	563.80	871.17	28555.37	1559.23
5	29482.33	7400	74	2944.67	620.12	37151.14	0.00	1981.73

and the profit signature:

t	P(in force)	$\Pr_t$	Π_t
0	1.0000000	−3700.00	−3700.00
1	1.0000000	419.58	461.53
2	0.9899877	738.24	793.20
3	0.9701749	1144.75	1148.49
4	0.9410561	1559.23	1472.98
5	0.9128102	1981.73	1799.42

We calculate the NPV for the values given

i	NPV
0.09903	373.66
0.10320	321.11
0.11382	191.63
0.12034	115.09

So (iv) $i = 0.12034$ is closest to the internal rate of return. [In fact the IRR is $i = 0.13051$.]

3. *A life aged 52 has an annual type A Universal life insurance policy that has been in effect for 12 years.*

- *The current account value is \$112,483.*
- *The annual premium is \$6,500.*
- *The expense charge is 1% of account value.*
- *The credited interest rate is* $i = 0.05$.
- *The total death benefit is \$300,000.*
- *The corridor factor requirement is 2.5.*
- *The insurance is priced using mortality rate* $q_{52} = 0.000412$ *and interest* $i = 0.03$.

Calculate the cost of insurance charge for the year.

If the cost of insurance is C, then the value after premium, expense charges and cost of insurance is $118983 \times 0.99 - C = 117793.17 - C$. After applying interest the value is $1.05(117793.17 - C) = 123682.83 - 1.05C$. The corridor factor applies if this is more than $\frac{300000}{2.5} = 120000$.

If the corridor factor does not apply, The additional death benefit is therefore $176317.17 + 1.05C$ and the cost of insurance is $(176317.17 + 1.05C)(1.03)^{-1} \times 0.000412 = 70.527 + 0.00042C$ We solve this to get $C = \frac{70.527}{1-0.00042} = \$70.56$. This means the account value is more than \$120,000, so the corridor factor applies.

Since the corridor factor applies, the additional death benefit is $1.5(123682.83 - 1.05C) = 185524.24 - 1.575C$. The cost of insurance is then $C = (185524.24 - 1.575C)(1.03)^{-1}0.000412 = 74.2097 - 0.00063C$. We solve this to get $C = \frac{74.2097}{1.00063} = \$74.16$.

Standard Questions

4. *Consider an annual type A universal life insurance policy with annual premiums of \$6,000, death benefit \$400,000 with no corridor factor requirement.*

Surrender charges and rates are

Year	*Charge*	*rate*
1	*\$2,200*	*2%*
2	*\$1,300*	*2%*
3	*\$800*	*2%*
4	*\$0*	*2%*
5	*\$0*	*100%*

Initial expenses are \$1,100, and renewal expenses are \$90. Cost of insurance is based on mortality $q_x = 0.000702$ *and* $i = 0.05$. *The insurance company makes an annual rate of return equal to* $i = 0.08$. *A competitor offers a comparable policy with expense charge 1% and credited interest rate* $i = 0.05$. *If the company wants to charge an expense charge of 2% and base Cost of Insurance on* $q_x = 0.00066$ *and* $i = 0.04$, *what credited interest rate should it charge so that the NPV of its policy at a risk discount rate of* $i = 0.12$ *is the same as the competitor's policy?*

(i) $i = 0.007917$

(ii) $i = 0.011423$

(iii) $i = 0.014042$

(iv) $i = 0.017162$

For the competitor's policy we project the account value

t	AV_{t-1}	P_t	EC_t	CoI_t	interest	AV_t
1	0.00	6000	60.00	263.44	283.83	5960.38
2	5960.38	6000	119.60	259.30	281.05	12160.56
3	12160.56	6000	181.61	254.99	278.17	18610.16
4	18610.16	6000	246.10	250.50	275.17	25319.24
5	25319.24	6000	313.19	245.83	272.05	32298.22

t	AV_{t-1}	P_t	E_t	I_t	EDB_t	ESB_t	EAV_t	$\Pr_t$
1	0.00	6000	0	480.00	280.80	75.15	5837.08	286.97
2	5960.38	6000	90	949.63	280.80	217.06	11908.98	413.18
3	12160.56	6000	90	1445.64	280.80	355.95	18225.16	654.29
4	18610.16	6000	90	1961.61	280.80	506.03	24795.43	899.51
5	25319.24	6000	90	2498.34	280.80	32275.55	0.00	1171.23

t	P(in force)	$\Pr_t$	Π_t
1	1.0000000	189.77	286.97
2	0.9793120	413.18	404.63
3	0.9590521	654.29	627.50
4	0.9392112	899.51	844.83
5	0.9197809	1171.23	1077.27

So the NPV of the competitor's policy (ignoring initial expenses) is \$2,173.61.

