ACSC/STAT 4720, Life Contingencies II
FALL 2021
Toby Kenney
Sample Final Examination

This Sample examination has more questions than the actual final, in order to cover a wider range of questions. Estimated times are provided after each question to help your preparation.

1. A life aged 38 wants to buy a 3-year term insurance policy. A life-table based on current-year mortality is:

x	l_x	d_x
38	10000.00	5.00
39	9995.00	5.14
40	9989.86	5.30
41	9984.56	5.47
42	9979.09	5.67
43	9973.42	5.87

The insurance company uses a single-factor scale function $q(x,t) = q(x,0)(1-\phi_x)^t$ to model changes in mortality. The insurance company uses the following values for ϕ_x:

x	ϕ_x
38	0.03
39	0.025
40	0.025
41	0.02
42	0.015
43	0.02

Calculate $A^{1}_{38:\overline{3}|}$ at interest rate $i = 0.06$, taking into account the change in mortality.

2. The following lifetable applied in 2016:

x	l_x	d_x
55	10000.00	10.63
56	9989.37	11.30
57	9978.07	12.02
58	9966.05	12.80
59	9953.25	13.66
60	9939.59	14.60

An insurance company uses the following mortality scale based on both age and year:

	t					
x	2017	2018	2019	2020	2021	2022
55	0.01	0.015	0.015	0.02	0.02	0.015
56	0.03	0.03	0.025	0.02	0.015	0.02
57	0.02	0.03	0.03	0.025	0.02	0.015
58	0.025	0.03	0.025	0.015	0.015	0.02
59	0.015	0.02	0.015	0.01	0.015	0.01
60	0.02	0.015	0.01	0.015	0.02	0.025

Use this mortality scale to calculate $A^{1}_{55:\overline{4}|}$ at interest rate $i = 0.03$.

3. A pensions company has the current mortality scale for 2017:

x	$\phi(x, 2017)$	$\frac{d\phi(x,t)}{dt}\Big\vert_{x,t=2017}$	$\frac{d\phi(x+t,t)}{dt}\Big\vert_{x,t=2017}$
51	0.016389776	0.00054272913	−0.0015000971
52	0.018738397	−0.00107674028	0.0012410504
53	0.028229446	0.00120650853	−0.0002976607
54	0.028011768	−0.00109930339	−0.0004183465
55	0.014334489	−0.00194027424	0.0023952205
56	0.016770205	0.00271342277	−0.0053102487

Mortality in 2016 is given in the following lifetable.

x	l_x	d_x
51	10000.00	15.29
52	9984.71	16.44
53	9968.27	17.70
54	9950.56	19.09
55	9931.48	20.60
56	9910.88	22.26

The company assumes that from 2030 onwards, we will have $\phi(x, t) = 0.01$ for all x and t. Calculate $q(54, 2018)$ using the average of age-based and cohort-based effects.

4. An insurance company uses a Lee-Carter model and fits the following parameters:

$$c = -0.6 \qquad \sigma_k = 1.4 \qquad K_{2017} = -4.83$$

And the following values of α_x and β_x:

x	α_x	β_x
34	−5.314675	0.2697754
35	−5.234098	0.2504377
36	−5.043921	0.1782635
37	−4.892803	0.2889967
38	−4.637988	0.1460634
39	−4.413315	0.1174245
40	−4.261060	0.2078267

The insurance company simulates the following values of Z_t:

t	Z_t
2018	0.2525295
2019	−0.6276655
2020	−0.6007807

Using these simulated values, calculate the probability that a life aged exactly 36 at the start of 2017 dies within the next 4 years.

5. An insurance company uses a Lee-Carter model. One actuary fits the following parameters:

$$c = -0.13 \qquad \sigma_k = 0.9 \qquad K_{2017} = -1.70 \qquad \alpha_{52} = -4.45 \qquad \beta_{52} = 0.49$$

A second actuary fits the parameters

$$c = -0.14 \qquad \sigma_k = 0.8 \qquad K_{2017} = -1.40 \qquad \alpha_{52} = -4.94 \qquad \beta_{52} = 0.37$$

The insurance company sets its life insurance premiums for 2025 so that under the first actuary's model, it has a 95% chance of an expected profit. What is the probability that these premiums lead to an expected profit under the second actuary's model?

