MATH/STAT 4720, Life Contingencies II
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In Class Examples

Long-Term Coverages in Health Insurance

- Disability Income Insurance (DII)
- Long-term Care Insurance (LTC)
- Critical Illness Insurance (CII)
- Chronic Illness Insurance
- Hospital Indemnity Insurance (HII)
- Continuing Care Retirement Communities

SN 1.1: Disability Income Insurance

Disability Income Insurance

- Also known as Income Protection Insurance
- Typically level premiums while working.
- Benefits paid during periods of disability
- Benefits based on salary, capped at 50–70% of lost salary.

Features of DII

- Waiting period or elimination period is time between start of disability and first benefit payment.
- Total disability benefits paid if policyholder is unable to perform their usual job (certified by medical practicioner) and is not otherwise employed.
- Partial disability benefits can be paid if the policyholder cannot work at full capacity, but is still working.
- Benefits may be reduced if policyholder has other income.

SN 1.1: Disability Income Insurance

Term of DII Payments

- Benefit payment may be limited term or up to retirement age.
- The term covers both full and partial payments (or a mixture).
- Limited term payments apply to each period of sickness.
- Multiple disibilities separated by less than the off period treated as a single disability (for both elimination period and term of benefits).
- Disability defined either by ability to perform current job, or any reasonable job given qualifications and experience.
- Employers may buy as group insurance, receiving cheaper rates.
- Benefits may increase in line with inflation.
- Common additional benefit: return to work assistance.

SN 1.2 Long Term Care Insurance

Long Term Care Insurance

- Typically level premiums while healthy.
- Typically 90 day waiting period before receiving benefits.
- Benefits paid when trigger conditions apply

Triggers of LTC (USA and Canada)

- Trigger usually determined by Activities of Daily Living (ADL)
- Six commonly used ADLs:
  - Bathing - Dressing - Eating
  - Toileting - Continence - Transferring
- Benefit triggered (and waiting period commences) when policyholder is unable to perform two or more of these ADLs.
- Alternative triggers often based on severe cognitive impairment.
- Some policies use 3 ADLs as trigger.

SN 1.2 Long Term Care Insurance

Other Features of LTC Payments

- Benefits can either be definite term or indefinite term.
- Benefit may either be reimbursement (up to a limit) or a fixed annuity.
- Payments or payment limits may increase with inflation.
- Reimbursed care benefits may be in-home or residential.
- Hybrid LTC and life insurance. Two approaches:
  - Return of premium: unused LTC premiums added to death benefit.
  - Accelerated benefits: deducts LTC payments from death benefits.
- In the USA, some LTC policies are tax-qualifying.
- Insurers may reserve the right to increase premiums during the policy (subject to regulatory approval).
- In other countries with government support for LTC costs, LTC insurance can still be used to top up government support.

SN 1.3 Critical Illness Insurance

Critical Illness Insurance (CII)

- Single lump-sum payout upon diagnosis with any of a prescribed list of illnesses.
- Policy expires upon first benefit.
- Level premiums paid thoughout the term. May stop at a specified age.
- May be incorporated into life insurance as an accelerated benefit rider. Some benefit paid immediately upon diagnosis, remaining benefit (if any) paid upon death.

Chronic Illness Insurance

- Like CII, pays benefit upon diagnosis of one of a number of conditions.
- Difference from CII is that conditions are chronic (policyholder will not recover) but not terminal.
- Illness must be sufficiently severe that policyholder cannot perform at least 2 ADLs (see LTC section).
- Benefit paid either as lump sum or as annuity.
- Typically offered as accelerated benefit rider on life insurance policy.

SN 1.5 Hospital Indemnity Insurance

Hospital Indemnity Insurance (HII)

- Lump sum payment for hospitalisation.
- May include daily stipend for hospital stays.
- May include additional benefits for outpatient admission.
- Differs from standard health insurance in that benefits are cash payments, rather than reimbursement of costs.
- Premiums increase each year.
- insurers often guarantee renewal. This means:
  - Renewal is not subject to medical examination.
  - Future premiums are not affected by claims in earlier years.

SN 1.6 Continuing Care Retirement Communities

Continuing Care Retirement Communities (CCRCs)

- Three or four levels of care offered
  - Independent Living Unit (ILU) — minimal support
  - Assisted Living Unit (ALU) — non-medical support
  - Skilled Nursing Facility (SNF) — ongoing medical care
  - Memory Care Units (MCU) — dementia or cognitive impairments
- Variety of funding options:
  - Full life care — large upfront fee & fixed monthly payments.
  - Modified life care — smaller upfront fee & monthly payments. Partially subsidised additional fees for further services.
  - Fee for service — Pay for services as needed.
- Full and modified life care options require medical examination.
- Direct entrants to ALU or SNF only eligible for Fee for service.
- Full and modified life care packages may include a partial refund if more expensive care facilities are not used.
- Some CCRCs offer partial ownership of the ILU.
- Joint CCRC membership for couples is common.

8 Multiple State Models

“Definition”

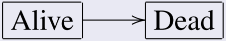
A Multiple State model has several different states into which individuals can be classified. These typically represent different payouts made under the policy.

Examples of Multiple State Models

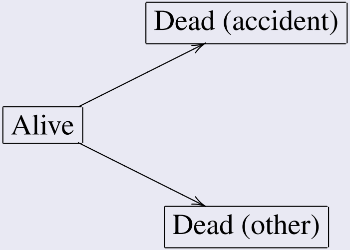
- Alive-Dead.
- Insurance with Increased Benefit for Accidental Death
- Permanent Disability Model.
- Disability Income Insurance Model

8.2 Examples of Multiple State Models

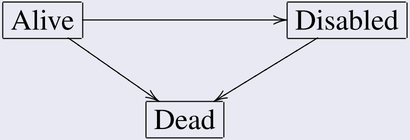
Alive-Dead



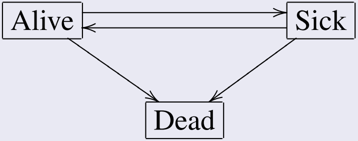
Accidental Death



Permanent Disability



Disability Income



8.4 Assumption and Notation

Assumption [Markov Property]

The probability of any future state depends only on the current state, and not on any information about the process before the present time. Formally:

$$P(Y(x+t) = n | Y(x)) = P(Y(x+t) = n | \{Y(z), z \leqslant x\})$$

Other Assumptions

- The probability of a given transition occuring in a time interval of length t is a differentiable function of t. Effectively, this means that the time at which a transition occurs is a continuous random variable, with no probability mass at any point.
- The probability of two transitions occuring within a time period t tends to zero faster than t.

Notation and Formulae

Notation

${}_tp_x^{ij}$	Probability of going from state i at age x to state j at age $x+t$
${}_tp_x^{\overline{ii}}$	Probability of remaining in state i for a period t for an individual aged x.
μ_x^{ij}	Rate of changing from state i to state j for an individual aged x in state i ($i \neq j$).

Formulae

- ${}_tp_x^{ij} = (Y(x+t) = j | Y(x) = i)$
- ${}_tp_x^{\overline{ii}} = P(Y(x+s) = i \text{ for all } 0 \leqslant s \leqslant t | Y(x) = i)$
- $\mu_x^{ij} = \lim_{t \to 0^+} \frac{{}_tP_x^{ij}}{t}$

8.4 Formulae for Probabilities

- ${}_tp_x^{\overline{ii}} = e^{-\int_x^{x+t} \sum_{j\neq i} \mu_y^{ij} dy}$
- ${}_{t+s}p_x^{ij} = \sum_k {}_tp_x^{ik} \, {}_sp_{x+t}^{kj}$
- $\dfrac{d}{dt} {}_tp_x^{ij} = \sum_{k\neq j} {}_tp_x^{ik} \mu_x^{kj} - {}_t p_x^{ij} \mu_x^{jk}$

Question 1

Under a permanent disability model, with transition intensities

$$\mu_x^{01} = 0.003$$
$$\mu_x^{02} = 0.001$$
$$\mu_x^{12} = 0.002$$

calculate the probability that an individual aged 27 is alive but permanently disabled at age 43.

Question 2

Under a permanent disability model, with transition intensities

$$\mu_x^{01} = 0.003 + 0.000002x$$
$$\mu_x^{02} = 0.001 + 0.000001x$$
$$\mu_x^{12} = 0.002 + 0.000002x$$

calculate the probability that an individual aged 32 is alive but permanently disabled at age 44.

Question 3

Under a disability income model, with transition intensities

$$\mu_x^{01} = 0.0003$$
$$\mu_x^{10} = 0.00003$$
$$\mu_x^{02} = 0.0001$$
$$\mu_x^{12} = 0.0002$$

calculate the probability that an individual aged 27 is alive but disabled at age 43.

Question 4

A disability income model has transition intensities

$\mu_x^{01} = 0.002 \quad \mu_x^{10} = 0.001 \quad \mu_x^{02} = 0.002 \quad \mu_x^{12} = 0.004$

State 0 is healthy, State 1 is sick and State 2 is dead. Three actuaries calculate different values for the transition probabilities and benefit values. Which one has calculated plausible values?

Value	Actuary I	Actuary II	Actuary III
${}_2p_{37}^{(00)}$	0.992036	0.992036	0.992036
${}_2p_{37}^{(01)}$	0.003960	0.003968	0.003964
${}_4p_{37}^{(01)}$	0.007857	0.007857	0.007857
${}_4p_{37}^{(02)}$	0.015857	0.008000	0.008000
${}_4p_{37}^{(12)}$	0.008000	0.015857	0.015857
${}_2p_{39}^{(01)}$	0.003960	0.003968	0.003964
${}_2p_{39}^{(11)}$	0.992054	0.992054	0.990054

8.6 Premiums

Benefit and Annuity functions

$\bar{a}_x^{ij}$	EPV of an annuity paying continuously at a rate of \$1 per year, whenever the life is in state j, to a life currently aged x and in state i	$\bar{a}_x^{ij} = \int_0^\infty e^{-\delta t} {}_tp_x^{ij} dt$
$\bar{A}_x^{ij}$	EPV of a benefit which pays \$1 immediately, whenever the life transitions into state j, to a life currently aged x and in state i	$\bar{A}_x^{ij} = \int_0^\infty \sum_{k \neq j} e^{-\delta t} {}_tp_x^{ik} \mu_{x+y}^{kj} dt$

Question 5

Under a permanent disability model, with transition intensities

$$\mu_x^{01} = 0.0003 + 0.000002x$$
$$\mu_x^{02} = 0.0001 + 0.000001x$$
$$\mu_x^{12} = 0.02$$

The interest rate is $\delta = 0.03$. Calculate the premium for a 5-year policy sold to a life aged 42, with premiums payable continuously while healthy, benefits at a rate of \$90,000 per year are payable while the life is sick, and a death benefit of \$100,000 payable immediately upon death.

Question 6

Under a disability income model, transition intensities are:

$$\mu_x^{01} = 0.0003 + 0.000002x$$
$$\mu_x^{10} = 0.00003 + 0.000001x$$
$$\mu_x^{02} = 0.0001 + 0.000001x^2$$
$$\mu_x^{12} = 0.0002 + 0.000002x$$

The interest rate is $i = 0.06$. Calculate the premium for a 10-year policy sold to a life aged 37, with premiums payable annually in advance while healthy, benefits of \$80,000 per year in arrear are payable if the life is sick at the end of a given year, and a death benefit of \$200,000 is payable at the end of the year of death.

Answer to Question 6

We calculate the probability that the life is in each state at the end of each year:

t	${}_tp_{37}^{00}$	${}_tp_{37}^{01}$	${}_tp_{37}^{02}$
0	1	0	0
1	0.99812	0.000375	0.001505
2	0.99617	0.000750	0.003083
3	0.99414	0.001127	0.004736
4	0.99203	0.001505	0.006464
5	0.98985	0.001884	0.008271
6	0.98758	0.002263	0.010156
7	0.98523	0.002644	0.012123
8	0.98280	0.003025	0.014171
9	0.98029	0.003407	0.016303
10	0.97769	0.003790	0.018519

Question 7

An insurance company is developing a new model for transition intensities in a disability income model. Under these transition intensities it calculates

$$\bar{A}^{02}_{34} = 0.14 \qquad \bar{A}^{02}_{44} = 0.19 \qquad \bar{A}^{12}_{44} = 0.21$$
$$\bar{a}^{00}_{34} = 22.07 \qquad \bar{a}^{00}_{44} = 19.30 \qquad \bar{a}^{10}_{44} = 0.11$$
$$\bar{a}^{01}_{34} = 0.64 \qquad \bar{a}^{01}_{44} = 0.43 \qquad \bar{a}^{11}_{44} = 17.32$$
$${}_{10}p^{00}_{34} = 0.934 \qquad {}_{10}p^{01}_{34} = 0.022 \qquad \delta = 0.03$$

Calculate the premium for a 10-year policy for a life aged 34, with continuous premiums payable while in the healthy state, which pays a continuous benefit while in the sick state, at a rate of $80,000 per year, and pays a death benefit of $280,000 immediately upon death.

