

ACSC/STAT 4720, Life Contingencies II

FALL 2021

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Homework Sheet 7

Due: Tuesday 30th November: 14:30

Basic Questions

1. An insurance company sells a 5-year life insurance policy to a life aged 36, for whom the lifetable below is appropriate.

x	l_x	d_x
52	10000.00	72.24
53	9927.76	81.27
54	9846.50	91.37
55	9755.12	102.66
56	9652.47	115.23
57	9537.24	129.19
58	9408.05	144.64

The annual gross premium is \$4,728.60. Initial expenses are \$6,090 plus 20% of the first premium. The death benefits are \$300,000. Renewal costs are 4% of each subsequent premium. The interest rate is $i = 0.03$

(a) Calculate the expected net cash-flows associated with this policy (assuming no reserve). [This is the profit vector for the policy.]

(b) Which of the following is the internal rate of return of the policy:

- (i) $i = 0.0962$
- (ii) $i = 0.1201$
- (iii) $i = 0.1449$
- (iv) $i = 0.3066$

2. An insurance company sells a 5-year endowment insurance policy to a life aged 58, for whom the lifetable below is appropriate.

x	l_x	d_x
58	10000.00	177.08
59	9822.92	199.46
60	9623.46	224.19
61	9399.27	251.32
62	9147.96	280.84

The annual gross premium is \$79,452. Initial expenses are \$2,640 plus 5% of the first premium. The death benefits are \$400,000. Renewal costs are

2% of each subsequent premium. The interest rate is $i = 0.04$. Reserves are calculated on the basis $i = 0.03$, with mortality following the table.

- (a) Calculate the reserves.
 - (b) Calculate the profit signature.
 - (c) Calculate the profit margin at a risk discount rate of $i = 0.04$.
3. For the policy in Question 2:
- (a) Calculate the reserves and profit signature for a general premium. [You may assume that P is such that the reserves are zero in Year 1]
 - (b) Calculate the premium that gives an internal rate of return of $i = 0.12$.
4. For a 5-year term insurance policy with death benefit \$650,000 sold to a life aged 52, with the following lifetable:

x	l_x	d_x
61	10000.00	129.66
62	9870.34	148.82
63	9721.53	170.44
64	9551.08	194.73
65	9356.36	221.83

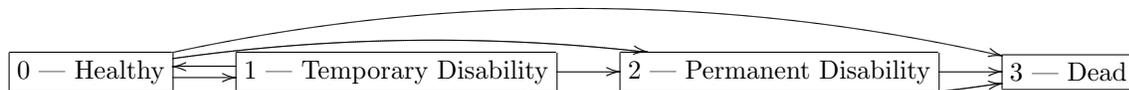
an actuary performs the following profit test without reserves:

Year	Premium	Expenses	Interest	Expected Death Benefits	Pr_t
0		200			-200
1	11300	0	452	8427.9	3324.1
2	11300	226	442.96	9800.3716184	1716.5883816
3	11300	226	442.96	11395.9428197	121.0171803
4	11300	226	442.96	13252.3756476	-1735.4156476
5	11300	226	442.96	15410.8542211	-3893.8942211

Calculate the reserves needed to ensure that all cash flows are non-negative.

Standard Questions

5. An insurer sells a 5-year disability income protection policy for a life aged 55. The policy has the following state diagram:



The transition probabilities are given in the following table:

x	p_x^{01}	p_x^{02}	p_x^{03}	p_x^{10}	p_x^{12}	p_x^{13}	p_x^{23}
55	0.05678216	0.01488069	0.009911805	0.01639633	0.08036063	0.04682560	0.1822381
56	0.09057920	0.02020286	0.011906520	0.01675757	0.12722420	0.06748682	0.2543372
57	0.13774081	0.02382313	0.016797271	0.01624059	0.17690254	0.12421022	0.4026589
58	0.18931439	0.03918915	0.023217273	0.01803712	0.28036575	0.14488319	0.6746820
59	0.22145329	0.04847626	0.028821625	0.01593516	0.35974325	0.19971552	0.8580170

The probability of being in each state at the end of each year is

t	${}_t p_{55}^{00}$	${}_t p_{55}^{01}$	${}_t p_{55}^{02}$	${}_t p_{55}^{03}$
1	0.91842535	0.05678216	0.01488069	0.009911805
2	0.80669658	0.1279647	0.03538679	0.02995188
3	0.66489142	0.1984697	0.05945463	0.0771842
4	0.50110424	0.2363644	0.08915138	0.1733800
5	0.35516526	0.2113329	0.07740444	0.3560974

The policy pays a benefit of \$79,000 at the end of any year if the life is disabled at that time (State 1 or State 2), and pays a death benefit of \$734,000 at the end of the year when the life dies (enters State 3). The interest rate is $i = 0.06$. Initial expenses are \$800 plus 25% of the first premium. Renewal expenses are 3% of each subsequent premium. The premium is \$126,060 at the start of each year. Use a profit test to calculate the reserves for each year in each state using a reserve rate of $i = 0.04$ and calculate the profit margin at a risk discount rate of $i = 0.12$.