6. An insurance company uses a Cairns-Blake-Dowd model with the following parameters:

$$\begin{array}{cccc} K^{(1)}_{2017} = -3.29 & K^{(2)}_{2017} = 0.38 & c^{(1)} = -0.17 & c^{(2)} = 0.01 \\ \sigma_{k_1} = 0.5 & \sigma_{k_2} = 0.08 & \rho = 0.3 & \overline{x} = 47 \end{array}$$

What is the probability that the mortality for an individual currently (in 2017) aged 39 will be higher in 2025 than in 2030?

7. An individual aged 42 has a current salary of \$76,000 for the coming year. The salary scale is $s_y = 1.05^y$. Estimate the individual's final average salary (average of last 3 years working) assuming the individual retires at exact age 65.

8. An employer sets up a DC pension plan for its employees. The target replacement ratio is 60% of final average salary for an employee who enters the plan at exact age 30, with the following assumptions:

   - At age 65, the employee will purchase a continuous life annuity, plus a continuous reversionary annuity for the employee's spouse, valued at 60% of the life annuity.
   - At age 65, the employee is married to someone aged 63.
   - The salary scale is $s_y = 1.04^y$.
   - Mortalities are independent and given by $\mu_x = 0.0000016(1.092)^x$. The value of the life annuity is based on $\delta = 0.045$. This gives $\overline{a}_{65} = 19.63036$, $\overline{a}_{63} = 19.83656$ and $\overline{a}_{65,63} = 18.7867$.
   - A fixed percentage of salary is payable annually in arrear.
   - Contributions earn an annual rate of 7%.

   Calculate the percentage of salary payable annually to achieve the target replacement rate under these assumptions.

9. The salary scale is given in the following table:

y	s_y	y	s_y	y	s_y	y	s_y
30	1.000000	39	1.350398	48	1.845766	57	2.553877
31	1.033333	40	1.397268	49	1.912422	58	2.649694
32	1.067933	41	1.445983	50	1.981785	59	2.749515
33	1.103853	42	1.496620	51	2.053975	60	2.853522
34	1.141149	43	1.549263	52	2.129115	61	2.961903
35	1.179879	44	1.604000	53	2.207337	62	3.074855
36	1.220103	45	1.660921	54	2.288777	63	3.192585
37	1.261887	46	1.720122	55	2.373580	64	3.315310
38	1.305295	47	1.781702	56	2.461894	65	3.443256

An employee aged 42 and 4 months has 12 years of service, and a current salary of \$106,000 (for the coming year). She has a defined benefit pension plan with $\alpha = 0.02$ and S_{Fin} is the average of her last 3 years' salary. The employee's mortality is given by $\mu_x = 0.00000195(1.102)^x$. The pension benefit is payable monthly in advance. The interest rate is $i = 0.05$. This results in $\ddot{a}_{65}^{(12)} = 17.15373$ and ${}_{22.66666667}p_{42.33333333} = 0.9901951$. There is no death benefit, and there are no exits other than death or retirement at age 65.

(a) Calculate the EPV of the accrued benefit using the projected unit method under the assumption that the employee retires at age 65. [Calculate the salary scale at non-integer ages by linear interpolation.]

(b) Calculate the employer's contribution for this employee for the year. [${}_{21.66666667}p_{43.33333333} = 0.9903189$.]

10. The service table is given below:

x	l_x	1	2	3
40	10000.00	118.76	0	0.51
41	9880.73	112.29	0	0.58
42	9767.86	107.16	0	0.65
43	9660.05	101.84	0	0.73
44	9557.49	96.80	0	0.82
45	9459.86	92.02	0	0.93
46	9366.91	87.50	0	1.04
47	9278.37	83.19	0	1.18
48	9193.99	80.11	0	1.32
49	9112.57	75.21	0	1.49
50	9035.87	71.48	0	1.68
51	8962.71	67.92	0	1.89
52	8892.90	64.51	0	2.12
53	8826.26	61.23	0	2.39
54	8762.64	58.07	0	2.69
55	8701.88	55.03	0	3.03
56	8643.83	52.06	0	3.41
57	8588.36	49.18	0	3.84
58	8535.34	46.37	0	4.32
59	8484.64	43.62	0	4.86
60^-	8484.64		1098.84	
60	7385.80	21.70	819.91	5.79
61	6538.40	18.30	611.98	6.38
62	5901.74	10.81	384.29	5.86
63	5500.78	9.14	639.20	6.15
64	4846.29	7.73	351.32	6.10
65^-	4481.14		4481.14	