Question 8

A disability income model has the following four states:

State	Meaning	State	Meaning
0	Healthy	2	Accidental Death
1	Sick	3	Other Death

The transition intensities are:

$$\mu_x^{01} = 0.001 \quad \mu_x^{02} = 0.002 \quad \mu_x^{03} = 0.001$$
$$\mu_x^{10} = 0.002 \quad \mu_x^{12} = 0.001 \quad \mu_x^{13} = 0.003$$

t years from the start of the policy, the probability that the life is healthy is $0.2113249e^{-0.006732051t} + 0.7886751e^{-0.003267949t}$; the probability that it is sick is $0.2886752e^{-0.003267949t} - 0.2886752e^{-0.006732051t}$.
Calculate the premium for a 5-year policy with premiums payable continuously while the life is in the healthy state, which pays no benefits while the life is in the sick state, but pays a benefit of \$200,000 in the event of accidental death and a benefit of \$100,000 in the event of other death. The interest rate is $\delta = 0.03$.

Question 9

(a) How does the premium in Question 8 change if there is a waiting period of 3 months but no off period for the sick state?
(b) How does the premium in Question 8 change if there is a waiting period of 3 months and an off period of 6 months for the sick state?

Thiele's Differential Equation

$$\frac{d}{dt}\,{}_tV^{(i)} = \delta\,{}_tV^{(i)} + P^{(i)} - B^{(i)} - \sum_{j \neq i} \mu_{x+t}^{(ij)} \left(S^{(ij)} + {}_tV^{(j)} - {}_tV^{(i)} \right)$$

where:

- δ is force of interest.
- ${}_tV^{(i)}$ is the policy value at time t if the life is in state i.
- $P^{(i)}$ is the rate at which premiums are paid while in state i.
- $B^{(i)}$ is the rate at which benefits are paid while in state i.
- $S^{(ij)}$ is the benefit which is paid upon every transition from state i to state j.

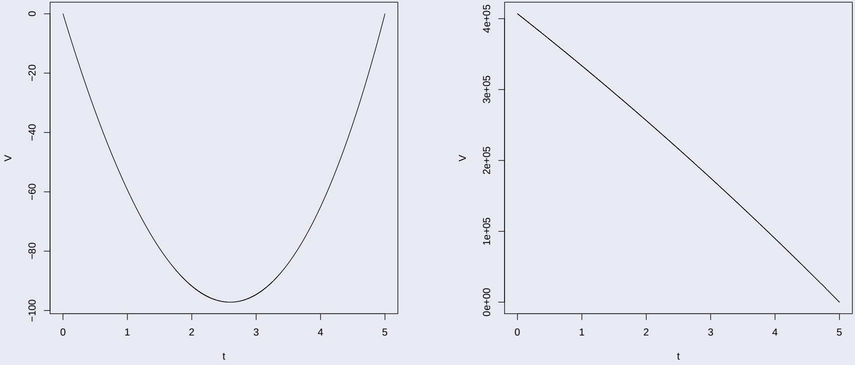
Question 10

Under a permanent disability model, with transition intensities

$$\mu_x^{01} = 0.0003 + 0.000002x$$
$$\mu_x^{02} = 0.0001 + 0.000001x^2$$
$$\mu_x^{12} = 0.02$$

The interest rate is $\delta = 0.03$. Recall (Question 5) that the continuous premium for a 5-year policy sold to a life aged 42 is \$98.54 per year; a benefit at a rate \$90,000 per year is payable while the life is disabled; and a benefit of \$100,000 is payable immediately upon death. Calculate the policy value of this policy in 3 years time, while the life is healthy, and while the life is disabled.

Answer to Question 10



(a) Healthy (b) Disabled

Question 11

In a certain life insurance policy, mortality is modelled as $\mu_x = 0.0003 + 0.00002x$, while policies lapse at a rate $\lambda_x = 0.002 - 0.00001x$. Force of interest is $\delta = 0.04$. Calculate the continuous premium for a 10-year policy with death benefits \$300,000, payable immediately on death sold to a life aged 36.

(a) If the insurer makes no payments to policies which lapse.

(b) If policies can be surrenderred for half the policy value. [Policy value is calculated under the assumption that the policy does not lapse.]

Question 12

A certain life insurance policy, pays double benefits for accidental death (state 1). Mortality is modelled as

$$\mu_x^{01} = 0.0003$$
$$\mu_x^{02} = 0.00002x$$
$$\mu_x^{03} = 0.002 - 0.00001x$$

Where state 1 represents accidental death, state 2 represents other deaths, and state 3 represents lapse. [The insurer makes no payments to policies which lapse.] Calculate the continuous premium for a 10-year policy with death benefits \$400,000 for accidental death, and \$200,000 for other deaths, payable immediately on death sold to a life aged 29, if force of interest is $\delta = 0.05$.

Question 13

The following is a multiple decrement table, giving probabilities of surrender, accidental death, and other death.

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$	$d_x^{(3)}$
40	10000.00	59.00	0.30	1.62
41	9939.08	58.65	0.29	1.70
42	9878.44	58.31	0.28	1.78
43	9818.06	57.96	0.27	1.89
44	9757.95	57.62	0.27	1.98
45	9698.08	57.28	0.26	2.10
46	9638.44	56.94	0.25	2.23
47	9579.02	56.61	0.24	2.36
48	9519.81	56.27	0.24	2.51
49	9460.78	55.94	0.23	2.68

Calculate the probability that a life who purchases a policy at age 42 surrenders it between ages 46 and 48.

Question 14

The following is a multiple decrement table, giving probabilities of surrender, accidental death, and other death.

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$	$d_x^{(3)}$
40	10000.00	59.00	0.30	1.62
41	9939.08	58.65	0.29	1.70
42	9878.44	58.31	0.28	1.78
43	9818.06	57.96	0.27	1.89
44	9757.95	57.62	0.27	1.98
45	9698.08	57.28	0.26	2.10

An annual 5-year term annual insurance policy pays benefits of \$200,000 in the case of accidental death, \$100,000 in the case of other death, and has no surrender value. Calculate the net premiums for this policy sold to a life aged 40 at interest rate $i = 0.03$.

Question 15

Recall the multiple decrement table from Question 13, giving probabilities of surrender, accidental death, and other death.

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$	$d_x^{(3)}$
40	10000.00	59.00	0.30	1.62
41	9939.08	58.65	0.29	1.70
42	9878.44	58.31	0.28	1.78
43	9818.06	57.96	0.27	1.89
44	9757.95	57.62	0.27	1.98
45	9698.08	57.28	0.26	2.10
46	9638.44	56.94	0.25	2.23
47	9579.02	56.61	0.24	2.36
48	9519.81	56.27	0.24	2.51
49	9460.78	55.94	0.23	2.68

Calculate the probability that a life who purchases a policy at age 42 and 4 months dies in an accident between ages 46 and 3 months and 47 and 5 months using:

(a) UDD

(b) Constant transition intensities.

Question 16

You want to update the multiple decrement table on the left below with the updated mortalities from the table on the right.

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
40	10000.00	59.00	1.92
41	9939.08	58.65	1.99
42	9878.44	58.31	2.06
43	9818.06	57.96	2.16
44	9757.95	57.62	2.25
45	9698.08	57.28	2.36
46	9638.44	56.94	2.48
47	9579.02	56.61	2.60

x	l_x	d_x
40	10000.00	1.10
41	9998.90	1.18
42	9997.72	1.26
43	9996.46	1.35
44	9995.11	1.45
45	9993.66	1.56
46	9992.10	1.67
47	9990.43	1.80

Construct the new multiple decrement table using:

(a) UDD in the Multiple Decrement table.
(b) Constant transition probabilities.
(c) UDD in the independent models

Answer to Question 16

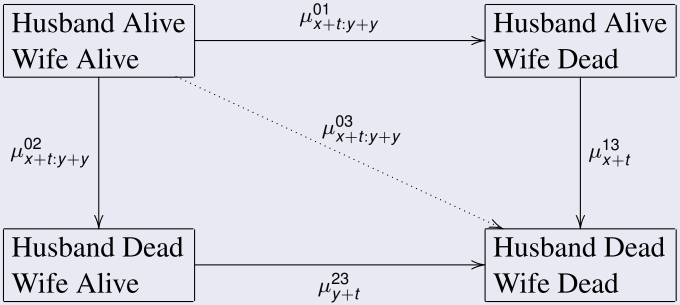
(a) and (b)

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
40	10000.00	59.00	1.10
41	9939.90	58.66	1.17
42	9880.07	58.32	1.24
43	9820.51	57.98	1.32
44	9761.21	57.64	1.41
45	9702.16	57.31	1.51
46	9643.34	56.97	1.61
47	9584.76	56.65	1.72

(c)

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
40	10000.00	59.00	1.10
41	9939.90	58.66	1.17
42	9880.07	58.32	1.24
43	9820.51	57.98	1.32
44	9761.21	57.64	1.41
45	9702.16	57.31	1.51
46	9643.34	56.97	1.61
47	9584.76	56.65	1.72

Model


Husband Alive
Wife Alive
$\mu^{01}_{x+t:y+y}$
Husband Alive
Wife Dead
$\mu^{02}_{x+t:y+y}$
$\mu^{03}_{x+t:y+y}$
μ^{13}_{x+t}
Husband Dead
Wife Alive
μ^{23}_{y+t}
Husband Dead
Wife Dead

9.2 Joint Life and Last Survivor Benefits

Joint Policies

Joint life annuity	a_{xy}	pays regular payments while both lives are alive.
Joint life insurance	A_{xy}	pays a death benefit upon the death of either life.
Last survivor annuity	$a_{\overline{xy}}$	pays regular payments while either life is still alive.
Last survivor insurance	$A_{\overline{xy}}$	pays a death benefit upon the death of both lifes.
Reversionary annuity	$a_{x\|y}$	pays regular payments while husband is dead and wife is alive.
Contingent insurance	A^{1}_{xy}	pays a death benefit upon the death of husband provided wife is alive.

Question 17

A couple want to receive a pension of $200,000 per year while both are alive. If the husband is alive, but the wife is not, he wants to receive $60,000 per year. If the wife is alive, but the husband is not, she wants to receive $220,000 per year. When they both die, they want to leave an inheritance of $700,000 to their children. Construct a collection of insurance and annuity policies that will achieve these objectives.

9.2 Joint Life and Last Survivor Benefits

Question 18

What are the advantages and disadvantages of a reversionary annuity over a standard life insurance policy, whose benefit could be used to purchase an annuity at the time the life dies.

9.2 Joint Life and Last Survivor Benefits

Formulae

$$a_{\overline{xy}} = a_x + a_y - a_{xy}$$

$$a_{x|y} = a_y - a_{xy}$$

$$A_{\overline{xy}} = A_x + A_y - A_{xy}$$

$$A_{x|y} + A_{y|x} = A_{xy}$$

$$\bar{a}_{xy} = \frac{1 - \bar{A}_{xy}}{\delta}$$

Assumptions

- While both husband and wife are alive, the probability of dying depends on both ages.
- Once one life has died, the probability of the other life dying depends on the age of that life and the fact that the other life has died, but not the time the other life died, or the age before they died.

