

ACSC/STAT 4720, Life Contingencies II

FALL 2021

Toby Kenney

Homework Sheet 6

Due: Thursday 25th November: 14:30

Basic Questions

1. An individual aged 39 has a current salary of \$71,000. The salary scale is $s_y = 1.05^y$. Estimate the individual's final average salary (average of last 3 years working) assuming the individual retires at exact age 65.
2. An employer sets up a DC pension plan for its employees. The target replacement ratio is 65% of final average salary for an employee who enters the plan at exact age 30, with the following assumptions:
 - At age 65, the employee will purchase a continuous life annuity, plus a continuous reversionary annuity for the employee's spouse, valued at 70% of the life annuity.
 - At age 65, the employee is married to someone aged 66.
 - The salary scale is $s_y = 1.03^y$.
 - Mortalities are independent and given by $\mu_x = 0.0000016(1.113)^x$.
 - A fixed percentage of salary is payable monthly in arrear.
 - Contributions earn an annual rate of 5%.
 - The value of the life annuity is based on $\delta = 0.045$.

Calculate the percentage of salary payable monthly to achieve the target replacement rate under these assumptions. [You may use numerical integration to compute the value of the annuities.]

3. The salary scale is given in the following table:

y	s_y	y	s_y	y	s_y	y	s_y
31	1.0000000000	40	1.46598394694	49	2.15275075482	58	3.16673115821
32	1.04333333333	41	1.52979439887	50	2.24688284912	59	3.30584525894
33	1.08856666667	42	1.59641582358	51	2.34518133040	60	3.45114616643
34	1.13578433333	43	1.66597358772	52	2.44783290930	61	3.60291246507
35	1.18507445667	44	1.73859871898	53	2.55503276764	62	3.76143543826
36	1.23652912243	45	1.81442816488	54	2.66698494742	63	3.92701965410
37	1.29024455921	46	1.89360506348	55	2.78390275783	64	4.09998357849
38	1.34632132704	47	1.97627902661	56	2.90600920129	65	4.28066021677
39	1.40486451487	48	2.06260643630	57	3.03353741915		

An employee aged 48 and 9 months has 21 years of service, and a current salary of \$144,000 (for the coming year). He has a defined benefit pension plan with $\alpha = 0.015$ and S_{Fin} is the average of his last 3 years' salary. The employee's mortality is given by $\mu_x = 0.0000019(1.119)^x$. The pension benefit is payable monthly in advance. The interest rate is $i = 0.07$. [This gives $\ddot{a}_{65}^{(12)} = 12.0863903952$.] Calculate the EPV of the accrued benefit under the assumption that the employee retires at age 65.

Standard Questions

4. An employee aged 58 has been working with a company for 29 years. The employee's salary last year was \$109,000. The salary scale is the same as for Question 3. The service table is given below:

t	${}_t p^{(00)}$	1	2	3
0	10000.00	42.04	0	10.26
1	9947.70	43.88	0	1.64
2 ⁻	9902.18		1327.14	
2	8575.04	42.41	206.70	0.36
3	8325.57	44.24	334.93	1.15
4	7945.25	48.30	592.74	1.85
5	7302.36	55.26	950.64	1.61
6	6294.85	64.11	1366.20	0.44
7 ⁻	4864.10		4864.10	

Mortality after exiting the plan follows a Gompertz model with $B = 0.0000114$ and $C = 1.095$. If the member withdraws, she receives a deferred monthly pension starting from age 65, with 4% COLA. The death benefit of the plan is three times the employee's final average salary if the employee is still working at the time of death. If the employee has withdrawn, the death benefit is three times final average salary with COLA of 4%. The accrual rate for the pension is 0.02. The interest rate is $i = 0.06$. Calculate the EPV of the accrued benefit. [You may assume that events happen in the middle of each year.]

You are given the following values:

x	$\ddot{a}_x^{(12)}$
60	14.09279
60.5	14.02004
61.5	13.87043
62.5	13.71527
63.5	13.55451
64.5	13.38806
65	13.3027

5. An individual aged 44 has 13 years of service, and last year's salary was \$56,000. The salary scale is $s_y = 1.06^y$. The accrual rate is 0.02. The interest rate is $i = 0.05$. There is no death benefit, and no exits other than death or retirement at age 65. The pension benefit is payable annually in advance. Mortality follows a Gompertz law with $B = 0.0000047$ and $C = 1.132$. You are given that $\ddot{a}_{65}^{(12)} = 10.1197028436$. Calculate this year's employer contribution to the plan using
- (a) The projected unit method.
 - (b) The traditional unit method.