MATH/STAT 4720, Life Contingencies II

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Formula Sheet

General Mathematics

- Quadratic Formula: Solution to $ax^2+bx+c=0$ is $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
- Gamma function: $\Gamma(\alpha)=\int_0^\infty x^\alpha e^{-x}dx$ satisfies $\Gamma(\alpha+1)=\alpha\Gamma(\alpha)$.

Non-parametric Estimators

Greenwood's formula

$$\mathrm{Var}(S_n(y_j))\approx S_n(y_j)^2\sum_{i=1}^{j}\frac{s_i}{r_i(r_i-s_i)}$$

where

- y_i is the ith observed data point in increasing order.
- s_i is the frequency of the observation y_i
- r_i is the size of the risk set at observation y_i.

Log-transformed Confidence intervals

$[S_n(x)^{\frac{1}{U}}, S_n(X)^U]$, where $U=e^{\Phi^{-1}\left(\frac{\alpha}{2}\right)\frac{\sigma}{S_n(x)\log(S_n(x))}}$.

- α is the confidence level (so for a 95% confidence interval, $\alpha=0.05$).
- σ is the standard deviation of the estimator $S_n(x)$.

Lifetables

Survival Probability of an Individual whose Spouse Dies

Probability of an individual surviving the year their spouse dies at a time unifomly distributed throughout the year.

$$(1-q_d)\left(\frac{q_a}{q_d}+\left(\frac{q_a-q_d}{{q_d}^2}\right)\log(1-q_d)\right)$$

- q_a is the probability of dying in the year if the spouse is alive for the whole year
- q_d is the probability of dying if the spouse is dead for the whole year.

Relation Between Multiple and Single Decrement Tables

We use

- p_x^{0i} is the probability that a life aged x who starts the year in state 0 ends in state i under the multiple decrement model
- $q_x^{(i)}$ is the probability of the ith decrement happening to a life aged x within a year, under a single decrement model.

UDD in the Individual Decrements

$$p_x^{00}=\prod(1-q_x^{(i)})$$
$$p_x^{0i}=q_x^{(i)}\int_0^1\prod_{j\neq i}(1-tq^{(j)})\,dt$$

For the two-decrement case:

$$p_x^{00}=\prod(1-q_x^{(i)})$$
$$p_x^{01}=q_x^{(1)}\left(1-\frac{q^{(2)}}{2}\right)$$
$$p_x^{02}=q_x^{(2)}\left(1-\frac{q^{(1)}}{2}\right)$$

UDD in the Multiple Decrement Table

$$p_x^{00} = \prod (1 - q_x^{(i)})$$
$$q_x^{(i)} = 1 - \left(p_x^{00}\right)^{\frac{p^{0i}}{\sum_{j \neq 0} p^{0j}}}$$

Stochastic Mortality Improvement Models

Lee-Carter Model

$$\log(m(x,t)) = \alpha_x + \beta_x K_t$$

where

- $m(x,t) = \frac{q(x,t)}{\int_0^1 {}_tp_x \, dt}$. Under UDD this gives $m(x,t) = \frac{q(x,t)}{1 - \frac{q(x,t)}{2}}$.
- K_t is given by the stochastic process $K_t = K_{t-1} + c + \sigma_k Z_t$.
- Z_t are independant standard normal distributions.

Cairns-Blake-Dowd (CBD) Model

$$\log\left(\frac{q(x,t)}{1 - q(x,t)}\right) = K_t^{(1)} + K_t^{(2)}(x - \overline{x})$$

where

- $K_t^{(i)}$ is given by the stochastic process $K_t^{(i)} = K_{t-1}^{(i)} + c^{(i)} + \sigma_{k_i} Z_t^{(i)}$.
- $(Z_t^{(1)}, Z_t^{(2)})$ are independant samples from a multivariate normal distribution with $\text{Var}(Z_t^{(i)}) = 1$ and $\text{Cov}(Z_t^{(1)}, Z_t^{(2)}) = \rho$.

Financial Mathematics

- Accumulated value of an annuity with n terms where payments grow at rate r, and interest is applied at rate i is
$$\frac{(1+i)^n - (1+r)^n}{i - r}$$
- Profit margin $= \frac{\text{EPV of net cash flows}}{\text{EPV of premiums received}}$