The salary scale is $s_y = 1.05^y$. The accrual rate is 0.02. The benefit for employees who withdraw is a deferred annual pension with COLA 2%, starting from age 65. For an individual aged 65, we have $\ddot{a}_{65} = 12.85$. The interest rate is $i = 0.04$. The lifetable for an individual who has withdrawn is

x	l_x	d_x
57	10000.00	7.54
58	9992.46	8.22
59	9984.24	8.95
60	9975.29	9.76
61	9965.52	10.65
62	9954.87	11.63
63	9943.25	12.69
64	9930.55	13.86
65	9916.69	15.15

Calculate the EPV of deferred pension benefits made to an individual aged exactly 57, with 16 years of service, whose salary for the past year was $121,000.

11. An insurance company sells a 5-year annual life insurance policy to a life aged 53, for whom the lifetable below is appropriate.

x	l_x	d_x
53	10000.00	49.24
54	9950.76	54.62
55	9896.14	60.60
56	9835.55	67.22
57	9768.32	74.56
58	9693.76	82.68

The annual gross premium is \$685. Initial expenses are \$400. The death benefits are \$90,000. Renewal costs are 2% of each subsequent premium. The interest rate is $i = 0.05$

(a) Calculate the profit vector for the policy.

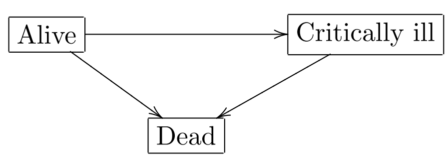
(b) Calculate the discounted payback period of the policy using a risk discount rate $i = 0.07$.

12. An insurance company sells a 5-year endowment insurance policy to a life aged 35 for whom the lifetable below is appropriate.

x	l_x	d_x
35	10000.00	8.74
36	9991.26	9.45
37	9981.81	10.24
38	9971.57	11.12
39	9960.45	12.11
40	9948.35	13.22

The benefit is \$300,000. The annual premium is \$60,000, and the interest rate is $i = 0.03$. Initial expenses are \$2,400 and renewal expenses are \$80 at the start of each year after the first. Use a profit test to calculate the reserves at the start of each year. There are no exits other than death or maturity.

13. An insurance company offers a 5-year critical illness insurance policy. The policy has 3 states — alive, critically ill, and dead. The possible transitions are as shown in the following diagram:



Premiums are payable at the start of each year while in the alive state.

For a life aged 37, transitions are as shown in the following lifetable:

age	Alive	Critically Ill	Death (direct)	Death (critically ill)	CI and Death
37	10000.00	0.00	6.95	0.00	0.03
38	9990.20	2.82	7.47	0.03	0.04
39	9979.47	6.01	8.03	0.08	0.03
40	9967.71	9.63	8.66	0.15	0.04
41	9954.78	13.75	9.36	0.22	0.03

At the end of 5 years, the expected number of lives who are critically ill is 18.42.

Initial expenses are 28% of the first premium, and renewal expenses are 4% of subsequent premiums while the life is in the alive state. There are also renewal expenses of \$80 at the start of each year if the life is in the critically ill state. Premiums are payable at the start of each year when the life is in the healthy state.

There is a death benfit of \$250,000 at the end of the year in which the life dies, and a benefit of \$100,000 at the end of the year in which the life becomes critically ill. (If the life becomes critically ill and then dies later in the same year, both benefits are payable at the end of the year.) The interest rate is $i = 0.04$. Use a profit test without reserves to determine the premium for this policy which achieves a profit margin of 5% at a risk discount rate of $i = 0.10$.

