ACSC/STAT 4720, Life Contingencies II
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Homework Sheet 7

Model Solutions

Basic Questions

1. *An equity-linked insurance policy has the following properties:*

   - *Annual premiums are \$6,000.*
   - *Expense charges are 10% of the first premium and 0.5% of subsequent premiums.*
   - *There is a year-end management fee of 1.5% of fund value.*
   - *There is a year-end death benefit of 150% of fund value.*
   - *Surrenders receive full fund value.*
   - *GMMB is the total of the premiums paid.*
   - *The annual return is 6%.*
   - *The insurer's initial expenses are \$700 plus 30% of the first premium.*
   - *The insurer's renewal expenses are 0.5% of each subsequent premium.*
   - *Mortality is given by $q_x = 0.0003 + 0.00002x$.*
   - *The policy is sold to a life aged 47.*
   - *The policy matures in 5 years.*
   - *Surrenders happen at a rate of 2% per year.*

   (a) *Calculate the projected fund value up to maturity of the policy.*

t	Alloc. Prem.	Start	Int.	Fund Before	Mmgt. Charge	Fund Value
1	5400	0.00	324.00	5724.00	85.86	5638.14
2	5970	5638.14	696.49	12304.63	184.57	12120.06
3	5970	12120.06	1085.40	19175.46	287.63	18887.83
4	5970	18887.83	1491.47	26349.30	395.24	25954.06
5	5970	25954.06	1915.44	33839.50	507.59	33331.91

   (b) *Calculate the profit signature of the policy.*

First we calculate the profit vector:

t	Unalloc. Prem.	Exp.	Int.	Mgmt. Charge	EDB	Pr_t
0		2500				-2500
1	600	0	36	85.86	3.50	718.36
2	30	30	0.0	184.57	7.64	176.93
3	30	30	0.0	287.63	12.09	275.54
4	30	30	0.0	395.24	16.87	378.37
5	30	30	0.0	507.59	22.00	485.59

This gives the profit signature:

P(in force)	Pr_t	Π_t	
0	1.0000000	-2500	-2500
1	1.0000000	718.36	718.36
2	0.9787848	176.93	173.18
3	0.9580005	275.54	263.97
4	0.9376388	378.37	354.77
5	0.9176914	485.59	445.63

(c) If the annual return is $i = 0.01$, what is the profit signature?

If the annual return is $i = 0.01$, the projected fund value is:

t	Alloc. Prem.	Start	Int.	Fund Before	Mmgt. Charge	Fund Value
1	5400	0.00	54.00	5454.00	81.81	5372.19
2	5970	5372.19	113.42	11455.61	171.83	11283.78
3	5970	11283.78	172.54	17426.32	261.39	17164.92
4	5970	17164.92	231.35	23366.27	350.49	23015.78
5	5970	23015.78	289.86	29275.63	439.13	28836.50

So the profit vector is

t	Unalloc. Prem.	Exp.	Int.	Mgmt. Charge	EDB	GMMB	Pr_t
0		2500					-2500
1	600	0	6	81.81	3.33	0.0	684.48
2	30	30	0	171.83	7.11	0.0	164.73
3	30	30	0	261.39	10.99	0.0	250.41
4	30	30	0	350.49	14.96	0.0	335.53
5	30	30	0	439.13	19.03	1163.5	-743.40

The profit signature is therefore

P(in force)	Pr_t	Π_t	
0	1.0000000	-2500	-2500
1	1.0000000	684.48	684.48
2	0.9787848	164.73	161.23
3	0.9580005	250.41	239.89
4	0.9376388	335.53	314.61
5	0.9176914	-743.40	-682.21

2. *For an equity-linked insurance policy with the following properties:*

   - *Annual premiums are \$10,000.*
   - *Expense charges are 6% of the first premium and 1% of subsequent premiums.*
   - *There is a year-end management fee of 0.6% of fund value.*
   - *There is a year-end death benefit of 120% of fund value.*
   - *Surrenders receive full fund value.*
   - *GMMB is the total of the premiums paid.*
   - *The insurer's initial expenses are \$600 plus 20% of the first premium.*

- *The insurer's renewal expenses are 0.4% of each subsequent premium.*
- *Mortality is given by $q_x = 0.0002 + 0.00001x$.*
- *The policy is sold to a life aged 52.*
- *The policy matures in 5 years.*
- *Surrenders happen at a rate of 1% per year.*

(a) Use the following random numbers from a uniform distribution to simulate 5 years of annual returns following a log-normal distribution with $\mu = 0.04$ and $\sigma = 0.07$.

0.42398186 0.82146466 0.88083835 0.38797765 0.05112565 0.59871460 0.51560631 0.96433468 0.10803186 0.70858266

t	U_t	$0.07\Phi^{-1}(U_t) + 0.04$	Return
1	0.42398186	0.02657979	0.02693619
2	0.82146466	0.10446728	0.11011907
3	0.88083835	0.12254317	0.13036791
4	0.38797765	0.02007843	0.02028136
5	0.05112565	-0.07438252	-0.07168347

(b) Use the simulated returns to calculate the account values for the next 5 years.

t	Alloc. Prem.	Start	Int.	Fund Before	Mmgt. Charge	Fund Value
1	9400	0.00	253.20	9653.20	57.92	9595.28
2	9900	9595.28	2146.80	21642.08	129.85	21512.23
3	9900	21512.23	4095.15	35507.38	213.04	35294.33
4	9900	35294.33	916.60	46110.94	276.67	45834.27
5	9900	45834.27	-3995.23	51739.04	310.43	51428.61

(c) Calculate the profit signature for the policy for these simulated returns.