Standard Notation for Joint Life Probabilities

Notation	Meaning	Multi-state
${}_tp_{xy}$	Probability both still alive at time t	${}_tp_{xy}^{00}$
${}_tq_{xy}$	Probability not both still alive at time t	$1 -{}_t p_{xy}^{00}$
${}_tq_{xy}^1$	Probability husband dies first before time t	
${}_tq_{xy}^2$	Probability husband dies second before time t	
${}_tp_{\overline{xy}}$	Probability at least one still alive at time t	$1 -{}_t p_{xy}^{03}$
${}_tq_{\overline{xy}}$	Probability both dead at time t	${}_tp_{xy}^{03}$

$$ {}_tq_{xy}^1 = {}_t p_{xy}^{02} + \int_0^t {}_sp_{xy}^{00} \mu_{x+s:y+s}^{02} {}_{t-s}p_y^{23} ds $$

$$ {}_tq_{xy}^2 = \int_0^t {}_sp_{xy}^{00} \mu_{x+s:y+s}^{01} {}_{t-s}p_x^{13} ds $$

Formulae

$$\bar{a}_{xy} = \int_0^\infty e^{-\delta t} {}_tp_{xy}^{00} dt$$

$$\bar{A}_{xy} = \int_0^\infty e^{-\delta t} {}_tp_{xy}^{00} (\mu_{x+t:y+t}^{01} + \mu_{x+t:y+t}^{02}) dt$$

$$\bar{a}_{\overline{xy}} = \int_0^\infty e^{-\delta t} ({}_tp_{xy}^{00} +_t p_{xy}^{01} +_t p_{xy}^{02}) dt$$

$$\bar{A}_{\overline{xy}} = \int_0^\infty e^{-\delta t} ({}_tp_{xy}^{00} \mu_{x+t:y+t}^{03} +_t p_{xy}^{01} \mu_{x+t}^{13} +_t p_{xy}^{02} \mu_{y+t}^{23}) dt$$

$$\bar{a}_{x|y} = \int_0^\infty e^{-\delta t} {}_tp_{xy}^{02} dt$$

$$\bar{A}_{xy}^{1} = \int_0^\infty e^{-\delta t} {}_tp_{xy}^{00} \mu_{x+t:y+t}^{02} dt$$

Question 19

A husband is 63. His wife is 62. Their mortalities both follow the lifetable below, and are assumed to be independent. They purchase a 10-year last survivor insurance policy with a death benefit of $2000,000. Annual Premiums are payable while both are alive. Calculate the net premiums using the equivalence principle and interest rate $i = 0.07$.

x	l_x	d_x
62	10000.00	1.70
63	9998.30	1.83
64	9996.47	1.98
65	9994.49	2.14
66	9992.35	2.31
67	9990.03	2.50

x	l_x	d_x
68	9987.53	2.70
69	9984.83	2.92
70	9981.91	3.16
71	9978.76	3.41
72	9975.34	3.69
73	9971.66	3.99

Question 20

A husband is 53. His wife is 64. Their independent mortalities both follow the lifetables below. They purchase a 7-year reversionary annuity. Annual Premiums are payable while both are alive. If the husband dies first, the policy will provide a life annuity to the wife with annual payments of $30,000. The lifetables are given below. Calculate the net premiums for this policy using the equivalence principle and an interest rate $i = 0.05$. For the wife, we have $\ddot{a}_{71} = 13.89755$.

x	l_x	d_x
53	10000.00	1.35
54	9998.65	1.48
55	9997.17	1.61
56	9995.55	1.76
57	9993.80	1.92
58	9991.87	2.10
59	9989.78	2.29

x	l_x	d_x
64	10000.00	2.64
65	9997.36	2.88
66	9994.48	3.14
67	9991.34	3.42
68	9987.91	3.73
69	9984.19	4.06
70	9980.12	4.43

Question 21

A husband is 72. His wife is 48. They purchase a last survivor annuity which pays $45,000 a year. The life-tables are below. Calculate the net premium for this insurance at $i = 0.06$. For the wife, $\ddot{a}_{68} = 16.1807$.

x	l_x	d_x
72	10000.00	576.84
73	9423.16	631.08
74	8792.08	683.61
75	8108.48	731.95
76	7376.52	773.08
77	6603.44	803.48
78	5799.96	819.33
79	4980.63	816.87
80	4163.76	792.84
81	3370.92	745.21
82	2625.71	673.92
83	1951.79	581.60
84	1370.19	474.03
85	896.16	359.95

x	l_x	d_x
86	536.21	250.05
87	286.16	154.93
88	131.23	82.49
89	48.74	35.57
90	13.17	11.16
91	2.01	1.98
92	0.03	0.03

x	l_x	d_x
48	10000.00	1.03
49	9998.97	1.10
50	9997.87	1.18
51	9996.69	1.26
52	9995.44	1.35
53	9994.09	1.44

x	l_x	d_x
54	9992.65	1.55
55	9991.10	1.66
56	9989.43	1.79
57	9987.65	1.92
58	9985.72	2.07
59	9983.65	2.23
60	9981.42	2.40
61	9979.02	2.59
62	9976.43	2.80
63	9973.63	3.02
64	9970.61	3.26
65	9967.35	3.52
66	9963.83	3.81
67	9960.02	4.12
68	9955.90	4.46

Question 22

A husband is 45. His wife is 76. Their lifetables are below. They purchase a 7-year joint life insurance policy with a death benefit of $850,000. If the interest rate is $i = 0.04$, calculate the monthly net premiums for this policy using the equivalence principle and the UDD assumption.

x	l_x	d_x
45	10000.00	1.80
46	9998.20	1.93
47	9996.26	2.08
48	9994.18	2.23
49	9991.95	2.40
50	9989.55	2.58
51	9986.97	2.78
52	9984.19	3.00

x	l_x	d_x
76	10000.00	16.51
77	9983.49	17.85
78	9965.64	19.29
79	9946.34	20.85
80	9925.49	22.54
81	9902.95	24.35
82	9878.60	26.31
83	9852.28	28.43

9.6 A Model with Dependent Future Lifetimes

Why are joint lives not independent?

- Broken heart syndrome.
- Common accident or illness.
- Similar lifestyles.

Question 23

A husband is 84. His wife is 39. Their mortalities while both are alive and the wife's mortality after the husband has died are shown below. What is the probability that the wife dies within 10 years?

x	l_x	d_x	x	l_x	d_x	x	l_x	d_x
84	10000.00	45.99	39	10000.00	1.00	39	10000.00	2.53
85	9954.01	49.84	40	9999.00	1.06	40	9997.47	2.75
86	9904.17	53.98	41	9997.94	1.13	41	9994.72	2.99
87	9850.19	58.44	42	9996.81	1.20	42	9991.74	3.25
88	9791.76	63.24	43	9995.60	1.28	43	9988.49	3.54
89	9728.52	68.40	44	9994.32	1.37	44	9984.95	3.85
90	9660.11	73.94	45	9992.94	1.47	45	9981.10	4.19
91	9586.17	79.89	46	9991.48	1.57	46	9976.91	4.57
92	9506.28	86.25	47	9989.91	1.68	47	9972.34	4.98
93	9420.03	93.05	48	9988.23	1.80	48	9967.36	5.43
94	9326.98	100.31	49	9986.44	1.93	49	9961.92	5.93

(a) Assuming changes to the wife's mortality apply at the end of the year of the husband's death.

(b) Using the UDD assumption.

Question 24

For the couple in Question 23 (lifetables recalled below). What is the premium for a 10-year annual life insurance policy for the wife with benefit $200,000 at interest rate $i = 0.04$. [Use the UDD assumption for changes to the wife's mortality at time of the Husband's death.]

x	l_x	d_x
84	10000.00	45.99
85	9954.01	49.84
86	9904.17	53.98
87	9850.19	58.44
88	9791.76	63.24
89	9728.52	68.40
90	9660.11	73.94
91	9586.17	79.89
92	9506.28	86.25
93	9420.03	93.05
94	9326.98	100.31

x	l_x	d_x
39	10000.00	1.00
40	9999.00	1.06
41	9997.94	1.13
42	9996.81	1.20
43	9995.60	1.28
44	9994.32	1.37
45	9992.94	1.47
46	9991.48	1.57
47	9989.91	1.68
48	9988.23	1.80
49	9986.44	1.93

x	l_x	d_x
39	10000.00	2.53
40	9997.47	2.75
41	9994.72	2.99
42	9991.74	3.25
43	9988.49	3.54
44	9984.95	3.85
45	9981.10	4.19
46	9976.91	4.57
47	9972.34	4.98
48	9967.36	5.43
49	9961.92	5.93

Question 25

A husband aged 25 and a wife aged 56 have the following transition intensities:

$$\mu_{xy}^{01} = 0.000001y^2 + 0.000000001x$$
$$\mu_{xy}^{02} = 0.000002x^2 + 0.000000002y$$
$$\mu_{xy}^{03} = 0.000042$$
$$\mu_{x}^{13} = 0.000003x^2$$
$$\mu_{y}^{23} = 0.000002y^2$$

Calculate the probability that in ten years time the husband is dead, and the wife is still alive.

Question 26

A husband aged 25 and a wife aged 56 have the following transition intensities:

$$\mu_{xy}^{01} = 0.000001y^2 + 0.000000001x$$
$$\mu_{xy}^{02} = 0.000002x^2 + 0.000000002y$$
$$\mu_{xy}^{03} = 0.000042$$
$$\mu_{x}^{13} = 0.000003x^2$$
$$\mu_{y}^{23} = 0.005$$

They wish to purchase a reversionary annuity, which will provide a continuous life annuity to the wife at a rate of $25,000 per year after the husband's death. The premiums are payable continuously while both are alive. The interest rate is $\delta = 0.04$. Calculate the rate of premiums.

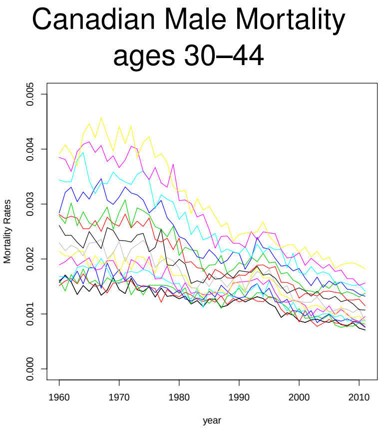
Question 27

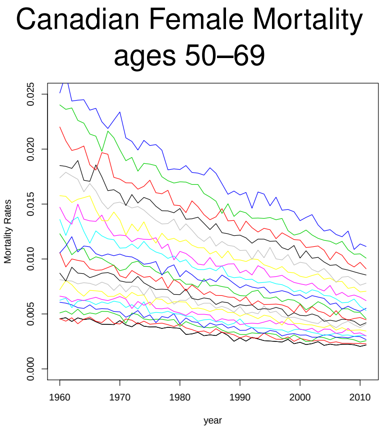
A husband aged 75 and a wife aged 29 have the following transition intensities:

$$\mu_{xy}^{01} = 0.001y + 0.000001x$$
$$\mu_{xy}^{02} = 0.002x + 0.000002y$$
$$\mu_{xy}^{03} = 0.012$$
$$\mu_{x}^{13} = 0.003x$$
$$\mu_{y}^{23} = 0.002y$$

They wish to purchase an annual whole-life last survivor insurance policy with benefit $300,000. The interest rate is $i = 0.06$.
(a) Calculate the annual premiums. (Premiums are payable while either life is still alive).
(b) Calculate the policy value after 10 years if the husband is dead, but the wife is alive.

SN 4.1 Mortality Improvement Modelling

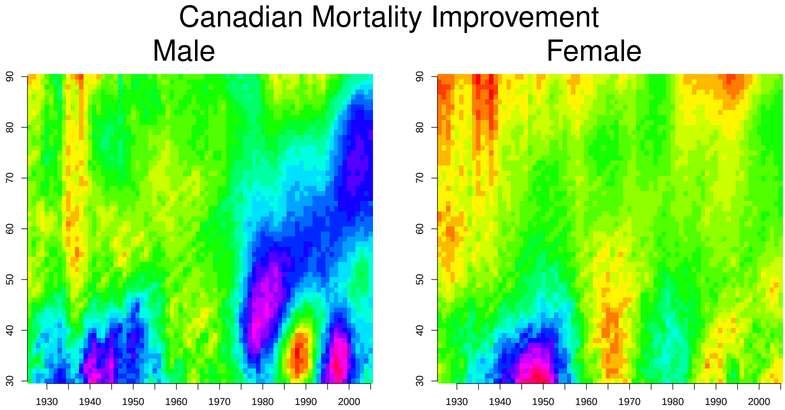




Notes

- General declining trend
- Large inter-year variation

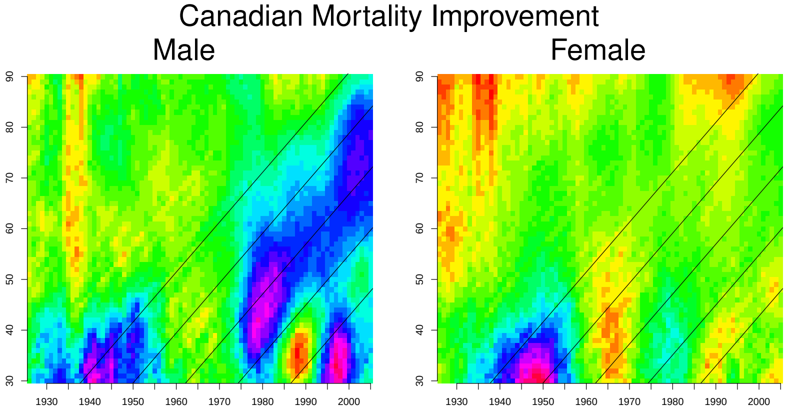
SN 4.1 Mortality Improvement Modelling



Type of Effect

- Year effects
- Age effects
- Cohort effects

SN 4.1 Mortality Improvement Modelling



Type of Effect

- Year effects
- Age effects
- Cohort effects

SN 4.2 Mortality Improvement Scales

Mortality Improvement Scales

- Deterministic function for $q(x, t)$ (Mortality at age x in year t).
- Generally obtained as product of $q(x, 0)$ and scale function.
- Simplest are single-factor scale functions depending only on age: $q(x, t) = q(x, 0)(1 - \phi_x)^t$
- Decreasing mortality increases cost of annuities, increases cost of life insurance.
- Longevity risk: risk of losses arising from misestimating future mortality.
- More advanced models have improvement factors which are a function of both age and year.

SN 4.2 Mortality Improvement Scales

Constructing Two-Dimensional Improvement Scales

- First calculate smoothed mortality improvment factors (based on smoothing log of mortality rate) for existing data.
- Calculate short term improvement factors from recent experience.
- Calculate long term improvement factors from whole experience.
- Decide when short term factors should revert to long term ones.
- Use smooth interpolation (e.g. cubic functions) to calculate intermediate term improvement factors.