14. A couple purchase a 5-year last survivor insurance policy. Annual Premiums of \$49,830 are payable while both are alive. If one life is dead, there are no premiums or benefits. If both lives die within the 5-year period, a benefit of \$1,000,000 is payable. The husband is 74 and the wife is 81. Their lifetables are given below. Assume both lives are independent.

x	l_x	d_x
74	10000.00	591.85
75	9408.15	628.62
76	8779.53	662.27
77	8117.26	691.27
78	7425.99	713.96
79	6712.03	728.54

x	l_x	d_x
81	10000.00	1113.81
82	8886.19	1114.43
83	7771.76	1097.45
84	6674.31	1061.21
85	5613.10	1004.92
86	4608.18	928.94

Initial expenses are \$3,000, and renewal expenses are \$80 at the start of each subsequent year while both are alive, and \$60 at the start of each year while only one is alive. The interest rate is $i = 0.04$.

An actuary computes the following profit tests without reserves in each state.

both alive:

t	Premium (at $t-1$)	Expenses	Interest	Expected Death Benefits	Net Cash Flow
0		3000.00			−3000.00
1	49830	0	1993.20	6592.08	45231.12
2	49830	80	1990.00	8379.56	43360.44
3	49830	80	1990.00	10651.95	41088.05
4	49830	80	1990.00	13540.45	38199.55
5	49830	80	1990.00	17212.67	34527.33

husband alive wife dead:

t	Premium (at $t-1$)	Expenses	Interest	Expected Death Benefits	Net Cash Flow
0		3000.00			−3000.00
1	0	0	0	59185.00	−59185.00
2	0	60	−2.40	66816.54	−66878.94
3	0	60	−2.40	75433.42	−75495.82
4	0	60	−2.40	85160.51	−85222.91
5	0	60	−2.40	96143.41	−96205.81

wife alive husband dead:

t	Premium (at $t-1$)	Expenses	Interest	Expected Death Benefits	Net Cash Flow
0		3000.00			-3000.00
1	0	0	0	111381.00	-111381.00
2	0	60	-2.40	125411.45	-125473.85
3	0	60	-2.40	141209.97	-141272.37
4	0	60	-2.40	158999.21	-159061.61
5	0	60	-2.40	179031.19	-179093.59

a) Calculate the reserves in the husband alive, wife dead state and in the husband dead, wife alive state.

b) Perform a new profit test with these reserves and use it to calculate the reserves in the both alive state.

c) Calculate the profit signature of the policy.

15. An insurance company collects the following data in a mortality study

i	d_i	x_i	u_i	i	d_i	x_i	u_i	i	d_i	x_i	u_i
1	68.0	68.1	-	8	68.8	68.9	-	15	69.4	-	69.6
2	68.0	71.8	-	9	68.8	-	69.0	16	69.6	71.2	-
3	68.0	-	68.3	10	69.0	69.0	-	17	70.1	-	72.9
4	68.0	-	69.5	11	69.1	69.2	-	18	70.5	70.5	-
5	68.0	75.7	-	12	69.2	69.3	-	19	70.6	-	70.6
6	68.4	68.6	-	13	69.3	69.4	-	20	70.7	70.9	-
7	68.6	68.7	-	14	69.4	69.5	-	21	70.7	-	71.1

Using a Nelson-Åalen estimator:

(a) estimate the probability that an individual aged 68 survives to age 71.

(b) Find a 95% log-transformed confidence interval for ${}_3p_{68}$

16. An insurance company collects the following data in a mortality study

i	y_i	s_i	r_i	i	y_i	s_i	r_i	i	y_i	s_i	r_i
1	42.1	4	103	7	42.8	4	97	13	43.5	9	91
2	42.2	8	85	8	42.9	8	96	14	43.6	3	70
3	42.3	4	105	9	43.1	6	72	15	43.7	7	105
4	42.5	4	99	10	43.2	4	79	16	43.8	9	98
5	42.6	6	95	11	43.3	8	102	17	43.9	7	88
6	42.7	3	88	12	43.4	4	88				

(a) Use a Kaplan-Meier product-limit estimator to estimate p_{42}.

(b) Find a 95% log-transformed confidence interval for p_{42}, using Greenwood's approximation to estimate the variance.

17. Using the following table:

Age	No. at start	enter	die	leave	No. at next age
65	0	46	8	15	23
66	23	38	14	20	27
67	27	53	22	27	31
68	31	44	22	18	35
69	35	38	28	24	21
70	21	32	27	26	0

Assume that events are uniformly distributed over the year. Estimate the probability ${}_2p_{67}$ of a life aged 67 surviving for 2 years using

(a) Exact exposure.

(b) Actuarial exposure.