ACSC/STAT 4720, Life Contingencies II

FALL 2021

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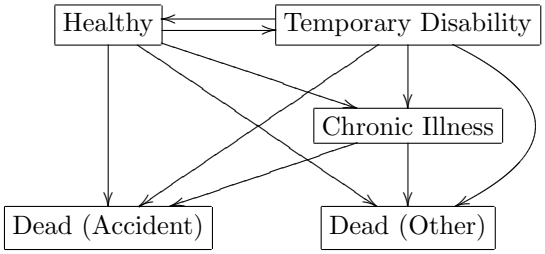
Homework Sheet 1

Due: Tuesday 28th September: 14:30

Basic Questions

1. An insurance company provides temporary disability insurance, chronic illness insurance and life insurance. It also pays an additional benefit for accidental death. It has the following model of the states:



Which of the following sequences of transitions are possible? (Indicate which parts of the transition sequence are not possible if the sequence is not possible.)

(i) Healthy–Temporary Disability – Dead (Accident)

(ii) Healthy–Temporary Disability – Healthy – Chronic Illness

(iii) Temporary Disability – Healthy – Dead (Other) – Chronic Illness

(iv) Temporary Disability – Healthy – Temporary Disability – Dead (Other)

(v) Healthy–Chronic Illness–Temporary Disability –Healthy

2. Consider a critical illness model with transition intensities

$$\begin{aligned}\mu_x^{01} &= 0.001 + 0.000001x\\ \mu_x^{02} &= 0.002 + 0.000006x\\ \mu_x^{12} &= 0.08 + 0.0002x\end{aligned}$$

where State 0 is healthy, State 1 is critically ill and State 2 is dead.

(a) Calculate the probability that a healthy individual aged 43 is still healthy at age 61.

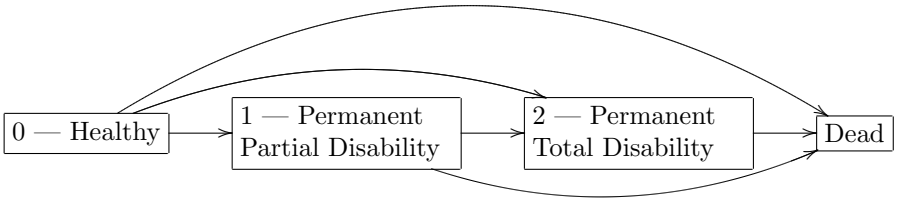
(b) Calculate the probability that a healthy individual aged 38 is dead by age 64.

3. Under a disability income model with transition intensities

$$\begin{aligned}\mu_x^{01} &= 0.001\\ \mu_x^{10} &= 0.016\\ \mu_x^{02} &= 0.003\\ \mu_x^{12} &= 0.009\end{aligned}$$

calculate the probability that a healthy individual is healthy 7 years later and has not had more than one period of disability during the 7 years. [State 0 is healthy, State 1 is sick and State 2 is dead.]

4. A permanent disability model has the following state diagram:



The transition intensities at age x are given by:

$$\begin{aligned}\mu_x^{01} &= 0.001\\ \mu_x^{02} &= 0.002\\ \mu_x^{03} &= 0.003 + 0.000006x\\ \mu_x^{12} &= 0.07\\ \mu_x^{13} &= 0.06\\ \mu_x^{23} &= 0.14 + 0.005x\end{aligned}$$

calculate the premium for a whole life policy sold to a life aged 42 with premiums payable continuously while the life is in the healthy state, which pays a benefit of \$110,000 upon entry into State 1 and a benefit of \$290,000 upon entry into State 2. The policy is sold to a life in the healthy state (State 0). The interest rate is $\delta = 0.05$

5. An insurer offers a life insurance policy with an additional benefit for accidental death. The possible exits from this policy are surrender, death (accident) and death (other). The transition intensities are

$$\mu_x^{01} = 0.06 - 0.001x$$
$$\mu_x^{02} = 0.007 - 0.00004x$$
$$\mu_x^{03} = 0.003 + 0.00008x$$

Calculate the probability that an individual aged 28 surrenders the policy before age 44. [State 0 is in force, State 1 is surrender, State 2 is death (accident) and State 3 is death (other).]

Standard Questions

6. An insurance company is developing a new model for transition intensities in a disability income model. Under these transition intensities it calculates

$$\overline{A}_{47}^{02} = 0.502563 \qquad \overline{A}_{65}^{02} = 0.749001 \qquad \overline{a}_{65}^{11} = 6.318455$$
$$\overline{a}_{47}^{00} = 13.204035 \qquad \overline{a}_{65}^{00} = 8.405827 \qquad \overline{a}_{65}^{10} = 1.482474$$
$${}_{18}p_{47}^{00} = 0.803335 \qquad {}_{18}p_{47}^{01} = 0.035908 \qquad \delta = 0.03$$

Calculate the premium for an 18-year policy for a life aged 47, with continuous premiums payable while in the healthy, which pays a continuous benefit while in the sick state, at a rate of \$80,000 per year, and pays a death benefit of \$500,000 immediately upon death.

[Hint: to calculate $\overline{a}_x^{01}$, consider how to extend the equation $\overline{a}_x = \frac{1-\overline{A}_x}{\delta}$ to the multiple state case by combining states 0 and 1.]