Incorporating Both Age and Cohort Effects

- Use the above interpolation method on age groups to get one improvement scale.
- Use the above interpolation method on cohort groups to get one improvement scale.
- Take the average of the two interpolated scales.

Question 28

For males, our smoothed mortality improvment is
$\phi(45, 2000) = 0.01863027$, $\left.\frac{\partial\phi(x,t)}{\partial t}\right|_{x=45,t=2000} = 0.001077732$,
$\phi(32, 2000) = 0.03146243$ and $\left.\frac{d\phi(x+t,t)}{dt}\right|_{x=32,t=2000} = 0.001847864$.
You decide that the long-term trend is $\phi(x, t) = 0.01$ for all x between 30 and 60, and that this trend should be used from 2025 onwards. Use the interpolation method from the previous slide to calculate $\phi(45, 2013)$

Stochastic Mortality Scales

Why use Stochastic Mortality Scales

- Mortality rates have a lot of random fluctuation.
- Deterministic mortality scales can give good expected present value, but don't give enough information about the risks.
- Longevity risks are non-diversifiable. This means the same mortality applies to all policyholders, so it cannot be reduced by selling more policies. (See Chapter 11 of textbook).

SN 4.4 The Lee Carter Model

Central Death Rates

$$m_x = \frac{q_x}{\int_0^1 {}_t p_x \, dt} = \frac{\int_0^1 {}_t p_x \mu_{x+t} \, dt}{\int_0^1 {}_t p_x \, dt}$$

Model

$$\log(m(x,t)) = \alpha_x + \beta_x K_t + \epsilon_{x,t}$$

- α_x and β_x depend only on x.
- K_t depends only on t.
- K_t is given by a stochastic process $K_t = K_{t-1} + c + \sigma_k Z_t$ where Z_t are independant standard normal random variables.
- c and σ_k are estimated from the data.
- $\epsilon_{x,t}$ is random fluctuation and is assumed negligible.
- We add the constraints $\sum \beta_x = 1$ and $\sum K_t = 0$.

Question 29

A Lee Carter model has been fitted to the data with the following parameters:

$$\alpha_{40} = -4.23 \qquad c = -0.8$$
$$\beta_{40} = 0.07 \qquad \sigma_k = 0.6$$
$$K_{2017} = -8.36$$

(a) Calculate the mean, standard dev and 5th percentile of $m(40, 2034)$
(b) A life currently aged 33 wants to buy a special life insurance policy where she pays a premium when she reaches age 40 (if still alive). If she dies while aged 40, she receives benefits of $300,000 at the end of that year. If she survives to age 41, the policy expires with no benefit. Using the UDD assumption for mortality between integer ages, and assuming an annual effective interest rate of 5%, what premium should the company charge under the equivalence principle?
(c) Using this premium, what is the probability that at the start of 2024, the policy has an EPVFL exceeding $50?

Problems with Lee Carter Model

- Does not fit data well.
- Does not account for cohort effects.
- Assumes effects perfectly correlated at different ages.

SN 4.5 Cairns-Blake-Dowd Models

Original Model

$$\log\left(\frac{q(x,t)}{1-q(x,t)}\right) = K_t^{(1)} + K_t^{(2)}(x-\overline{x})$$

- $K_t^{(i)} = K_{t-1}^{(i)} + c^{(i)} + \sigma_{k_i} Z_t^{(i)}$
- $Z_t^{(i)}$ and $Z_s^{(j)}$ are independent if $t \neq s$.
- $Z_t^{(i)} \sim N(0,1)$
- $\mathrm{Cov}(Z_t^{(1)}, Z_t^{(2)}) = \rho$.

Question 30

Suppose mortality is projected to follow a CBD model with

$$K^{(1)}_{2017} = -4.36 \qquad K^{(2)}_{2017} = 0.13 \qquad c^{(1)} = -0.2 \qquad c^{(2)} = -0.01$$
$$\sigma_{k_1} = 0.4 \qquad \sigma_{k_2} = 0.03 \qquad \rho = 0.3 \qquad \bar{x} = 52$$

(a) Calculate the median and 95th percentile of $q(33, 2048)$.
(b) A life aged 26 wants to buy a 5-year annual term insurance policy with a death benefit of $400,000. The insurance company simulates the following values of $Z^{(i)}_t$:

t	2018	2019	2020	2021
$Z^{(1)}_t$	-1.609243	0.2000999	-2.070148	-2.3544103
$Z^{(2)}_t$	-1.108664	-1.5800675	-1.561336	-0.7512827

If the interest rate is 6% under these simulated values, and using the equivalence principle, what annual premium should they charge for this policy?

SN 4.5 Cairns-Blake-Dowd Models

The CBD M7 model

$$\log\left(\frac{q(x,t)}{1-q(x,t)}\right) = K_t^{(1)} + K_t^{(2)}(x-\overline{x}) + K_t^{(3)}((x-\overline{x})^2 - {s_x}^2) + G_{t-x}$$

Remarks

- First two terms are the same as for the original CBD model.
- Third term adds a quadratic age dependance.
- Fourth term gives a cohort effect time series. $t-x$ is the year of birth of the cohort.
- G_{t-x} is usually fitted using an ARIMA model.

SN 4.7 Notes on Mortality Improvement Modelling

Remarks

- Typically use Monte Carlo Simulation to assess longevity risk.
- For each set of simulated $q(x, t)$ values, we can calculate the actuarial value of annuities and benefits. We therefore get a sample of actuarial values.
- Typically, we take the value for the whole portfolio, because changes to mortality are correlated between different age groups.
- Importance of age, year and cohort effects can vary between populations.
- This type of modelling requires a lot of data. It can be difficult to model for subpopulations.
- Policyholders tend to be wealthier and thus have first access to improvements in mortality.

SN 4.7 Notes on Mortality Improvement Modelling

Further Remarks

- Whole-population models may underestimate longevity risk if policyholder mortality improves faster than the population.
- Conversely, they may underestimate longevity risk if improvements to policyholder mortality slow down before the rest of the population.
- Can generate deterministic scales from mean or median of stochastic model.
- Other time series models can be used.
- Model selection is challenging, there is large parameter uncertainty, and all models fit badly.

Question 31

Calculate the empirical distribution and cumulative hazard rate function for the following data set:

4	3	6	0	3	7	0	0	2	1	1	3	6	3	4

Question 32

For the data set from the previous question,

4 3 6 0 3 7 0 0 2 1 1 3 6 3 4

compute a Nelson-Åalen estimate for the probability that a random sample is larger than 5.

Question 33

An insurance company collects the following data on life insurance policies:

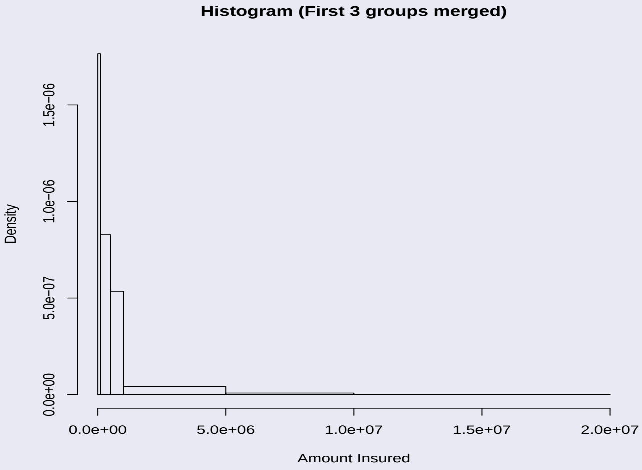
Amount Insured	Number of Policies
Less than \$5,000	30
\$5,000–\$20,000	52
\$20,000–\$100,000	112
\$100,000–\$500,000	364
\$500,000–\$1,000,000	294
\$1,000,000–\$5,000,000	186
\$5,000,000–\$10,000,000	45
More than \$10,000,000	16

The government is proposing a tax on insurance policies for amounts larger than \$300,000. Using the ogive to estimate the empirical distribution function, what is the probability that a random policy is affected by this tax?

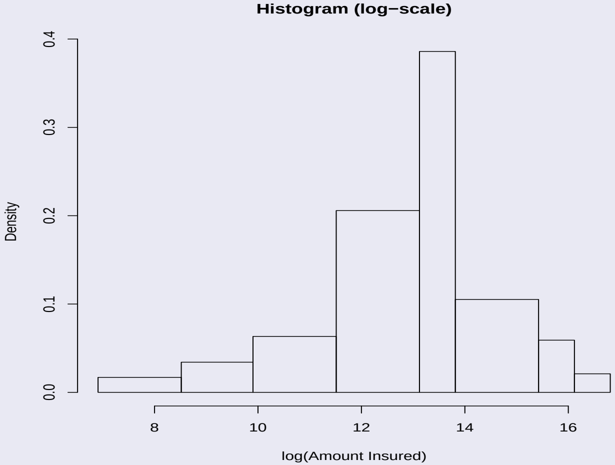
Question 34

Draw the histogram for the data from Question 33

Answer to Question 34



Answer to Question 34



Question 35

A sample of size 2,000 contains 1,700 observations that are at most 6,000; 30 that are between 6,000 and 7,000; and 270 that are more than 7,000. The total of the 30 observations between 6,000 and 7,000 is 200,000. The value of $\mathbb{E}(X \wedge 6000)$ under the empirical distribution obtained from this data is 1,810. Calculate the value of $\mathbb{E}(X \wedge 7000)$ under the empirical distribution obtained from this data.

Question 36

A random sample of unknown size includes 36 observations between 0 and 50, x observations between 50 and 150, y observations between 150 and 250, 84 observations between 250 and 500, 80 observations between 500 and 1,000, and no observations above 1,000.
The ogive includes the values $F_n(90) = 0.21$ and $F_n(210) = 0.51$.
Calculate x and y.

Question 37

Calculate the variance of the empirical survival function for grouped data using the ogive.

LM 12.2 The Empirical Distribution for Grouped Data

Question 38

An insurance company receives 4,356 claims, of which 2,910 are less than $10,000, and 763 are between $10,000 and $100,000. Calculate a 95% confidence interval for the probability that a random claim is larger than $50,000.

LM 12.3 & 12.5 Empirical Estimation for Modified Data — Truncation and Censoring

Definition

Truncated from below	Observations $\leqslant d$ are not recorded
Truncated from above	Observations $\geqslant u$ are not recorded
Censored from below	Observations $\leqslant d$ are recorded only as $\leqslant d$
Censored from above	Observations $\geqslant u$ are recorded only as $\geqslant u$

We will by default assume truncation is on the left (deductible) and censoring is on the right (policy limit).

Example Data Set

i	d_i	x_i	u_i	i	d_i	x_i	u_i	i	d_i	x_i	u_i
1	0	0.4	-	8	0	-	1.8	15	1.2	-	1.4
2	0	1.6	-	9	0	1.4	-	16	0.5	-	1.3
3	0	-	2.4	10	0	-	1.2	17	0.5	2.2	-
4	0	0.7	-	11	0	1.3	-	18	0.9	-	2.3
5	0	-	0.4	12	0	-	1.1	19	0.8	1.2	-
6	0	1.9	-	13	0.4	1.4	-	20	0.6	-	1.5
7	0	1.1	-	14	0.7	1.7	-	21	1.1	1.8	-

Notes

- d_i is the left truncation point (deductible). 0 indicates no truncation.
- x_i indicate complete data. That is the exact value of x_i is known.
- u_i indicate censored data. All that is known is that $x_i \geqslant u_i$.
- There should be an entry in exactly one of the third and fourth columns.

Kaplan-Meier Product-Limit Estimator

Notation

- y_j Unique uncensored values sorted in increasing order $y_1 < \ldots < y_k$
- s_j Number of times y_j occurs in the sample
- r_j Size of risk set at y_j. That is the number of samples i such that $d_i < y_j < x_i$ or $d_i < y_j < u_i$

Formulae

$$r_j = |\{i|x_i \geqslant r_j\}| + |\{i|u_i \geqslant r_j\}| - |\{i|d_i \geqslant r_j\}|$$
$$r_j = |\{i|d_i < r_j\}| - |\{i|u_i < r_j\}| - |\{i|x_i < r_j\}|$$

Kaplan-Meier Product Limit-Estimator

For $y_{j-1} < t \leqslant y_j$:

$$S(t) = \prod_{i=1}^{j-1} \left(1 - \frac{s_i}{r_i}\right)$$

LM 12.3 & 12.5 Empirical Estimation for Modified Data

i	d_i	x_i	u_i	i	d_i	x_i	u_i	i	d_i	x_i	u_i
1	0	0.4	-	8	0	-	1.8	15	1.2	-	1.4
2	0	1.6	-	9	0	1.4	-	16	0.5	-	1.3
3	0	-	2.4	10	0	-	1.2	17	0.5	2.2	-
4	0	0.7	-	11	0	1.3	-	18	0.9	-	2.3
5	0	-	0.4	12	0	-	1.1	19	0.8	1.2	-
6	0	1.9	-	13	0.4	1.4	-	20	0.6	-	1.5
7	0	1.1	-	14	0.7	1.7	-	21	1.1	1.8	-

Summary of Dataset

i	y_i	s_i	r_i	i	y_i	s_i	r_i	i	y_i	s_i	r_i
1	0.4	1	12	5	1.3	1	13	9	1.8	1	6
2	0.7	1	14	6	1.4	2	11	10	1.9	1	4
3	1.1	1	16	7	1.6	1	8	11	2.2	1	3
4	1.2	1	15	8	1.7	1	7				

Question 39

Using the summary of the dataset above, and the Kaplan-Meier product-limit estimator, estimate the probability that a randomly chosen observation is more than 1.6.