First we calculate the profit vector:

t	Unalloc. Prem.	Exp.	Int.	Mgmt. Charge	EDB	ESB	Pr_t
0		2600				-2600	
1	600	0	16.16	57.92	1.38	0	672.70
2	100	40	6.61	129.85	3.14	0	193.32
3	100	40	7.82	213.04	5.22	0	275.64
4	100	40	1.22	276.67	6.88	0	331.01
5	100	40	-4.30	310.43	7.82	0	358.32

This gives the profit signature:

P(in force)	Pr_t	Π_t	
0	1.0000000	-2600	-2600
1	1.0000000	672.70	672.70
2	0.9892872	193.32	191.25
3	0.9786794	275.64	269.77
4	0.9681756	331.01	320.47
5	0.9577750	358.32	343.19

Standard Questions

3. *An equity-linked insurance policy has the following properties:*

   - *Annual premiums are \$11,000.*
   - *Expense charges are 10% of the first premium and 1% of subsequent premiums.*
   - *There is a year-end management fee of 1.3% of fund value.*
   - *There is a year-end death benefit of 150% of fund value.*
   - *Surrenders receive full fund value.*
   - *GMMB is 110% of the total of the premiums paid.*
   - *The insurer's initial expenses are \$200 plus 20% of the first premium.*
   - *The insurer's renewal expenses are 0.5% of each subsequent premium.*
   - *Mortality is given by* $q_x = 0.0002 + 0.00003x$.
   - *The policy is sold to a life aged 55.*
   - *The policy matures in 5 years.*
   - *Surrenders happen at a rate of 2% per year.*
   - *Annual returns are log-normally distributed with* $\mu = 0.04$ *and* $\sigma = 0.18$.

   Simulate 5000 sets of 5-years' worth of annual returns. [Please include your code with your answer.]

   (a) Calculate the expected NPV of the policy at a risk discount rate of 10%.

   My simulation gives the value

$$-\$3,217.23$$

   (b) Calculate the value of the Management expense fee needed to ensure that the probability of a net loss (negative NPV) is at most 10%, and the expected NPV is at least \$500.

   For my simulation, a management fee of 79% is needed to ensure the probability of net loss is at most 10%. This was easily enough to ensure that the expected NPV is at least \$500.

4. *For the policy in the previous question, suppose the fund value at the beginning of year 4 (before premiums are received) is \$39,230. Use a simulation to calculate a 95% quantile reserve at the start of year 4, if the reserve makes an annual return of* $i = 0.03$.

   We simulate 5,000 sets of returns in Years 4 and 5. If the returns are i_4 and i_5 respectively, then the account value is

t	4	5
Alloc. Prem.	10890	10890
Start	39230	$49468.44(1+i_4)$
Fund Before	$50120(1+i_4)$	$(10890+49468.44(1+i_4))(1+i_5)$
Mmgt. Charge	$651.56(1+i_4)$	$141.57(1+i_5)+643.09(1+i_4)(1+i_5)$
Fund Value	$49468.44(1+i_4)$	$10748.43(1+i_5)+48825.35(1+i_4)(1+i_5)$

   This gives a profit vector:

t	4	5
Unalloc. Prem.	110	110
Exp.	55	55
Int.	$55i_4$	$55i_5$
Mgmt. Charge	$651.56(1+i_4)$	$141.57(1+i_5)+771.40(1+i_4)(1+i_5)$
EDB	$28.79063(1+i_4)$	$6.352322(1+i_5)+28.85578(1+i_4)(1+i_5)$
GMMB		$(56500-10748.43(1+i_5)-48825.35(1+i_4)(1+i_5))_+$
Pr_t	$787.3241(1+i_4)$	$300.22(1+i_5)+742.54422(1+i_4)(1+i_5)-GMMB$

The profit signature is

t	4	5
P(in force)	1.0000000	0.9780988
Pr_t	$787.3241(1+i_4)$	$300.22(1+i_5)+742.54422(1+i_4)(1+i_5)-GMMB$
Π_t	$787.3241(1+i_4)$	$293.645(1+i_5)+726.2816(1+i_4)(1+i_5)-0.9780988GMMB$

The NPV of the policy at the rate $i = 0.03$ earned by the reserves is therefore

$$\begin{aligned}&(1.03)^{-1}787.3241(1+i_4)+(1.03)^{-2}\left(293.645(1+i_5)+726.2816(1+i_4)(1+i_5)-0.9780988GMMB\right)\\=&764.3923301(1+i_4)+276.7885757(1+i_5)+684.5900650(1+i_4)-0.9219519GMMB\end{aligned}$$

The reserve should therefore be -1 times the 5th percentile of this NPV. We note that if the GMMB is zero, then this NPV is positive, so the reserve is actually 0. Therefore, we can assume the GMMB is positive, and instead take the 5th percentile of

$$\begin{aligned}&(1.03)^{-1}787.3241(1+i_4)+(1.03)^{-2}\left(293.645(1+i_5)+726.2816(1+i_4)(1+i_5)-0.9780988GMMB\right)\\=&764.3923301(1+i_4)+276.7885757(1+i_5)+684.5900650(1+i_4)(1+i_5)-0.9219519(56500-10748.43(1+i_5)-48825.35(1+i_4)(1+i_5))\\=&764.3923301(1+i_4)+10186.32(1+i_5)+45699.21(1+i_4)(1+i_5)-55778.09\end{aligned}$$

For my simulation, from 5,000 simulations, the 250th value is -13424.44 and the 251st is -13404.41, so the quantile reserve is $\frac{13424.44+13404.41}{2} = \$13,414.43$.