ACSC/STAT 4720, Life Contingencies II

FALL 2021

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Homework Sheet 5

Due: Thursday 28th October: 14:30

Basic Questions

1. A disability income insurance company collects the following claim data (in thousands):

i	d_i	x_i	u_i	i	d_i	x_i	u_i	i	d_i	x_i	u_i
1	0.0	1.8	-	8	0.0	-	10	15	0.2	2.0	-
2	0.0	2.1	-	9	0.1	1.2	-	16	0.2	2.2	-
3	0.0	2.9	-	10	0.1	2.6	-	17	0.4	3.3	-
4	0.0	3.4	-	11	0.1	3.2	-	18	0.8	-	5
5	0.0	4.0	-	12	0.1	3.5	-	19	0.9	-	10
6	0.0	-	5	13	0.1	4.6	-	20	1.4	7.9	-
7	0.0	-	10	14	0.1	8.3	-	21	1.9	-	5

   Using a Kaplan-Meier product-limit estimator:

   (a) estimate the probability that a random loss exceeds 2.7.

   (b) estimate the 60th percentile of the distribution.

   (c) Use a Nelson-Åalen estimator to estimate the 60th percentile of the distribution.

2. For the data in Question 1, use Greenwood's approximation to obtain a 95% confidence interval for the probability that a random loss exceeds 2.7, based on the Kaplan-Meier estimator.

   (a) Using a normal approximation

   (b) Using a log-transformed confidence interval.

3. An insurance company records the following data in a mortality study:

entry	death	exit	entry	death	exit	entry	death	exit
66.9	75.1	-	72.2	81.1	-	74.1	-	74.7
72.5	-	74.1	73.3	-	80.3	69.3	75.8	-
73.7	-	76.2	72.2	74.3	-	74.1	-	75.6
72.0	-	75.6	72.5	74.6	-	74.5	-	80.3
70.5	-	84.6	73.7	-	78.9	74.2	-	74.5
74.7	-	75.0	74.1	76.7	-	72.9	81.8	-
67.7	-	76.3	71.9	-	74.4	74.6	74.9	-
71.6	76.6	-	74.6	-	85.1	73.4	-	75.2
72.1	-	74.7	74.2	79.7	-	74.8	-	75.5
71.8	-	74.1	74.9	-	76.4	66.8	-	74.8
71.4	76.2	-	68.0	-	74.3	74.8	-	82.9
69.9	-	76.0	73.4	-	74.8	65.6	76.4	-

Estimate the probability of an individual currently aged exactly 74 dying within the next year using:

(a) the exact exposure method.

(b) the actuarial exposure method.

4. Using the following table:

Age	No. at start	enter	die	leave	No. at next age
82	38	41	8	24	47
83	47	20	10	24	33
84	33	17	11	18	21
85	21	11	10	14	8
86	8	9	8	6	3

Estimate the probability that an individual aged 83 withdraws from the policy within the next year, conditional on surviving to the end of the year.

5. In a mortality study of 40 individuals in a disability income policy, an insurance company observes the following transitions, where state H is healthy, D is disabled, S is surrendered and X is dead.

Entry	State	Time	State	Time	State	Exit	Entry	State	Time	State	Exit		
46.0	D	46.3	S			46.3	46.0	D	46.1	S			46.1
46.0	H					47.0	46.0	D					47.0
46.0	H					47.0	46.0	H					47.0
46.0	H	46.5	D	47.0	H	47.0	46.0	D	46.3	X			47.0
46.0	H	46.2	D			47.0	46.2	H	46.5	D	46.6	X	46.6
46.0	H	46.7	X			46.7	46.3	H					47.0
46.0	H					47.0	46.5	H					47.0
46.0	H					47.0	46.5	H	46.9	D			47.0
46.0	H					47.0	46.5	D					47.0
46.0	H	46.2	S			46.2	46.6	H	47.0	D			47.0
46.0	H					47.0	46.7	D					47.0
46.0	H					47.0	46.8	H					47.0
46.0	H	46.8	S			46.8	46.8	H					47.0
46.0	H	46.6	D			47.0	46.8	D					47.0
46.0	D	46.6	S			46.6	46.8	H	46.8	D			47.0
46.0	H	46.2	S			46.2	46.9	D	46.9	S			46.9
46.0	D					47.0	46.9	H					47.0
46.0	H					47.0	46.9	H					47.0
46.0	D					47.0	47.0	D	47.0	H			47.0
46.0	D	46.3	S			46.3	47.0	H					47.0

Based on these data, estimate the probability that an individual aged 46.4 who is disabled becomes healthy and later dies before reaching age 47.

Standard Questions

6. For the study in Question 3, use the exact exposure method, and assume that the number of deaths follows a negative binomial distribution with $r = 1$ and β equal to the exposure multiplied by probability of dying, to find a 95% confidence interval for q_{74}.