Question 40

Using the data in the table:

i	d_i	x_i	u_i	i	d_i	x_i	u_i	i	d_i	x_i	u_i
1	0	1.3	-	8	0	0.8	-	15	0.2	0.6	-
2	0	0.1	-	9	0	-	0.7	16	0.6	1.4	-
3	0	-	0.4	10	0.2	-	0.8	17	0.2	0.3	-
4	0	0.2	-	11	0.3	1.4	-	18	0.1	-	1.0
5	0	-	0.3	12	0.3	-	0.8	19	0.1	0.3	-
6	0	0.8	-	13	0.2	-	0.5	20	0.3	1.3	-
7	0	-	0.1	14	0.4	-	0.5	21	0.3	1.4	-

Using the Kaplan-Meier product-limit estimator, estimate the median of the distribution.

Answer to Question 40

i	y_i	s_i	r_i
1	0.1	1	12+9-12= 9
2	0.2	1	11+8-10= 9
3	0.3	2	10+8- 6=12
4	0.6	1	8+4- 1=11
5	0.8	2	7+3- 0=10
6	1.3	2	5+0- 0= 5
7	1.4	3	3+0- 0= 3

Question 41

For the same data as in Question 39, summarised in the following table:

i	y_i	s_i	r_i	i	y_i	s_i	r_i	i	y_i	s_i	r_i
1	0.4	1	12	5	1.3	1	13	9	1.8	1	6
2	0.7	1	14	6	1.4	2	11	10	1.9	1	4
3	1.1	1	16	7	1.6	1	8	11	2.2	1	3
4	1.2	1	15	8	1.7	1	7				

using a Nelson-Åalen estimator, estimate the probability that a random observation is larger than 1.6.

Question 42

From the data in Question 40 summarised below:

i	y_i	s_i	r_i
1	0.1	1	12+9-12= 9
2	0.2	1	11+8-10= 9
3	0.3	2	10+8- 6=12
4	0.6	1	8+4- 1=11
5	0.8	2	7+3- 0=10
6	1.3	2	5+0- 0= 5
7	1.4	3	3+0- 0= 3

Estimate the probability that a random observation that is known to be more than 0.5 is at most 1. Use a Kaplan-Meier estimator.

Question 43

Show that under the assumption that the sizes of the risk set and the possible dying times are fixed, the Kaplan-Meier product-limit estimate is unbiassed and calculate its variance.

Greenwood's Approximation

Approximation

If $a_1, \ldots, a_n$ are all small, then

$$(1 + a_1)\cdots(1 + a_n) \approx 1 + a_1 + a_2 + \cdots + a_n$$

Formula

$$\mathrm{Var}(S_n(y_j)) \approx \left(\frac{S(y_j)}{S(y_0)}\right)^2 \sum_{i=1}^{j} \frac{S(y_{i-1}) - S(y_i)}{r_i S(y_i)}$$

Since $\frac{r_i - s_i}{r_i}$ is an estimate of $\frac{S(y_i)}{S(y_{i-1})}$, we can estimate this by

$$\mathrm{Var}(S_n(y_j)) \approx \hat{S}(y_j)^2 \sum_{i=1}^{j} \frac{s_i}{r_i(r_i - s_i)}$$

Question 44

Recall from Question 40 (data summarised below) that using a Kaplan Meier estimator, we have

$$S_n(1) = \frac{8}{9} \times \frac{8}{9} \times \frac{10}{12} \times \frac{10}{11} \times \frac{8}{10} = \frac{1280}{2673}$$

Use Greenwood's formula to find a 95% confidence interval for $S_n(1)$

i	y_i	s_i	r_i
1	0.1	1	12+9-12= 9
2	0.2	1	11+8-10= 9
3	0.3	2	10+8- 6=12
4	0.6	1	8+4- 1=11
5	0.8	2	7+3- 0=10
6	1.3	2	5+0- 0= 5
7	1.4	3	3+0- 0= 3

Log-transformed Confidence Interval

Problem

The usual method for constructing a confidence interval for $S(x)$ may lead to impossible values (negative or more than 1).

Solution

Instead find a confidence interval for

$$\log(-\log(S(x)))$$

which has no impossible values.

Log-transformed Confidence Interval (Continued)

Method

By the delta method, if $S_n(x)$ is approximately normal with mean μ and small variance σ^2, then for any smooth function $g(x)$, we have that $g(S_n(x))$ is approximately normal with mean $g(\mu)$ and variance $g'(\mu)^2\sigma^2$.
In particular, when $g(x) = \log(-\log(S(x)))$, we have $g'(x) = \frac{1}{S(x)\log(S(x))}$.

Definition

The log-transformed confidence interval for $S(X)$ is given by $[S_n(x)^{\frac{1}{U}}, S_n(X)^U]$, where $U = e^{\Phi^{-1}\left(\frac{\alpha}{2}\right)\frac{\sigma}{S_n(x)\log(S_n(x))}}$.

Question 45

Recall from Question 44 that for the following data set

i	y_i	s_i	r_i
1	0.1	1	12+9-12= 9
2	0.2	1	11+8-10= 9
3	0.3	2	10+8- 6=12
4	0.6	1	8+4- 1=11
5	0.8	2	7+3- 0=10
6	1.3	2	5+0- 0= 5
7	1.4	3	3+0- 0= 3

The Kaplan-Meier estimator is $S_n(1) = \frac{1280}{2673} = 0.4788627$ and Greenwood's formula gives the variance as 0.0180089. Find a 95% log-transformed confidence interval for $S(1)$.

Question 46

An insurance company observes the following claims

Claim size	Frequency	r_i
1	226	1641
2	387	1415
3	290	1028
4	215	738
5	176	523
7	144	347
9	97	203
> 9	106	

Use a Nelson Åalen estimator to obtain a 95% log-transformed confidence interval for the probability that a random claim is more than 5.

Aim

Construct a lifetable from data in a mortality study. For each individual this data includes:

- Age at entry. (This might either be when the policy was purchased or when the study started if the policy was purchased before this time.)
- Age at exit.
- Reason for exit (death or other). Other exits might be surrender or termination of policy or end of study period.

Two Methods

Exact Exposure

- Exposure e_i of each observation during an age range is the proportion of that age range for which the individual was in the study.
- $\frac{d_i}{e_i}$ (deaths divided by exposures) is the estimator for hazard rate.
- The probability of dying within the age range is $1 - e^{-\frac{d_i}{e_i}t}$ where t is the length of the age range.

Actuarial Exposure

- Exposure e_i is the proportion of the age range for which the individual was either in the study or dead.
- That is, individuals who die are assumed to remain in the study until the end of the age range.
- $\frac{d_i}{e_i}t$ is the estimator for the probability of dying within the age range.

Question 47

An insurance company records the following data in a mortality study:

entry	death	exit	entry	death	exit	entry	death	exit
61.4	-	64.4	61.9	-	64.9	62.1	-	63.5
62.4	-	63.7	60.6	-	63.4	62.6	63.1	-
62.7	-	64.4	61.3	-	63.8	63.1	65.3	-
61.0	-	63.2	63.8	-	64.8	63.4	65.6	-
63.2	-	66.2	62.2	-	64.4	61.8	63.2	-
62.7	-	65.0	61.8	-	63.4	62.2	63.4	-
63.6	-	66.6	62.6	-	65.6			

Estimate the probability of an individual currently aged exactly 63 dying within the next year using:
(a) the exact exposure method.
(b) the actuarial exposure method.

LM 12.7 Approximations for Large Data Sets

Insuring Ages

- Premiums based on whole ages only.
- q_{36} — the probability of an individual aged 36 dying within a year — is not for an individual aged exactly 36, but rather for an average individual aged 36.
- Now an individuals age is changed slightly so that their birthday is adjusted to match the date on which they purchased the policy.

Anniversary-based Mortality Studies

- Policyholders enter the study on the first policy anniversary following the start of the study.
- Policyholders leave the study on the last policy anniversary before the scheduled end of the study or their surrender.

Question 48

Recall the following data from Question 47 (with an additional column showing the age at which the individuals purchased their policy):

purchase	entry	death	exit	purchase	entry	death	exit
58.2	61.4	-	64.4	63.8	63.8	-	64.8
53.7	62.4	-	63.7	56.4	62.2	-	64.4
59.3	62.7	-	64.4	60.4	61.8	-	63.4
48.9	61.0	-	63.2	56.0	62.6	-	65.6
59.4	63.2	-	66.2	61.8	61.8	63.2	-
62.7	62.7	-	65.0	62.2	62.2	63.4	-
61.0	63.6	-	66.6	61.7	63.4	65.6	-
55.2	61.9	-	64.9	55.0	62.1	-	63.5
38.4	60.6	-	63.4	52.4	62.6	63.1	-
49.9	61.3	-	63.8	60.3	63.1	65.3	-

(a) Calculate the estimate for q_{63} using insuring ages.

(b) Now recalculate q_{63} on an anniversary-to-anniversary basis.

Question 49

Recall the data from Question 47:

entry	death	exit	entry	death	exit	entry	death	exit
61.4	-	64.4	61.9	-	64.9	62.1	-	63.5
62.4	-	63.7	60.6	-	63.4	62.6	63.1	-
62.7	-	64.4	61.3	-	63.8	63.1	65.3	-
61.0	-	63.2	63.8	-	64.8	63.4	65.6	-
63.2	-	66.2	62.2	-	64.4	61.8	63.2	-
62.7	-	65.0	61.8	-	63.4	62.2	63.4	-
63.6	-	66.6	62.6	-	65.6			

Rewrite the information from this table showing only the events by age interval.

Answer to Question 49

Age	Number at start	enter	die	leave	Number at next age
60	0	1	0	0	1
61	2	5	0	0	7
62	7	8	0	0	15
63	15	5	3	6	11
64	11	0	0	5	6
65	5	0	2	1	2
66	2	0	0	2	0

Question 50

Using the above table estimate q_{63} (the probability that an individual aged exactly 63 dies within one year). Assuming events are uniformly distributed over the year and use:

(a) exact exposure.

(b) actuarial exposure.

Question 51

Using the table from Question 50:

Age	Number at start	enter	die	leave	Number at next age
60	0	1	0	0	1
61	2	5	0	0	7
62	7	8	0	0	15
63	15	5	3	6	11
64	11	0	0	5	6
65	5	0	2	1	2
66	2	0	0	2	0

Estimate the probability that an individual aged exactly 63 withdraws from their policy within the next year conditional on surviving to age 64.

Question 52

In a mortality study of 20 individuals in a disability income policy, an insurance company observes the following transitions.

Entry	50	50	50	50	50	50	50	50	50	50.2
State	H	H	H	H	H	H	H	H	H	H
Time						50.4	50.3	50.4	50.2	
State						D	S	S	X	
Exit	51	51	51	50.6	50.8	51	50.3	50.4	50.2	51

E	50	50.8	50.3	50.3	50.4	50.9	50	50	50	50
S	H	H	H	H	H	H	D	D	D	D
T	50.6		50.9	50.6	50.9	51			50.6	50.5
S	D		X	D	S	X			H	X
T	50.9			50.8						
S	X			H						
E	50.9	51	50.9	51	50.9	51	50.6	51	51	50.5

Estimate the transition intensities.

Reasons for Employers Offering Pensions

- Competition for new employees
- Facilitate retirement of older employees.
- Provide an incentive for employees to remain with the organisation.
- Pressure from trade unions.
- Tax efficiency
- Social Responsibility

Types of Pension Plan

Defined Contribution

- Employer contributions specified.
- Employee contributions may be permitted, and may influence employer contributions according to some formula (e.g. matching contributions)
- Contributions held in an account.
- Employee receives account upon retirement.
- Retirement benefits depend on state of the account when employees retire.
- Contributions may be designed to achieve a target level of retirement benefits. Actual benefits may be different from target benefits.

Types of Pension Plan

Defined Benefit

- Retirement benefit specified according to a formula usually based on:
  - Final or average salary
  - Years of service
- Contributions may need to be adjusted according to performance of investment and mortality experience.
- Funding is monitored on a regular basis to assess whether contributions need to be changed.

