ACSC/STAT 4720, Life Contingencies II

Formulae

$$\int_l^u (t^2+bt+c)e^{-s(t^2+2dt)}\,dt = e^{sd^2}\left(\frac{\left(l+\frac{b}{2}\right)e^{-s(l+d)^2}-\left(u+\frac{b}{2}\right)e^{-s(u+d)^2}}{s} + \left(c-bd+d^2-\frac{1}{s}\right)\sqrt{\frac{\pi}{s}}\left(\Phi\left(\sqrt{2s}(u+d)\right)-\Phi\left(\sqrt{2s}(l+d)\right)\right)\right)$$

$$\int_l^u (t+b)e^{-s(t^2+2dt)}\,dt = e^{sd^2}\left(\left(\frac{e^{-s(l+d)^2}-e^{-s(u+d)^2}}{2s}\right)+(b-d)\sqrt{\frac{\pi}{s}}\left(\Phi\left(\sqrt{2s}(u+d)\right)-\Phi\left(\sqrt{2s}(l+d)\right)\right)\right)$$

Derivations

$$\begin{aligned}\int_a^b t^2e^{-st^2}\,dt &= \left[-\frac{t}{s}e^{-st^2}\right]_a^b+\int_a^b\frac{e^{-st^2}}{s}\,dt\\ &= \frac{ae^{-sa^2}-be^{-sb^2}}{s}-\sqrt{\frac{\pi}{s^3}}\left(\Phi\left(\sqrt{2s}b\right)-\Phi\left(\sqrt{2s}b\right)\right)\end{aligned}$$

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\begin{aligned}
\int_l^u (t^2+bt+c)e^{-s(t^2+2dt)}\,dt &= \int_l^u (t^2+bt+c)e^{-s(t+d)^2+sd^2}\,dt \\
&= \int_l^u \left((t+d)^2+(b-2d)(t+d)+(c-bd+d^2)\right)e^{-s(t+d)^2+sd^2}\,dt \\
&= e^{sd^2}\int_l^u (t+d)^2e^{-s(t+d)^2}+(b-2d)(t+d)e^{-s(t+d)^2}+(c-bd+d^2)e^{-s(t+d)^2}\,dt \\
&= e^{sd^2}\int_{l+d}^{u+d} v^2e^{-sv^2}+(b-2d)ve^{-sv^2}+(c-bd+d^2)e^{-sv^2}\,dv \\
&= e^{sd^2}\left(\frac{(l+d)e^{-s(l+d)^2}-(u+d)e^{-s(u+d)^2}}{s}-\sqrt{\frac{\pi}{s^3}}\left(\Phi\left(\sqrt{2s}(u+d)\right)-\Phi\left(\sqrt{2s}(l+d)\right)\right)\right. \\
&\quad \left.+(b-2d)\left(\frac{e^{-s(l+d)^2}-e^{-s(u+d)^2}}{2s}\right)+(c-bd+d^2)\sqrt{\frac{\pi}{s}}\left(\Phi\left(\sqrt{2s}(u+d)\right)-\Phi\left(\sqrt{2s}(l+d)\right)\right)\right) \\
&= e^{sd^2}\left(\frac{\left(l+\frac{b}{2}\right)e^{-s(l+d)^2}-\left(u+\frac{b}{2}\right)e^{-s(u+d)^2}}{s}\right. \\
&\quad \left.+\left(c-bd+d^2-\frac{1}{s}\right)\sqrt{\frac{\pi}{s}}\left(\Phi\left(\sqrt{2s}(u+d)\right)-\Phi\left(\sqrt{2s}(l+d)\right)\right)\right)
\end{aligned}
$$

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\begin{aligned}
\int_l^u (t+b)e^{-s(t^2+2dt)}\,dt &= \int_l^u (t+b)e^{-s(t+d)^2+sd^2}\,dt \\
&= \int_l^u \left((t+d)+(b-d)\right)e^{-s(t+d)^2+sd^2}\,dt \\
&= e^{sd^2}\int_l^u (t+d)e^{-s(t+d)^2}+(b-d)e^{-s(t+d)^2}\,dt \\
&= e^{sd^2}\int_{l+d}^{u+d} ve^{-sv^2}+(b-d)e^{-sv^2}\,dv \\
&= e^{sd^2}\left(\left(\frac{e^{-s(l+d)^2}-e^{-s(u+d)^2}}{2s}\right)+(b-d)\sqrt{\frac{\pi}{s}}\left(\Phi\left(\sqrt{2s}(u+d)\right)-\Phi\left(\sqrt{2s}(l+d)\right)\right)\right)
\end{aligned}
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