If the company sets its credited interest so that the account values are A_i, then

t	AV_{t-1}	P_t	E_t	I_t	EDB_t	ESB_t	EAV_t	$\Pr_t$
1	0.00	6000	0	480	280.80	$0.0199868A_1 - 43.97$	$0.9793532A_1$	$6199.20 - 0.99934A_1$
2	A_1	6000	90	$472.80 + 0.08A_1$	280.80	$0.0199868A_2 - 25.98$	$0.9793532A_2$	$6102 + 1.08A_1 - 0.99934A_2$
3	A_2	6000	90	$472.80 + 0.08A_2$	280.80	$0.0199868A_3 - 15.99$	$0.9793532A_3$	$6102 + 1.08A_2 - 0.99934A_3$
4	A_3	6000	90	$472.80 + 0.08A_3$	280.80	$0.0199868A_4$	$0.9793532A_4$	$6102 + 1.08A_3 - 0.99934A_4$
5	A_4	6000	90	$472.80 + 0.08A_4$	280.80	$0.99934A_5$	0.00	$6102 + 1.08A_4 - 0.99934A_5$

t	P(in force)	$\Pr_t$	Π_t
1	1.0000000	$6199.20 - 0.99934A_1$	$6199.20 - 0.99934A_1$
2	0.9793120	$6102 + 1.08A_1 - 0.99934A_2$	$5975.762 + 1.0576570A_1 - 0.9786657A_2$
3	0.9590521	$6102 + 1.08A_2 - 0.99934A_3$	$5852.135 + 1.0357762A_2 - 0.9584190A_3$
4	0.9392112	$6102 + 1.08A_3 - 0.99934A_4$	$5731.066 + 1.0143480A_3 - 0.9385912A_4$
5	0.9197809	$6102 + 1.08A_4 - 0.99934A_5$	$5612.502 + 0.9933632A_4 - 0.9191737A_5$

At a risk discount rate of $i = 0.12$ the NPV is

$$21291.16 - 0.05500343A_1 - 0.05386552A_2 - 0.05275115A_3 - 0.05165983A_4 - 0.5841515A_5$$

The company therefore wants to choose the credited interest rate so that

$$21291.16 - 0.05500343A_1 - 0.05386552A_2 - 0.05275115A_3 - 0.05165983A_4 - 0.5841515A_5 = 2173.61$$

The insurance company has $EC_t = 0.02(A_{t-1} + 6000)$, $CoI_t = \frac{0.000702}{1.04}(400000 - A_t) =$ and

$$\begin{aligned}
A_t &= 0.98(A_{t-1} + 6000 - CoI_t)(1+i) \\
A_t &= 0.98(A_{t-1} + 6000 - 270 + 0.000675A_t)(1+i) \\
A_t(1 - 0.0006615(1+i)) &= 0.98(1+i)A_{t-1} + 5615.4(1+i) \\
A_t &= \frac{0.98(1+i)A_{t-1} + 5615.4(1+i)}{(1 - 0.0006615(1+i))}
\end{aligned}$$

If we let $a = \frac{0.98(1+i)}{(1-0.0006615(1+i))}$ and $b = \frac{5615.4(1+i)}{(1-0.0006615(1+i))}$ then we get

$$
\begin{aligned}
A_1 &= b \\
A_2 &= ab + b \\
A_3 &= a^2b + ab + b \\
A_4 &= a^3b + a^2b + ab + b \\
A_5 &= a^4b + a^3b + a^2b + ab + b
\end{aligned}
$$

$$0.5841515a^4b + 0.6358113a^3b + 0.6885625a^2b + 0.7424280ab + 0.7974314b = 19117.55$$

Furthermore $b = \frac{5615.4}{0.98}a = 5730a$

$$
\begin{aligned}
3347.188a^5 + 3643.199a^4 + 3945.463a^3 + 4254.112a^2 + 4569.282a &= 19117.55 \\
3347.188a^5 + 3643.199a^4 + 3945.463a^3 + 4254.112a^2 + 4569.282a - 19117.55 &= 0
\end{aligned}
$$

Numerically, we find that the solution is $a = 0.98841755$, which gives

$$
\begin{aligned}
\frac{0.98(1+i)}{(1 - 0.0006615(1+i))} &= 0.98841755 \\
0.98(1+i) &= 0.98841755(1 - 0.0006615(1+i)) \\
0.9806538(1+i) &= 0.98841755 \\
(1+i) &= 1.007917
\end{aligned}
$$

So the credited interest rate is (i) $i = 0.007917$.