10.3 The Salary Scale Function

Estimating Future Salary

- Salary scale is given by a function s_y.
- If salary at age x is P, salary at age $y > x$ for an employee who remains employed at the company between ages x and y is $\frac{s_y}{s_x}P$.
- In practice, salary is more uncertain, but this model is widely used.
- It is important to make a distinction between salary in the year between ages x and $x+1$ and salary rate at age x. The latter is usually approximated as the salary received between age $x-0.5$ and age $x+0.5$.

Question 53

An individual aged 42 has a current salary of $60,000 (i.e. salary in the year from age 42 to 43 is $60,000). Estimate her final average salary (average over last 3 years working) assuming she retires at age 65 if:
(a) The salary scale is given by $s_y = 1.03^y$.
(b) The salary scale at integer ages is as shown in the table below:

x	s_x	x	s_x	x	s_x	x	s_x
42	1.000	49	1.391	56	1.827	63	2.335
43	1.036	50	1.424	57	1.904	64	2.400
44	1.092	51	1.470	58	1.982		
45	1.164	52	1.515	59	2.056		
46	1.228	53	1.583	60	2.120		
47	1.290	54	1.679	61	2.187		
48	1.334	55	1.748	62	2.261		

(c) What if the individual is currently aged 42 and 4 months?

Question 54

An employer sets up a DC pension plan for its employees. The target replacement ratio is 60% of final average salary for an employee who enters the plan at age exactly 30. Under the following assumptions:

- At age 65, the employee will purchase a continuous life annuity, plus a continuous reversionary annuity valued at 50% of the life annuity.
- At age 65, the employee is married to someone aged 62.
- The salary scale is $s_y = 1.03^y$.
- Mortalities are independent and given by $\mu_x = 0.000002(1.093)^x$.
- A fixed percentage of salary is payable monthly in arrear.
- Contributions earn an annual rate of return of 6%.
- The value of a life annuity is based on a rate of interest of 4%.

Calculate the percentage of salary payable monthly.

Question 55

Recall from Question 54, that the rate of contribution was 20.74%. Calculate the actual replacement ratio achieved if the following changes are made to the assumptions:

(a) At age 65, the employee is not married.

(b) At age 65, the employee's spouse is aged 73.

(c) The rate of return on contributions is 7%.

(d) Salary increases continuously at an annual rate of 5%.

(e) At age 65, the employee purchases a whole life annuity, plus a reversionary annuity for only 30% of the value.

(f) The life annuities are valued using an interest rate of 3%.

(g) The employee is in poor health at retirement, and has mortality given by $\mu_x = 0.000002(1.143)^x$. [The employee's spouse still has mortality given by $\mu_x = 0.000002(1.093)^x$.]

10.5 The Service Table

Reasons for Early Exit

- Withdrawl — Leaving to take another job (or for other reasons).
- Early retirement.
- Disability retirement.
- Death.

Question 56

For a multiple decrement model with the following states and transition intensities:

0 — Employed

1 — Withdrawn

2 — Disability retirement

3 — Age retirement

4 — Death

$$\mu_x^{(01)} = e^{-0.07x}$$

$$\mu_x^{(02)} = 0.0004$$

$$\mu_x^{(03)} = 0.08 \text{ for } 60 < x < 65$$

$$\mu_x^{(04)} = 0.000002 \times 1.102^x$$

In addition, 25% of employees who reach age 60 retire then, 30% of employees still employed at age 62 retire then, and all employees still working at age 65 retire then.

(a) Construct a service table for ages from 30 to 65.

(b) What is the probability that an employee currently aged exactly 37 retires while aged 63.

Answer to Question 56

t	$_tp^{(00)}$	1	2	3	4
0	10000.00	1182.45	4.00	0	0.39
1	8813.17	971.66	3.53	0	0.38
2	7837.61	805.68	3.14	0	0.37
3	7028.43	673.65	2.81	0	0.36
4	6351.59	567.63	2.54	0	0.36
5	5781.07	481.71	2.31	0	0.36
6	5296.68	411.51	2.12	0	0.37
7	4882.68	353.70	1.95	0	0.37
8	4526.66	305.74	1.81	0	0.38
9	4218.72	265.68	1.69	0	0.39
10	3950.97	231.99	1.58	0	0.40
11	3716.99	203.50	1.49	0	0.42
12	3511.58	179.26	1.40	0	0.44
13	3330.48	158.52	1.33	0	0.46
14	3170.18	140.69	1.27	0	0.48
15	3027.74	125.28	1.21	0	0.50
16	2900.75	111.91	1.16	0	0.53
17	2787.14	100.26	1.11	0	0.56
18	2685.21	90.06	1.07	0	0.60

t	$_tp^{(00)}$	1	2	3	4
19	2593.47	81.11	1.04	0	0.64
10	2510.69	73.21	1.00	0	0.68
21	2435.80	66.22	0.97	0	0.72
22	2367.88	60.02	0.95	0	0.78
23	2306.13	54.51	0.92	0	0.83
24	2249.87	49.58	0.90	0	0.90
25	2198.49	45.17	0.88	0	0.96
26	2151.48	41.22	0.86	0	1.04
27	2108.36	37.66	0.84	0	1.12
28	2068.73	34.46	0.83	0	1.21
29	2032.23	31.56	0.81	0	1.31
30^-	1998.54			499.64	
30	1498.90	21.70	0.60	119.91	1.07
31	1355.62	18.30	0.55	108.44	1.06
32^-	1227.26			368.18	
32	859.02	10.81	0.34	68.73	0.74
33	778.45	9.14	0.31	62.28	0.74
34	705.99	7.73	0.28	56.48	0.74
35^-	640.76		640.76		

Annual Pension Benefit

$$nS_{\mathrm{Fin}}\alpha$$

- n is the number of years of service. (Possibly capped by some upper bound).
- S_{Fin} is the final average salary.
- α is the accrual rate (usually between 0.01 and 0.02).

For an individual aged y who joined the pension at age x, the estimated benefits are often given as

$$(R-x)\hat{S}_{\mathrm{Fin}}\alpha = (y-x)\hat{S}_{\mathrm{Fin}}\alpha + (R-y)\hat{S}_{\mathrm{Fin}}\alpha$$

where R is the normal retirement age for the individual. The first term $(y-x)\hat{S}_{\mathrm{Fin}}\alpha$ is called the accrued benefit. Only accrued benefits are considered liabilities for valuation purposes.

Projected vs. Current Unit Method

- Projected Unit Method uses estimated future salary at retirement.
- Traditional or Current Unit Method uses current final average salary.

Question 57

The salary scale is $s_y = 1.04^y$. A defined benefit pension plan has $\alpha = 0.01$ and S_{Fin} is the average of the last 3 years' salary. A member's mortality follows a Gompertz model with $B = 0.0000023$, $C = 1.12$. The member is currently aged 46, has 13 years of service and the member's annual salary for the coming year is \$76,000. The interest rate is $i = 0.05$. The pension benefit is paid monthly in advance. Calculate the EPV of the accrued benefit under the assumption that:

(a) The individual retires at age 65.

(b) The individual retires at age 60.

(c) The individual's retirement happens between ages 60 and 65. The probability of retirement at 60 is 0.3. Between ages 60 and 65, $\mu_x^{(03)} = 0.15$, and there are no other decrements between these ages. [Calculate the conditional EPV conditioning on the member exiting through retirement. You may use the approximation that retirements not at an exact age happen in the middle of the year of retirement.]

Question 58

An employee aged 43 has been working for a company for 15 years. The salary scale is $s_y = 1.05^y$. The employee's salary last year was $75,000. If the employee withdraws from the pension plan, he receives a deferred pension based on accrual rate 2%, with COLA of 2% per year. He receives the pension starting from age 65 with payments monthly in advance. The individual's mortality is given by $\mu_x^{(04)} = 0.000002 \times 1.102^x$. The interest rate is $i = 0.04$.
(a) Calculate the EPV of the pension benefits if he withdraws now.
(b) Calculate the EPV of the accrued withdrawl benefits if the rate of withdrawl is $\mu_x^{(01)} = e^{-0.07x}$.

Question 59

Let the salary scale be $s_y = 1.04^y$. A pension plan has benefit defined by $\alpha = 0.015$ and S_{Fin} is the average of the last 3 years' salary. Suppose a member's mortality follows a Gompertz model with $B = 0.0000023$, $C = 1.12$. The member is currently aged 46 and has 13 years of service, and a current annual salary of \$45,000. The rate of withdrawl from the pension plan is $\mu_x^{(01)} = e^{-0.07x}$. The individual will retire at age 60 with probability 0.3; will retire at rate $\mu_x^{(03)} = 0.06$ between ages 60 and 65; and will retire at age 65 if still employed at that age. The interest rate is $i = 0.06$ while the employee is employed. Once the employee exits the plan, the benefits are calculated at an interest rate $i = 0.05$. The pension benefit is paid monthly in advance. Upon withdrawl, the employee receives a deferred pension with COLA 2%. There is no death benefit. Calculate the EPV of the accrued benefit of the employee.

Question 60

A pension plan offers a benefit of 4% of career average earnings per year of service. The benefit is payable monthly in advance. Mortality follows a Gompertz model with $B = 0.0000023$, $C = 1.12$. The salary scale is $s_y = 1.04^y$. One plan member aged 44 joined the plan 6 years ago with a starting salary rate of \$180,000. Withdrawls receive a deferred pension benefit from age 65, with COLA of 2%. The rate of withdrawl from the pension plan is $\mu_x^{(01)} = e^{-0.07x}$. The individual will retire at age 60 with probability 0.3; will retire at rate $\mu_x^{(03)} = 0.06$ between ages 60 and 65; and will retire at age 65 if still employed at that age. The interest rate is $i = 0.06$ while the employee is employed. Once the employee exits the plan, the benefits are calculated at an interest rate $i = 0.05$. There is no death benefit. Calculate the EPV of the accrued benefit.

Funding DB Pension Plans

- Employee pays fixed contribution (as percentage of salary).
- Employer pays the remaining costs of benefits.
- Employer contributions not usually specified in contract. Employer has an incentive to keep its contributions smooth and predictable.
- Employer will usually establish a reserve level equal to the EPV of accrued liabilities, called Actuarial Liability.

Normal contribution C_t at start of year satisfies

$$ {}_tV + C_t = \text{EPV of benefits for exits during the year} + (1+i)^{-1}\, {}_1p_x^{00}\, {}_{t+1}V $$

Question 61

An individual aged 45 has 26 years of service, and a last year's salary of $47,000. The salary scale is $s_y = 1.05^y$, and the accrual rate is 0.02. The interest rate is $i = 0.04$. There is no death benefit. There are no exits other than death or retirement at age 65. Mortality follows a Gompertz model with $B = 0.0000076$, $C = 1.087$. Calculate this year's employer contribution to the plan using:
(a) The Projected Unit Method.
(b) The Traditional Unit Method.

Question 62

Annual Pension benefits are 1% of final average salary over 3 years per year of service. The salary scale is $s_y = 1.06^y$. Mortality follows a Gompertz model with $B = 0.00000187$, $C = 1.130$. The rate of withdrawl is $\mu_x^{01} = 0.2e^{-0.04x}$. Withdrawl benefits take the form of a deferred pension with COLA 2%, beginning at age 65. The benefit for death while in service is 3 times the last year's annual salary. Pension benefits are guaranteed for 5 years. Interest rates are 5%. Members alive at age 60 retire then with probability 0.08. Members aged between 60 and 65 retire at a rate $\mu_x^{03} = 0.1$. Members who are still employed at age 65 all retire then. If a member aged 46 has 12 years of service and last year's salary $87,000, and makes an annual contribution of 4% of annual salary, calculate the employer's annual contribution to the pension plan on behalf of this member.

SN 5 Retiree Health Benefits

Differences Between Retiree Health and Pension Benefits

- Retiree Health Benefits are typically only offered to individuals who retire from the company.
- Usually Minimal Service Requirements
- Usually not legally binding.
- Sometimes prefunded, sometimes paid from current income.
- Health benefits are not a linear function of years of service.
- Benefits do not depend on Salary.

Remarks on Retiree Health Benefits

- Usually tops up government health-care provision.
- Usually claims are dealt with by a health insurance company. Costs to employers are the premiums.
- Premiums increase with age, and over time, usually above inflation.

SN 5 Retiree Health Benefits

Valuing Retirement Health Benefits

- Benefits are a whole-life annuity with annual payments $B(x,t)$ the premium for an individual aged x in year t.
- We will assume interest rate is i, premiums are subject to inflation j and premiums increase by a factor c with age.
- We let
  $\ddot{a}_B(x_r,t) = 1 + p_{x_r}(1+j)c(1+i)^{-1} + {}_2p_{x_r}(1+j)^2c^2(1+i)^{-2} + \cdots$
- If we define $1+i^* = \frac{1+i}{c(1+j)}$, then $\ddot{a}_B(x_r,t) = \ddot{a}_{x_r|i^*}$
- If costs increase above the interest rate, i^* will be negative.
- For an individual retiring at age x_r at time t, the EPV of the benefits is $B(x_r,t)\ddot{a}_{x_r|i^*}$.
- We can use a linear method to calculate accrued benefit.
- We calculate the total accrued benefit by taking the expectation over time of retirement (using the service table).

Question 63

Why do we calculate the accrued benefits for each retirement age, rather than calculate total benefits first then calculate accrued value?

Question 64

An individual aged 53 has 7 years of service. From the service table, this individual's probability of retiring at each age and some values of $\ddot{a}_{x_r|i^*}$ are as given in the following tables:

age	probability $\frac{r_x}{l_{53}}$
60	0.185
60–61	0.032
61–62	0.039
62–63	0.044
63–64	0.051
64–65	0.016
65	0.580

	i^*		
x_r	-0.01027	0.01027	0.02971
60	42.26993	28.73036	21.22565
60.5	41.61225	28.40999	21.05377
61.5	40.30700	27.76851	20.70700
62.5	39.02199	27.12570	20.35422
63.5	37.75674	26.48191	19.99559
64.5	36.51172	25.83743	19.63124
65	35.89451	25.51505	19.44766

We have $c = 1.02$, $j = 0.05$, $i = 0.06$. The current annual premium for an individual aged 60 is \$984. Calculate the employer's annual contribution to this individual's retirement health benefit fund.

Question 65

An insurance company sells a 10-year annual life insurance policy to a life aged 34, for whom the lifetable below is appropriate. The interest rate is $i = 0.04$. The death benefits are $180,000. The initial expenses are $300 plus 20% of the first premium. The renewal costs are 4% of each annual premium.

x	l_x	d_x
34	10000.00	3.13
35	9996.87	3.29
36	9993.58	3.47
37	9990.10	3.67
38	9986.44	3.88

x	l_x	d_x
39	9982.56	4.11
40	9978.45	4.36
41	9974.10	4.62
42	9969.47	4.92
43	9964.55	5.23

Calculate the cashflows associated with the policy if the annual premium is $90.

Answer to Question 65

t	Premium (at $t-1$)	Expenses	Interest	Expected Death Benefits	Net Cash Flow
0		160			-160.00
1	90	0.0	3.60	56.34	37.26
2	90	3.6	3.46	59.24	30.62
3	90	3.6	3.46	62.50	27.36
4	90	3.6	3.46	66.13	23.73
5	90	3.6	3.46	69.93	19.93
6	90	3.6	3.46	74.11	15.75
7	90	3.6	3.46	78.65	11.21
8	90	3.6	3.46	83.38	6.48
9	90	3.6	3.46	88.83	1.03
10	90	3.6	3.46	94.47	-4.61

Question 66

Repeat Question 65 including a reserve, where the reserve is the net premium reserve, calculated on the reserve basis $i = 0.03$, and mortality higher than in the table by a constant rate 0.004. This gives the following reserves: Premium=$767.4278.

t	${}_tV$
0	0
1	15.89511
2	29.38556
3	40.07908
4	47.51575

t	${}_tV$
5	51.39936
6	51.24223
7	46.53873
8	36.94503
9	21.56219

Answer to Question 66

t	${}_{t-1}V$	P	E_t	I	Death Benefits	${}_tVp_{34+t-1}$	Net Profit
0			160				-160.00
1	0.00	90	0.0	3.60	56.34	15.90	21.36
2	15.90	90	3.6	4.09	59.24	29.39	17.76
3	29.39	90	3.6	4.63	62.50	40.08	17.84
4	40.08	90	3.6	5.18	66.13	47.52	18.01
5	47.52	90	3.6	5.36	69.93	51.40	17.95
6	51.40	90	3.6	5.51	74.11	51.24	17.96
7	51.24	90	3.6	5.51	78.65	46.54	17.96
8	46.54	90	3.6	5.32	83.38	36.95	17.93
9	36.95	90	3.6	4.93	88.83	21.56	17.89
10	21.56	90	3.6	4.32	94.47	0.00	17.81

Profit Signatures

- The above profits can be calculated using one of the formulae:

$$\mathrm{Pr}_t = ({}_{t-1}V + P_t - E_t)(1+i) - S_t q_{x+t-1} - {}_tV p_{x+t-1}$$
$$\mathrm{Pr}_t = (P_t - E_t)(1+i) - S_t q_{x+t-1} - \Delta_t V$$

  where $\Delta_t V = {}_{t-1}V(1+i) - {}_tV p_{x+t-1}$ is the change in reserve.
- The final column Pr_t is called the profit vector of the contract. Pr_t is the expected end-of-year profit conditional on the contract still being in force at time $t-1$.
- The profit signature Π_t is the expected profit realised at time t, given by $\Pi_0 = \mathrm{Pr}_0$ and $\Pi_t = \mathrm{Pr}_t\, {}_{t-1}p_x$ for $t > 0$.
- We can then apply various profit measures to the profit signature to determine how profitable the contract is.

Question 67

Calculate the profit signatures for the contract in Question 65, both for the original case and the case (Question 66) with reserves.

Answer to Question 67

t	Without Reserves	With Reserves
0	−160.00	−160.00
1	37.26	21.36
2	30.61	17.75
3	27.34	17.83
4	23.71	17.99
5	19.90	17.93
6	15.72	17.93
7	11.19	17.92
8	6.46	17.88
9	1.03	17.84
10	−4.59	17.75

Notes on Profit Testing

- Easy to adapt this to Multiple Decrement Models.
- Profit testing is usually applied to a portfolio of policies, rather than a single policy.
- We have replaced random variables by their expected values. This is called deterministic profit testing.
- The profit signature is used to assess profitability. The profit vector is used for policies already in force.
- We will cover stochastic profit testing and profit testing for multiple-state models later.

Profit Measures

Net Present Value	Present value of profit signature at risk discount rate
Profit Margin	NPV as a proportion of EPV of premiums received
Partial NPV	$\mathrm{NPV}(t)$ is the NPV of all cash-flows up to time t
Internal Rate of Return	Interest rate at which NPV is zero
Discounted Payback Period	First time at which partial NPV is at least 0

Question 68

Calculate these profit measures for the policy in Question 65, both with and without reserves. Use risk discount rates of 1%, 5%, and 10% where appropriate. The profit signatures are recalled below:

t	Without Reserves	With Reserves
0	−160.00	−160.00
1	37.26	21.36
2	30.61	17.75
3	27.34	17.83
4	23.71	17.99
5	19.90	17.93
6	15.72	17.93
7	11.19	17.92
8	6.46	17.88
9	1.03	17.84
10	−4.59	17.75

Answer to Question 68

discount rate	Profit Measure	No Reserves	Reserves
1%	NPV	3.151168	12.69993
	Profit Margin	0.003666031	0.01477495
	Partial NPV(5)	-24.8471	-69.79779
	DPP	7 years	10 years
5%	NPV	-16.13285	-18.69238
	Profit Margin	-0.02214158	-0.02565442
	Partial NPV(5)	-38.03435	-79.3061
	DPP		
10%	NPV	-35.44164	-47.02866
	Profit Margin	-0.0583403	-0.07741364
	Partial NPV(5)	-51.73822	-89.09592
	DPP		
	IRR	1.60%	2.48%

Question 69

For the policy in Question 65, calculate the premium that achieves a risk discount rate of 10%.

Answer to Question 69

t	Premium (at $t-1$)	Expenses	Interest	Death Benefits	Net Cash Flow
0		160			-160.00
1	P	0.0	$0.04P$	56.34	$1.04P - 56.34$
2	P	$0.04P$	$0.0396P$	59.24	$0.9996P - 59.24$
3	P	$0.04P$	$0.0396P$	62.50	$0.9996P - 62.50$
4	P	$0.04P$	$0.0396P$	66.13	$0.9996P - 66.13$
5	P	$0.04P$	$0.0396P$	69.93	$0.9996P - 69.93$
6	P	$0.04P$	$0.0396P$	74.11	$0.9996P - 74.11$
7	P	$0.04P$	$0.0396P$	78.65	$0.9996P - 78.65$
8	P	$0.04P$	$0.0396P$	83.38	$0.9996P - 83.38$
9	P	$0.04P$	$0.0396P$	88.83	$0.9996P - 88.83$
10	P	$0.04P$	$0.0396P$	94.47	$0.9996P - 94.47$

Answer to Question 69

t	Pr_t	${}_{t-1}p_{34}$	Π_t	EPV at 10%
0	-160.00	1	-160.00	-160.00
1	$1.04P - 56.34$	1	$1.04P - 56.34$	$0.94545P - 51.22$
2	$0.9996P - 59.24$	0.99969	$0.99929P - 59.22$	$0.82586P - 48.94$
3	$0.9996P - 62.50$	0.99936	$0.99896P - 62.46$	$0.75053P - 46.93$
4	$0.9996P - 66.13$	0.99901	$0.99861P - 66.06$	$0.68206P - 45.12$
5	$0.9996P - 69.93$	0.99864	$0.99824P - 69.84$	$0.61983P - 43.36$
6	$0.9996P - 74.11$	0.99826	$0.99786P - 73.98$	$0.56326P - 41.76$
7	$0.9996P - 78.65$	0.99785	$0.99745P - 78.48$	$0.51185P - 40.27$
8	$0.9996P - 83.38$	0.99741	$0.99701P - 83.16$	$0.46511P - 38.80$
9	$0.9996P - 88.83$	0.99695	$0.99655P - 88.56$	$0.42263P - 37.56$
10	$0.9996P - 94.47$	0.99646	$0.99606P - 94.14$	$0.38402P - 36.29$
tot				$6.1706P - 590.25$

$$P = \$95.66$$

Question 70

Calculate the reserves for the policy in Question 65 so that no year has a negative cash flow.

12.7 Using the Profit Test to Calculate Reserves

Recall that for Question 65, we calculated the following cash-flows.

t	Premium (at $t-1$)	Expenses	Interest	Expected Death Benefits	Net Cash Flow
0		160			-160.00
1	90	0.0	3.60	56.34	37.26
2	90	3.6	3.46	59.24	30.62
3	90	3.6	3.46	62.50	27.36
4	90	3.6	3.46	66.13	23.73
5	90	3.6	3.46	69.93	19.93
6	90	3.6	3.46	74.11	15.75
7	90	3.6	3.46	78.65	11.21
8	90	3.6	3.46	83.38	6.48
9	90	3.6	3.46	88.83	1.03
10	90	3.6	3.46	94.47	-4.61

Question 71

Recall Question 6, where a life insurance company sells a 10-year term disability income policy to a life aged 37. The transition intensities are

$$\mu_x^{01} = 0.0003 + 0.000002x$$
$$\mu_x^{10} = 0.00003 + 0.000001x$$
$$\mu_x^{02} = 0.0001 + 0.000001x^2$$
$$\mu_x^{12} = 0.0002 + 0.000002x$$

Premiums are payable annually in advance while healthy. Benefits of \$80,000 per year in arrear are payable if the life is sick at the end of a given year. A death benefit of \$200,000 is payable at the end of the year of death. Suppose the initial expenses are \$200. The net annual premium for this policy using $i = 0.06$ is \$489.45. Use a profit test to calculate the reserves and the internal rate of return of the policy if the

Answer to Question 71

t	p_{37+t}^{01}	p_{37+t}^{02}	p_{37+t}^{10}	p_{37+t}^{12}
0	0.0003746	0.0015050	0.0000674	0.0002750
1	0.0003766	0.0015808	0.0000684	0.0002770
2	0.0003785	0.0016587	0.0000694	0.0002790
3	0.0003805	0.0017385	0.0000704	0.0002810
4	0.0003825	0.0018204	0.0000714	0.0002830
5	0.0003845	0.0019042	0.0000724	0.0002850
6	0.0003865	0.0019900	0.0000734	0.0002870
7	0.0003884	0.0020778	0.0000744	0.0002890
8	0.0003904	0.0021676	0.0000754	0.0002910
9	0.0003924	0.0022594	0.0000764	0.0002930

Answer to Question 71 — Profit Vector in Healthy state

t	Premium	Exp	Interest	Expected Disability Benefit	Expected Death Benefit	Net Cash Flow
0		200				−200
1	489.45		34.26	29.96666	300.9938	192.75108
2	489.45		34.26	30.12527	316.1673	177.41896
3	489.45		34.26	30.28383	331.7389	161.68881
4	489.45		34.26	30.44234	347.7084	145.56071
5	489.45		34.26	30.60081	364.0759	129.03478
6	489.45		34.26	30.75924	380.8412	112.11110
7	489.45		34.26	30.91762	398.0041	94.78978
8	489.45		34.26	31.07596	415.5646	77.07092
9	489.45		34.26	31.23425	433.5226	58.95464
10	489.45		34.26	31.39249	451.8780	40.44103

Answer to Question 71 — Profit Vector in Sick state

t	Premium	Exp	Interest	Expected Disability Benefit	Expected Death Benefit	Net Cash Flow
0		200				−200
1				79972.61	55.00074	−80027.61
2				79972.37	55.40126	−80027.77
3				79972.13	55.80181	−80027.93
4				79971.89	56.20238	−80028.09
5				79971.65	56.60299	−80028.25
6				79971.41	57.00362	−80028.41
7				79971.17	57.40429	−80028.57
8				79970.93	57.80498	−80028.73
9				79970.69	58.20571	−80028.90
10				79970.45	58.60646	−80029.06

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
Sick	2		−80027.77		
	3		−80027.93		
	4		−80028.09		
	5		−80028.25		
	6		−80028.41		
	7		−80028.57		
	8		−80028.73		
	9		−80028.90		
	10		−80029.06	0	0
Healthy	1		192.75		
	2		177.42		
	3		161.69		
	4		145.56		
	5		129.03		
	6		112.11		
	7		94.79		
	8		77.07		
	9		58.95		
	10		40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
	2		−80027.77		
	3		−80027.93		
	4		−80028.09		
	5		−80028.25		
Sick	6		−80028.41		
	7		−80028.57		
	8		−80028.73		
	9		−80028.90		
	10	74793.51	−80029.06	0	0
	1		192.75		
	2		177.42		
	3		161.69		
	4		145.56		
	5		129.03		
Healthy	6		112.11		
	7		94.79		
	8		77.07		
	9		58.95		
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
Sick	2		−80027.77		
	3		−80027.93		
	4		−80028.09		
	5		−80028.25		
	6		−80028.41		
	7		−80028.57		
	8		−80028.73		
	9		−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
Healthy	1		192.75		
	2		177.42		
	3		161.69		
	4		145.56		
	5		129.03		
	6		112.11		
	7		94.79		
	8		77.07		
	9		58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
Sick	2		−80027.77		
	3		−80027.93		
	4		−80028.09		
	5		−80028.25		
	6		−80028.41		
	7		−80028.57		
	8		−80028.73		
	9	144668.23	−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
Healthy	1		192.75		
	2		177.42		
	3		161.69		
	4		145.56		
	5		129.03		
	6		112.11		
	7		94.79		
	8		77.07		
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
Sick	2		−80027.77		
	3		−80027.93		
	4		−80028.09		
	5		−80028.25		
	6		−80028.41		
	7		−80028.57		
	8		−80028.73	0	144615.66
	9	144668.23	−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
Healthy	1		192.75		
	2		177.42		
	3		161.69		
	4		145.56		
	5		129.03		
	6		112.11		
	7		94.79		
	8		77.07	0	56.1891
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
Sick	2		-80027.77		
	3		-80027.93		
	4		-80028.09		
	5		-80028.25		
	6		-80028.41		
	7		-80028.57		
	8	209948.03	-80028.73	0	144615.66
	9	144668.23	-80028.90	0	74766.11
	10	74793.51	-80029.06	0	0
Healthy	1		192.75		
	2		177.42		
	3		161.69		
	4		145.56		
	5		129.03		
	6		112.11		
	7		94.79		
	8	0	77.07	0	56.1891
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
Sick	2		−80027.77		
	3		−80027.93		
	4		−80028.09		
	5		−80028.25		
	6		−80028.41		
	7		−80028.57	0	209872.36
	8	209948.03	−80028.73	0	144615.66
	9	144668.23	−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
Healthy	1		192.75		
	2		177.42		
	3		161.69		
	4		145.56		
	5		129.03		
	6		112.11		
	7		94.79	0	81.1449
	8	0	77.07	0	56.1891
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
	2		−80027.77		
	3		−80027.93		
	4		−80028.09		
	5		−80028.25		
Sick	6		−80028.41		
	7	270935.45	−80028.57	0	209872.36
	8	209948.03	−80028.73	0	144615.66
	9	144668.23	−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
	1		192.75		
	2		177.42		
	3		161.69		
	4		145.56		
	5		129.03		
Healthy	6		112.11		
	7	0	94.79	0	81.1449
	8	0	77.07	0	56.1891
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
Sick	2		−80027.77		
	3		−80027.93		
	4		−80028.09		
	5		−80028.25		
	6		−80028.41	0	270838.62
	7	270935.45	−80028.57	0	209872.36
	8	209948.03	−80028.73	0	144615.66
	9	144668.23	−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
Healthy	1		192.75		
	2		177.42		
	3		161.69		
	4		145.56		
	5		129.03		
	6		112.11	0	104.1747
	7	0	94.79	0	81.1449
	8	0	77.07	0	56.1891
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
	2		−80027.77		
	3		−80027.93		
	4		−80028.09		
	5		−80028.25		
Sick	6	327913.11	−80028.41	0	270838.62
	7	270935.45	−80028.57	0	209872.36
	8	209948.03	−80028.73	0	144615.66
	9	144668.23	−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
	1		192.75		
	2		177.42		
	3		161.69		
	4		145.56		
	5		129.03		
Healthy	6	0	112.11	0	104.1747
	7	0	94.79	0	81.1449
	8	0	77.07	0	56.1891
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
	2		−80027.77		
	3		−80027.93		
	4		−80028.09		
	5		−80028.25	0	327796.90
Sick	6	327913.11	−80028.41	0	270838.62
	7	270935.45	−80028.57	0	209872.36
	8	209948.03	−80028.73	0	144615.66
	9	144668.23	−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
	1		192.75		
	2		177.42		
	3		161.69		
	4		145.56		
	5		129.03	0	125.4268
Healthy	6	0	112.11	0	104.1747
	7	0	94.79	0	81.1449
	8	0	77.07	0	56.1891
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
Sick	2		−80027.77		
	3		−80027.93		
	4		−80028.09		
	5	381145.00	−80028.25	0	327796.90
	6	327913.11	−80028.41	0	270838.62
	7	270935.45	−80028.57	0	209872.36
	8	209948.03	−80028.73	0	144615.66
	9	144668.23	−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
Healthy	1		192.75		
	2		177.42		
	3		161.69		
	4		145.56		
	5	0	129.03	0	125.4268
	6	0	112.11	0	104.1747
	7	0	94.79	0	81.1449
	8	0	77.07	0	56.1891
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
	2		−80027.77		
	3		−80027.93		
	4		−80028.09	0	381011.07
	5	381145.00	−80028.25	0	327796.90
Sick	6	327913.11	−80028.41	0	270838.62
	7	270935.45	−80028.57	0	209872.36
	8	209948.03	−80028.73	0	144615.66
	9	144668.23	−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
	1		192.75		
	2		177.42		
	3		161.69		
	4		145.56	0	145.0257
	5	0	129.03	0	125.4268
Healthy	6	0	112.11	0	104.1747
	7	0	94.79	0	81.1449
	8	0	77.07	0	56.1891
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
	2		−80027.77		
	3		−80027.93		
	4	430877.72	−80028.09	0	381011.07
	5	381145.00	−80028.25	0	327796.90
Sick	6	327913.11	−80028.41	0	270838.62
	7	270935.45	−80028.57	0	209872.36
	8	209948.03	−80028.73	0	144615.66
	9	144668.23	−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
	1		192.75		
	2		177.42		
	3		161.69		
	4	0	145.56	0	145.0257
	5	0	129.03	0	125.4268
Healthy	6	0	112.11	0	104.1747
	7	0	94.79	0	81.1449
	8	0	77.07	0	56.1891
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
	2		−80027.77		
	3		−80027.93	0	430727.60
	4	430877.72	−80028.09	0	381011.07
	5	381145.00	−80028.25	0	327796.90
Sick	6	327913.11	−80028.41	0	270838.62
	7	270935.45	−80028.57	0	209872.36
	8	209948.03	−80028.73	0	144615.66
	9	144668.23	−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
	1		192.75		
	2		177.42		
	3		161.69	0	163.0872
	4	0	145.56	0	145.0257
	5	0	129.03	0	125.4268
Healthy	6	0	112.11	0	104.1747
	7	0	94.79	0	81.1449
	8	0	77.07	0	56.1891
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
Sick	2		−80027.77		
	3	477341.61	−80027.93	0	430727.60
	4	430877.72	−80028.09	0	381011.07
	5	381145.00	−80028.25	0	327796.90
	6	327913.11	−80028.41	0	270838.62
	7	270935.45	−80028.57	0	209872.36
	8	209948.03	−80028.73	0	144615.66
	9	144668.23	−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
Healthy	1		192.75		
	2		177.42		
	3	1.3069	161.69	0	163.0872
	4	0	145.56	0	145.0257
	5	0	129.03	0	125.4268
	6	0	112.11	0	104.1747
	7	0	94.79	0	81.1449
	8	0	77.07	0	56.1891
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
Sick	2		−80027.77	0.00008939	477176.74
	3	477341.61	−80027.93	0	430727.60
	4	430877.72	−80028.09	0	381011.07
	5	381145.00	−80028.25	0	327796.90
	6	327913.11	−80028.41	0	270838.62
	7	270935.45	−80028.57	0	209872.36
	8	209948.03	−80028.73	0	144615.66
	9	144668.23	−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
Healthy	1		192.75		
	2		177.42	1.3044	179.7669
	3	1.3069	161.69	0	163.0872
	4	0	145.56	0	145.0257
	5	0	129.03	0	125.4268
	6	0	112.11	0	104.1747
	7	0	94.79	0	81.1449
	8	0	77.07	0	56.1891
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
Sick	2	520751.88	−80027.77	0.00008939	477176.74
	3	477341.61	−80027.93	0	430727.60
	4	430877.72	−80028.09	0	381011.07
	5	381145.00	−80028.25	0	327796.90
	6	327913.11	−80028.41	0	270838.62
	7	270935.45	−80028.57	0	209872.36
	8	209948.03	−80028.73	0	144615.66
	9	144668.23	−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
Healthy	1		192.75		
	2	3.4133	177.42	1.3044	179.7669
	3	1.3069	161.69	0	163.0872
	4	0	145.56	0	145.0257
	5	0	129.03	0	125.4268
	6	0	112.11	0	104.1747
	7	0	94.79	0	81.1449
	8	0	77.07	0	56.1891
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
Sick	2	520751.88	−80027.77	0.00008939	477176.74
	3	477341.61	−80027.93	0	430727.60
	4	430877.72	−80028.09	0	381011.07
	5	381145.00	−80028.25	0	327796.90
	6	327913.11	−80028.41	0	270838.62
	7	270935.45	−80028.57	0	209872.36
	8	209948.03	−80028.73	0	144615.66
	9	144668.23	−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
Healthy	1		192.75	3.4069	195.0737
	2	3.4133	177.42	1.3044	179.7669
	3	1.3069	161.69	0	163.0872
	4	0	145.56	0	145.0257
	5	0	129.03	0	125.4268
	6	0	112.11	0	104.1747
	7	0	94.79	0	81.1449
	8	0	77.07	0	56.1891
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves

	t	Reserve	Cash Flow	Exp. Healthy Res.	Exp. Sick Res.
Sick	2	520751.88	−80027.77	0.00008939	477176.74
	3	477341.61	−80027.93	0	430727.60
	4	430877.72	−80028.09	0	381011.07
	5	381145.00	−80028.25	0	327796.90
	6	327913.11	−80028.41	0	270838.62
	7	270935.45	−80028.57	0	209872.36
	8	209948.03	−80028.73	0	144615.66
	9	144668.23	−80028.90	0	74766.11
	10	74793.51	−80029.06	0	0
Healthy	1	5.3547	192.75	3.4069	195.0737
	2	3.4133	177.42	1.3044	179.7669
	3	1.3069	161.69	0	163.0872
	4	0	145.56	0	145.0257
	5	0	129.03	0	125.4268
	6	0	112.11	0	104.1747
	7	0	94.79	0	81.1449
	8	0	77.07	0	56.1891
	9	0	58.95	0	29.1994
	10	0	40.44	0	0

Answer to Question 71 — Reserves in Healthy state

t	Res.	Prem.	Int.	Exp. Dis. Ben.	Exp. Death Ben.	Exp. Res. Sick	Exp. Res.	Net CF
1	5.35	489.45	34.64	29.97	300.99	195.07	3.41	0
2	3.41	489.45	34.50	30.13	316.17	179.77	1.30	0
3	1.31	489.45	34.35	30.28	331.74	163.09		0
4		489.45	34.26	30.44	347.71	145.03		0.53
5		489.45	34.26	30.60	364.08	125.47		3.56
6		489.45	34.26	30.76	380.84	104.17		7.94
7		489.45	34.26	30.92	398.00	81.14		13.65
8		489.45	34.26	31.08	415.56	56.19		20.88
9		489.45	34.26	31.23	433.52	29.20		29.76
10		489.45	34.26	31.39	451.88			40.44

Answer to Question 71 — Profit Signature

t	Pr_t (Healthy)	Prob Healthy	Pr_t Sick	Prob sick	Π_t
0	−225.35	1	0	0	−225.35
1	0	1	0	0	0
2	0	0.99812	0	0.000375	0
3	0	0.99617	0	0.000750	0
4	0.53	0.99414	0	0.001127	0.53
5	3.56	0.99203	0	0.001505	3.53
6	7.94	0.98985	0	0.001884	7.86
7	13.65	0.98758	0	0.002263	13.48
8	20.88	0.98523	0	0.002644	20.57
9	29.76	0.98280	0	0.003025	29.25
10	40.44	0.98029	0	0.003407	39.64